

Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.2-c-x^m-a-x^j+b-xⁿ^p

Nasser M. Abbasi

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3.117	$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x+ax}}} dx$	621
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3.127	$\int \frac{1}{\sqrt{x}(b\sqrt{x+ax})^{3/2}} dx$	665
3.128	$\int \frac{1}{x^{3/2}(b\sqrt{x+ax})^{3/2}} dx$	669
3.129	$\int \frac{1}{x^{5/2}(b\sqrt{x+ax})^{3/2}} dx$	673
3.130	$\int \frac{1}{x^{7/2}(b\sqrt{x+ax})^{3/2}} dx$	678
3.131	$\int x^3 \sqrt{b\sqrt[3]{x+ax}} dx$	684
3.132	$\int x^2 \sqrt{b\sqrt[3]{x+ax}} dx$	689
3.133	$\int x \sqrt{b\sqrt[3]{x+ax}} dx$	695

3.134	$\int \sqrt{b\sqrt[3]{x} + ax} dx$	700
3.135	$\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x} dx$	705
3.136	$\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^2} dx$	710
3.137	$\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^3} dx$	715
3.138	$\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^4} dx$	720
3.139	$\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^5} dx$	726
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3.148	$\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^6} dx$	775
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3.150	$\int \frac{x^3}{\sqrt{b\sqrt[3]{x} + ax}} dx$	785
3.151	$\int \frac{x^2}{\sqrt{b\sqrt[3]{x} + ax}} dx$	791
3.152	$\int \frac{x}{\sqrt{b\sqrt[3]{x} + ax}} dx$	796
3.153	$\int \frac{1}{\sqrt{b\sqrt[3]{x} + ax}} dx$	801
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3.155	$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x} + ax}} dx$	811
3.156	$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x} + ax}} dx$	816
3.157	$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x} + ax}} dx$	822

3.158	$\int \frac{x^4}{(b\sqrt[3]{x+ax})^{3/2}} dx$	827
3.159	$\int \frac{x^3}{(b\sqrt[3]{x+ax})^{3/2}} dx$	834
3.160	$\int \frac{x^2}{(b\sqrt[3]{x+ax})^{3/2}} dx$	839
3.161	$\int \frac{x}{(b\sqrt[3]{x+ax})^{3/2}} dx$	845
3.162	$\int \frac{1}{(b\sqrt[3]{x+ax})^{3/2}} dx$	850
3.163	$\int \frac{1}{x(b\sqrt[3]{x+ax})^{3/2}} dx$	855
3.164	$\int \frac{1}{x^2(b\sqrt[3]{x+ax})^{3/2}} dx$	860
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3.166	$\int \frac{1}{x^4(b\sqrt[3]{x+ax})^{3/2}} dx$	871
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3.168	$\int x^2 \sqrt{bx^{2/3} + ax} dx$	883
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3.170	$\int \sqrt{bx^{2/3} + ax} dx$	892
3.171	$\int \frac{\sqrt{bx^{2/3} + ax}}{x} dx$	896
3.172	$\int \frac{\sqrt{bx^{2/3} + ax}}{x^2} dx$	899
3.173	$\int \frac{\sqrt{bx^{2/3} + ax}}{x^3} dx$	903
3.174	$\int \frac{\sqrt{bx^{2/3} + ax}}{x^4} dx$	907
3.175	$\int \frac{\sqrt{bx^{2/3} + ax}}{x^5} dx$	912
3.176	$\int x^2 (bx^{2/3} + ax)^{3/2} dx$	917
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3.178	$\int (bx^{2/3} + ax)^{3/2} dx$	927
3.179	$\int \frac{(bx^{2/3} + ax)^{3/2}}{x} dx$	931
3.180	$\int \frac{(bx^{2/3} + ax)^{3/2}}{x^2} dx$	935
3.181	$\int \frac{(bx^{2/3} + ax)^{3/2}}{x^3} dx$	939
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3.184	$\int \frac{(bx^{2/3} + ax)^{3/2}}{x^6} dx$	952

3.185	$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}} dx$	958
3.186	$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}} dx$	963
3.187	$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}} dx$	968
3.188	$\int \frac{x}{\sqrt{bx^{2/3}+ax}} dx$	972
3.189	$\int \frac{1}{\sqrt{bx^{2/3}+ax}} dx$	976
3.190	$\int \frac{1}{x\sqrt{bx^{2/3}+ax}} dx$	979
3.191	$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}} dx$	983
3.192	$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}} dx$	987
3.193	$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}} dx$	992
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3.196	$\int \frac{x^2}{(bx^{2/3}+ax)^{3/2}} dx$	1007
3.197	$\int \frac{x}{(bx^{2/3}+ax)^{3/2}} dx$	1011
3.198	$\int \frac{1}{(bx^{2/3}+ax)^{3/2}} dx$	1015
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3.201	$\int \frac{1}{x^3(bx^{2/3}+ax)^{3/2}} dx$	1028
3.202	$\int \frac{1}{x^4(bx^{2/3}+ax)^{3/2}} dx$	1033
3.203	$\int x^2(ax^2 + bx^3) dx$	1039
3.204	$\int x(ax^2 + bx^3) dx$	1042
3.205	$\int (ax^2 + bx^3) dx$	1045
3.206	$\int \frac{ax^2+bx^3}{x} dx$	1048
3.207	$\int \frac{ax^2+bx^3}{x^2} dx$	1051
3.208	$\int x^2(ax^2 + bx^3)^2 dx$	1054
3.209	$\int x(ax^2 + bx^3)^2 dx$	1057
3.210	$\int (ax^2 + bx^3)^2 dx$	1060
3.211	$\int \frac{(ax^2+bx^3)^2}{x} dx$	1063
3.212	$\int \frac{(ax^2+bx^3)^2}{x^2} dx$	1066

3.213	$\int \frac{x^6}{ax^2+bx^3} dx$1069
3.214	$\int \frac{x^5}{ax^2+bx^3} dx$1073
3.215	$\int \frac{x^4}{ax^2+bx^3} dx$1076
3.216	$\int \frac{x^3}{ax^2+bx^3} dx$1079
3.217	$\int \frac{x^2}{ax^2+bx^3} dx$1082
3.218	$\int \frac{x}{ax^2+bx^3} dx$1085
3.219	$\int \frac{1}{ax^2+bx^3} dx$1088
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3.221	$\int \frac{1}{x^2(ax^2+bx^3)} dx$1094
3.222	$\int \frac{x^8}{(ax^2+bx^3)^2} dx$1098
3.223	$\int \frac{x^7}{(ax^2+bx^3)^2} dx$1102
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3.228	$\int \frac{x^2}{(ax^2+bx^3)^2} dx$1118
3.229	$\int \frac{x}{(ax^2+bx^3)^2} dx$1121
3.230	$\int \frac{1}{(ax^2+bx^3)^2} dx$1125
3.231	$\int \frac{1}{x(ax^2+bx^3)^2} dx$1129
3.232	$\int x^2 \sqrt{ax^2 + bx^3} dx$1133
3.233	$\int x \sqrt{ax^2 + bx^3} dx$1137
3.234	$\int \sqrt{ax^2 + bx^3} dx$1141
3.235	$\int \frac{\sqrt{ax^2+bx^3}}{x} dx$1144
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3.241	$\int x(ax^2 + bx^3)^{3/2} dx$	1167
3.242	$\int (ax^2 + bx^3)^{3/2} dx$	1171
3.243	$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx$	1175
3.244	$\int \frac{(ax^2+bx^3)^{3/2}}{x^2} dx$	1179
3.245	$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx$	1183
3.246	$\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx$	1186
3.247	$\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx$	1190
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3.250	$\int \frac{(ax^2+bx^3)^{3/2}}{x^8} dx$	1202
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3.258	$\int \frac{1}{x^2\sqrt{ax^2+bx^3}} dx$	1231
3.259	$\int \frac{1}{x^3\sqrt{ax^2+bx^3}} dx$	1235
3.260	$\int \frac{x^6}{(ax^2+bx^3)^{3/2}} dx$	1239
3.261	$\int \frac{x^5}{(ax^2+bx^3)^{3/2}} dx$	1243
3.262	$\int \frac{x^4}{(ax^2+bx^3)^{3/2}} dx$	1247
3.263	$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx$	1250
3.264	$\int \frac{x^2}{(ax^2+bx^3)^{3/2}} dx$	1253
3.265	$\int \frac{x}{(ax^2+bx^3)^{3/2}} dx$	1257
3.266	$\int \frac{1}{(ax^2+bx^3)^{3/2}} dx$	1261

3.267	$\int \frac{1}{x(ax^2+bx^3)^{3/2}} dx$	1265
3.268	$\int \frac{1}{x^2(ax^2+bx^3)^{3/2}} dx$	1269
3.269	$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}} dx$	1273
3.270	$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} dx$	1277
3.271	$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}} dx$	1281
3.272	$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}} dx$	1285
3.273	$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx$	1289
3.274	$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}} dx$	1292
3.275	$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}} dx$	1295
3.276	$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}} dx$	1299
3.277	$\int x^{1-3n} (ax^2 + bx^3)^n dx$	1303
3.278	$\int x^{-3n} (ax^2 + bx^3)^n dx$	1307
3.279	$\int x^{-1-3n} (ax^2 + bx^3)^n dx$	1311
3.280	$\int x^{-2-3n} (ax^2 + bx^3)^n dx$	1315
3.281	$\int x^{-3-3n} (ax^2 + bx^3)^n dx$	1318
3.282	$\int x^{-4-3n} (ax^2 + bx^3)^n dx$	1321
3.283	$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx$	1325
3.284	$\int \frac{x^9}{\sqrt{ax^2+bx^5}} dx$	1328
3.285	$\int \frac{x^6}{\sqrt{ax^2+bx^5}} dx$	1332
3.286	$\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx$	1335
3.287	$\int \frac{1}{\sqrt{ax^2+bx^5}} dx$	1338
3.288	$\int \frac{1}{x^3\sqrt{ax^2+bx^5}} dx$	1342
3.289	$\int \frac{x^4}{\sqrt{ax^2+bx^5}} dx$	1346
3.290	$\int \frac{x}{\sqrt{ax^2+bx^5}} dx$	1350
3.291	$\int \frac{1}{x^2\sqrt{ax^2+bx^5}} dx$	1354
3.292	$\int \frac{x^5}{\sqrt{ax^2+bx^5}} dx$	1358
3.293	$\int \frac{x^2}{\sqrt{ax^2+bx^5}} dx$	1363
3.294	$\int \frac{1}{x\sqrt{ax^2+bx^5}} dx$	1368
3.295	$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}} dx$	1373

3.296	$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}} dx$	1379
3.297	$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} dx$	1385
3.298	$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}} dx$	1390
3.299	$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}} dx$	1396
3.300	$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}} dx$	1402
3.301	$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}} dx$	1406
3.302	$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}} dx$	1410
3.303	$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}} dx$	1417
3.304	$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}} dx$	1420
3.305	$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}} dx$	1426
3.306	$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}} dx$	1433
3.307	$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}} dx$	1436
3.308	$\int \frac{x}{ax^3+bx^4} dx$	1442
3.309	$\int \frac{1}{ax^3+bx^4} dx$	1445
3.310	$\int \frac{x^4}{\sqrt{ax^3+bx^4}} dx$	1448
3.311	$\int \frac{x^3}{\sqrt{ax^3+bx^4}} dx$	1452
3.312	$\int \frac{x^2}{\sqrt{ax^3+bx^4}} dx$	1456
3.313	$\int \frac{x}{\sqrt{ax^3+bx^4}} dx$	1460
3.314	$\int \frac{1}{\sqrt{ax^3+bx^4}} dx$	1464
3.315	$\int \frac{1}{x\sqrt{ax^3+bx^4}} dx$	1467
3.316	$\int \frac{1}{x^2\sqrt{ax^3+bx^4}} dx$	1470
3.317	$\int \frac{1}{x^3\sqrt{ax^3+bx^4}} dx$	1473
3.318	$\int \frac{1}{x^4\sqrt{ax^3+bx^4}} dx$	1477
3.319	$\int \frac{1}{x^3+bx^5} dx$	1481
3.320	$\int \frac{1}{-x^3+bx^5} dx$	1484
3.321	$\int \frac{1}{ax+bx} dx$	1487
3.322	$\int \frac{1}{(ax+bx)^2} dx$	1490
3.323	$\int \frac{1}{(ax+bx)^3} dx$	1493
3.324	$\int \frac{1}{ax^2+bx^2} dx$	1496
3.325	$\int \frac{1}{ax^n+bx^n} dx$	1499

3.326	$\int \frac{1}{(ax^n+bx^n)^2} dx$1502
3.327	$\int \frac{1}{(ax^n+bx^n)^3} dx$1506
3.328	$\int (ax + bx^{14})^{12} dx$1510
3.329	$\int x^{12} (ax + bx^{26})^{12} dx$1514
3.330	$\int x^{24} (ax + bx^{38})^{12} dx$1518
3.331	$\int x^{12(-1+m)} (ax + bx^{2+12m})^{12} dx$1522
3.332	$\int (ax + bx^{14})^{12} dx$1526
3.333	$\int (ax^2 + bx^{27})^{12} dx$1530
3.334	$\int (ax^3 + bx^{40})^{12} dx$1534
3.335	$\int (ax^m + bx^{1+13m})^{12} dx$1538
3.336	$\int (ax^m + bx^{1+6m})^5 dx$1542
3.337	$\int \frac{1}{(bx^{1-2m}+ax^m)^3} dx$1545
3.338	$\int \frac{1}{\frac{b}{x}+ax} dx$1548
3.339	$\int \frac{1}{\frac{b}{x^2}+ax} dx$1551
3.340	$\int \frac{1}{\frac{b}{x^3}+ax} dx$1554
3.341	$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3} dx$1557
3.342	$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx$1560
3.343	$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx$1563
3.344	$\int \left(\frac{a}{x} + bx\right)^2 dx$1566
3.345	$\int \left(\frac{a}{x} + bx\right)^3 dx$1569
3.346	$\int \left(\frac{a}{x} + bx\right)^4 dx$1572
3.347	$\int \frac{1}{\frac{1}{x^2}+x^3} dx$1575
3.348	$\int x^p (ax^n + bx^{1+13n+p})^{12} dx$1581
3.349	$\int x^{12} (a + bx^{13})^{12} dx$1585
3.350	$\int x^{12} (ax + bx^{26})^{12} dx$1588
3.351	$\int x^{12} (ax^2 + bx^{39})^{12} dx$1592
3.352	$\int x^{24} (a + bx^{25})^{12} dx$1596
3.353	$\int x^{24} (ax + bx^{38})^{12} dx$1599

3.354	$\int x^{36} (a + bx^{37})^{12} dx$	1603
3.355	$\int \frac{1}{ax+bx^n} dx$	1606
3.356	$\int \frac{1}{ax+bx^{1+n}} dx$	1609
3.357	$\int \frac{1}{ax+bx^{1-n}} dx$	1613
3.358	$\int \frac{1}{2x+3x^{1+n}} dx$	1616
3.359	$\int \frac{1}{2x+3x^{1-n}} dx$	1620
3.360	$\int \frac{1}{-\sqrt{x+x}} dx$	1623
3.361	$\int \frac{1}{-x^{3/5}+x} dx$	1626
3.362	$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} dx$	1629
3.363	$\int \frac{1}{x+x\sqrt{2}} dx$	1632
3.364	$\int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$	1636
3.365	$\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$	1640
3.366	$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}} dx$	1644
3.367	$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2} dx$	1648
3.368	$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}} dx$	1652
3.369	$\int \frac{\sqrt{a+bx^n}}{cx} dx$	1656
3.370	$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}} dx$	1660
3.371	$\int \sqrt{\frac{a}{x^2} + bx^n} dx$	1664
3.372	$\int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx$	1668
3.373	$\int (cx)^{-1-\frac{3j}{2}} (ax^j + bx^n)^{3/2} dx$	1672
3.374	$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx$	1676
3.375	$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4} dx$	1680
3.376	$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} dx$	1684
3.377	$\int \frac{(a+bx^n)^{3/2}}{cx} dx$	1688
3.378	$\int \sqrt{cx} \left(\frac{a}{x} + bx^n\right)^{3/2} dx$	1692
3.379	$\int c^2x^2 \left(\frac{a}{x^2} + bx^n\right)^{3/2} dx$	1696
3.380	$\int (cx)^{7/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2} dx$	1700
3.381	$\int c^5x^5 \left(\frac{a}{x^4} + bx^n\right)^{3/2} dx$	1704
3.382	$\int \sqrt{\frac{a+bx}{x^2}} dx$	1708

3.383	$\int \sqrt{\frac{a+bx^2}{x^2}} dx$.1712
3.384	$\int \sqrt{\frac{a+bx^3}{x^2}} dx$.1716
3.385	$\int \sqrt{\frac{a+bx^n}{x^2}} dx$.1720
3.386	$\int \sqrt{\frac{-a+bx}{x^2}} dx$.1724
3.387	$\int \sqrt{\frac{-a+bx^2}{x^2}} dx$.1728
3.388	$\int \sqrt{\frac{-a+bx^3}{x^2}} dx$.1732
3.389	$\int \sqrt{\frac{-a+bx^n}{x^2}} dx$.1736
3.390	$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}} dx$.1740
3.391	$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}} dx$.1744
3.392	$\int \frac{1}{\sqrt{ax^2+bx^n}} dx$.1748
3.393	$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx$.1751
3.394	$\int \frac{1}{cx\sqrt{a+bx^n}} dx$.1755
3.395	$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}} dx$.1759
3.396	$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}} dx$.1763
3.397	$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}} dx$.1767
3.398	$\int \frac{(cx)^{-1+\frac{j}{2}}}{(ax^j+bx^n)^{3/2}} dx$.1771
3.399	$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}} dx$.1775
3.400	$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}} dx$.1779
3.401	$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}} dx$.1783
3.402	$\int \frac{1}{cx(ax+bx^n)^{3/2}} dx$.1787
3.403	$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}} dx$.1791
3.404	$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}} dx$.1795
3.405	$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}} dx$.1799
3.406	$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}} dx$.1803

3.407	$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}} dx$	1807
3.408	$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx$	1811
3.409	$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} dx$	1815
3.410	$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} dx$	1819
3.411	$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx$	1823
3.412	$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} dx$	1827
3.413	$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx$	1831
3.414	$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}} dx$	1835
3.415	$\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}} dx$	1839
3.416	$\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}} dx$	1843
3.417	$\int \frac{1}{\sqrt{x(bx+ax^{-1+n})}} dx$	1847
3.418	$\int \frac{1}{\sqrt{x^n(a-bx^{2-n})}} dx$	1851
3.419	$\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}} dx$	1855
3.420	$\int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}} dx$	1859
3.421	$\int (cx)^m (ax^j + bx^n)^{3/2} dx$	1863
3.422	$\int (cx)^m \sqrt{ax^j + bx^n} dx$	1867
3.423	$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} dx$	1871
3.424	$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}} dx$	1875
3.425	$\int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}} dx$	1879
3.426	$\int (ax^j + bx^n)^{3/2} dx$	1883
3.427	$\int \sqrt{ax^j + bx^n} dx$	1887
3.428	$\int \frac{1}{\sqrt{ax^j + bx^n}} dx$	1891
3.429	$\int \frac{1}{(ax^j + bx^n)^{3/2}} dx$	1895
3.430	$\int \frac{1}{(ax^j + bx^n)^{5/2}} dx$	1899

3.431	$\int \sqrt{\frac{1+x}{x^5}} dx$.1903
3.432	$\int \sqrt{x+x^{5/2}} dx$.1906
3.433	$\int \frac{1}{\sqrt{x+x^{3/2}}} dx$.1909
3.434	$\int x\sqrt{x^2(a+bx^3)} dx$.1912
3.435	$\int x\sqrt{ax^2+bx^5} dx$.1915
3.436	$\int \sqrt{x^4(a+bx^3)} dx$.1918
3.437	$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x+bx^{2/3}}}} dx$.1921
3.438	$\int \frac{1}{(a\sqrt[3]{x+bx^{2/3}})^{2/3}} dx$.1928
3.439	$\int x^m(ax^j+bx^n)^p dx$.1934
3.440	$\int x^{-1-pq}(bx^n+ax^q)^p dx$.1938
3.441	$\int x^{-1-np}(bx^n+ax^q)^p dx$.1942
3.442	$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p dx$.1946
3.443	$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p dx$.1950
3.444	$\int (ax^m+bx^{1+m+mp})^p dx$.1954
3.445	$\int (x^m(a+bx^{1+mp}))^p dx$.1957
3.446	$\int x^n(x^m(a+bx^{1+n+mp}))^p dx$.1960
3.447	$\int x^n(ax^m+bx^{1+m+n+mp})^p dx$.1963
3.448	$\int \sqrt{x^{2(-1+n)}(a+bx^n)} dx$.1966
3.449	$\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)} dx$.1969
3.450	$\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)} dx$.1972
3.451	$\int (x^{(-1+n)p}(a+bx^n))^{\frac{1}{p}} dx$.1975
3.452	$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx$.1978
3.453	$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q dx$.1981
3.454	$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q dx$.1984

4 Listing of Grading functions

1987

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [454]. This is test number [30].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (454)	% 0. (0)
Mathematica	% 100. (454)	% 0. (0)
Maple	% 84.8 (385)	% 15.2 (69)
Maxima	% 29.96 (136)	% 70.04 (318)
Fricas	% 56.39 (256)	% 43.61 (198)
Sympy	% 24.89 (113)	% 75.11 (341)
Giac	% 51.54 (234)	% 48.46 (220)

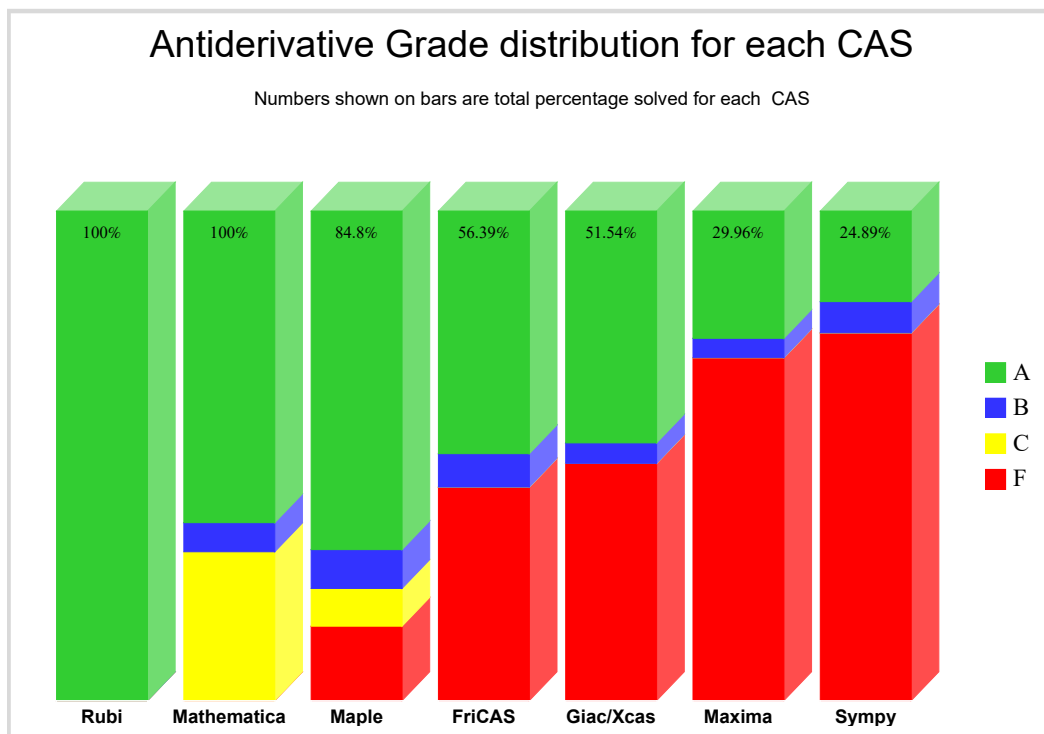
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

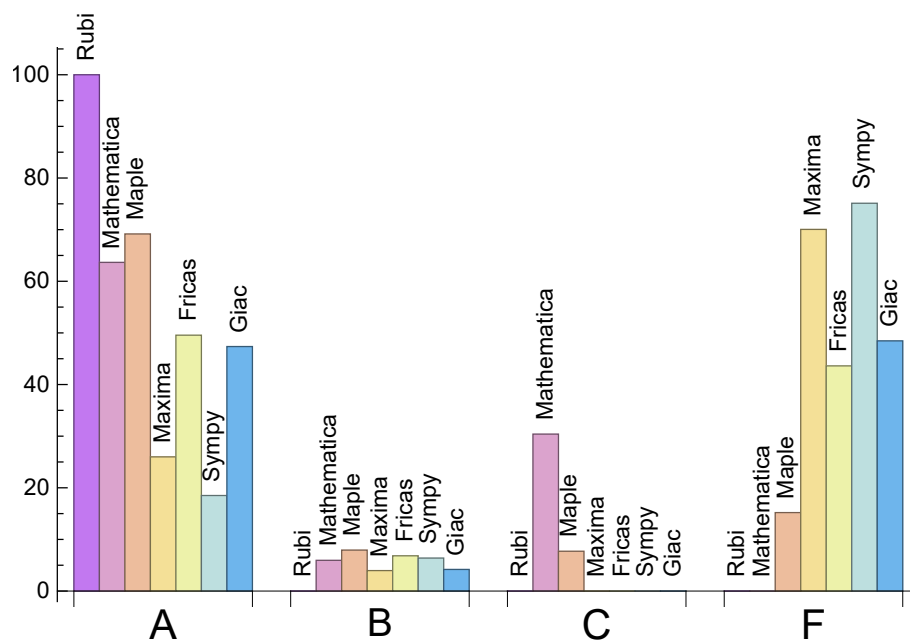
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	63.66	5.95	30.4	0.
Maple	69.16	7.93	7.71	15.2
Maxima	25.99	3.96	0.	70.04
Fricas	49.56	6.83	0.	43.61
Sympy	18.5	6.39	0.	75.11
Giac	47.36	4.19	0.	48.46

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	119.28	1.	74.	1.
Mathematica	0.04	62.66	1.08	55.	0.89
Maple	0.02	192.7	1.96	79.	0.94
Maxima	1.07	48.33	2.02	32.	1.2
Fricas	1.09	184.34	4.45	128.	2.8
Sympy	0.59	46.24	2.15	26.	0.88
Giac	1.24	91.34	1.8	61.5	1.2

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {347}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

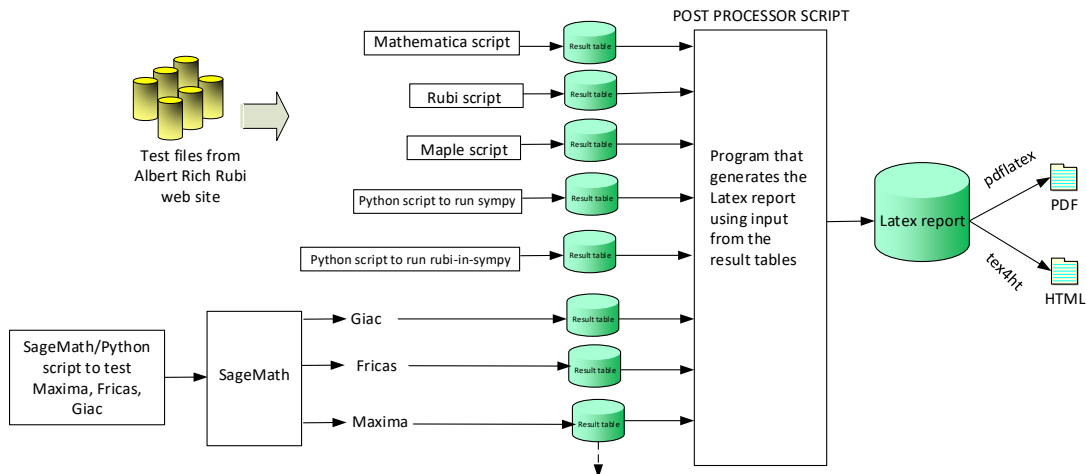
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 89, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 194, 195, 196, 197, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 252, 253, 254, 255, 256, 257, 260, 261, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 407, 408, 409, 411, 412, 413, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 26, 28, 30, 32, 34, 328, 329, 330, 332, 333, 334, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 86, 88, 90, 97, 98, 99, 100, 101, 102, 110, 111, 112, 124, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 191, 192, 193, 198, 199, 200, 201, 202, 238, 239, 247, 249, 250, 251, 258, 259, 264, 265, 266, 267, 268, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 402, 403, 404, 405, 406, 437, 438 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 103, 105, 117, 118, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246,

247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 303, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 358, 360, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 32, 86, 104, 110, 111, 112, 119, 120, 124, 125, 126, 127, 272, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 357, 359, 361, 387, 412 }

C grade: { 92, 93, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 407, 411 }

F grade: { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 30, 32, 283, 327, 328, 329, 330, 332, 333, 334, 337, 341, 342, 343, 350, 351, 353 }

C grade: { }

F grade: { 13, 15, 17, 19, 21, 23, 25, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 336, 348, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 28, 30, 32, 34, 79, 84, 127, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 277, 278, 279, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 325, 326, 327, 338, 339, 340, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 362, 369, 377, 394, 433 }

B grade: { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 328, 329, 330, 332, 333, 334, 341, 342, 349, 350, 351, 352, 353, 354, 361, 363, 402 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154,

155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 336, 337, 343, 348, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 113, 117, 118, 119, 120, 121, 122, 123, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 255, 256, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 283, 287, 288, 297, 300, 303, 306, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 383, 384, 386, 387, 388, 432, 433, 434, 435, 436 }

B grade: { 26, 28, 30, 32, 245, 328, 329, 330, 332, 333, 334, 335, 336, 348, 350, 351, 353, 362, 431 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 110, 111, 112, 114, 115, 116, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 252, 253, 254, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 277, 278, 279, 280, 281, 282, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 310, 311, 325, 326, 327, 331, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.	1.06	1.184	0.053	1.125

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.004	0.001	0.001	1.079	1.22	0.053	1.181

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.003	0.	0.002	1.063	1.271	0.052	1.177

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.003	0.	0.	1.041	1.345	0.052	1.239

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	30	10	19
normalized size	1	1.	1.	0.92	1.15	2.31	0.77	1.46
time (sec)	N/A	0.005	0.001	0.002	1.036	1.334	0.077	1.184

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.015	0.001	0.001	1.059	1.204	0.061	1.131

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.02	0.001	0.001	1.029	1.177	0.059	1.166

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.011	0.002	0.001	1.089	1.201	0.06	1.209

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	32	55	24	32
normalized size	1	1.	1.	1.56	2.	3.44	1.5	2.
time (sec)	N/A	0.007	0.002	0.002	1.103	1.342	0.068	1.17

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.012	0.001	0.002	0.996	1.319	0.06	1.22

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	49	130	42	49
normalized size	1	1.	1.	0.8	1.07	2.83	0.91	1.07
time (sec)	N/A	0.016	0.002	0.002	1.042	1.208	0.062	1.253

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	49	20	32
normalized size	1	1.	1.	0.89	1.15	1.81	0.74	1.19
time (sec)	N/A	0.023	0.004	0.001	1.064	1.385	0.29	1.201

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	165	56	35
normalized size	1	1.	1.	0.87	0.	5.32	1.81	1.13
time (sec)	N/A	0.016	0.009	0.004	0.	1.412	0.288	1.247

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.007	0.002	0.002	1.052	1.367	0.101	1.271

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.009	0.004	0.003	0.	1.41	0.123	1.162

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	49	15	32
normalized size	1	1.	1.	0.95	1.23	2.23	0.68	1.45
time (sec)	N/A	0.012	0.004	0.005	1.061	1.409	0.178	1.182

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	173	65	39
normalized size	1	1.	1.	0.88	0.	5.09	1.91	1.15
time (sec)	N/A	0.017	0.012	0.005	0.	1.398	0.312	1.271

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	80	31	58
normalized size	1	1.	1.	0.91	1.2	2.29	0.89	1.66
time (sec)	N/A	0.026	0.006	0.004	1.048	1.423	0.4	1.305

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	0	234	87	54
normalized size	1	1.	1.	0.91	0.	5.44	2.02	1.26
time (sec)	N/A	0.026	0.019	0.005	0.	1.398	0.369	1.256

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	59	108	42	77
normalized size	1	1.	1.	0.9	1.2	2.2	0.86	1.57
time (sec)	N/A	0.033	0.007	0.006	1.07	1.438	0.453	1.215

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	261	78	47
normalized size	1	1.	1.	0.8	0.	5.8	1.73	1.04
time (sec)	N/A	0.015	0.024	0.004	0.	1.425	0.345	1.263

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	46	108	34	63
normalized size	1	1.	0.87	0.92	1.21	2.84	0.89	1.66
time (sec)	N/A	0.028	0.013	0.009	1.008	1.408	0.415	1.202

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	288	90	63
normalized size	1	1.	0.95	0.81	0.	5.05	1.58	1.11
time (sec)	N/A	0.018	0.034	0.008	0.	1.46	0.436	1.183

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	68	157	49	69
normalized size	1	1.	0.84	0.94	1.39	3.2	1.	1.41
time (sec)	N/A	0.039	0.035	0.012	1.11	1.422	0.515	1.195

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	359	114	80
normalized size	1	1.	0.99	0.87	0.	5.28	1.68	1.18
time (sec)	N/A	0.029	0.037	0.01	0.	1.466	0.516	1.24

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	29	22	28	65	19	31
normalized size	1	1.	2.23	1.69	2.15	5.	1.46	2.38
time (sec)	N/A	0.011	0.004	0.003	1.067	1.505	0.094	1.197

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	24	39	14	20
normalized size	1	1.	0.9	0.95	1.2	1.95	0.7	1.
time (sec)	N/A	0.015	0.003	0.002	1.036	1.561	0.075	1.324

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	22	17	22	51	14	24
normalized size	1	1.	3.67	2.83	3.67	8.5	2.33	4.
time (sec)	N/A	0.008	0.003	0.001	1.042	1.686	0.083	1.231

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	18	26	8	20
normalized size	1	1.	1.	1.17	1.5	2.17	0.67	1.67
time (sec)	N/A	0.006	0.002	0.003	1.023	1.61	0.074	1.271

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	18	45	12	20
normalized size	1	1.	9.5	1.5	9.	22.5	6.	10.
time (sec)	N/A	0.004	0.002	0.001	1.074	1.608	0.083	1.236

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	38	10	22
normalized size	1	1.	1.	1.07	1.33	2.53	0.67	1.47
time (sec)	N/A	0.008	0.002	0.006	1.028	1.445	0.088	1.221

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	24	19	24	55	15	27
normalized size	1	1.	3.	2.38	3.	6.88	1.88	3.38
time (sec)	N/A	0.008	0.003	0.005	1.1	1.434	0.093	1.193

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	65	17	35
normalized size	1	1.	1.	0.95	1.23	2.95	0.77	1.59
time (sec)	N/A	0.015	0.003	0.007	1.047	1.385	0.097	1.27

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	24	34	80	24	36
normalized size	1	1.	2.07	1.6	2.27	5.33	1.6	2.4
time (sec)	N/A	0.01	0.004	0.005	1.097	1.406	0.108	1.231

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	36	78	22	45
normalized size	1	1.	1.	0.9	1.24	2.69	0.76	1.55
time (sec)	N/A	0.016	0.003	0.007	1.141	1.405	0.112	1.236

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	41	12	24
normalized size	1	1.	1.	0.93	1.2	2.73	0.8	1.6
time (sec)	N/A	0.009	0.003	0.005	1.035	1.385	0.123	1.215

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	20	39	12	24
normalized size	1	1.	1.	0.89	1.11	2.17	0.67	1.33
time (sec)	N/A	0.01	0.003	0.003	1.051	1.408	0.122	1.235

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	168	0	0	0	0
normalized size	1	1.	0.58	1.03	0.	0.	0.	0.
time (sec)	N/A	0.188	0.05	0.018	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	80	197	0	0	0	0
normalized size	1	1.	0.28	0.7	0.	0.	0.	0.
time (sec)	N/A	0.264	0.032	0.011	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	79	146	0	0	0	0
normalized size	1	1.	0.58	1.07	0.	0.	0.	0.
time (sec)	N/A	0.122	0.027	0.012	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	51	175	0	0	0	0
normalized size	1	1.	0.2	0.69	0.	0.	0.	0.
time (sec)	N/A	0.173	0.011	0.011	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	0	0	0
normalized size	1	1.	0.42	1.1	0.	0.	0.	0.
time (sec)	N/A	0.086	0.011	0.012	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	51	177	0	0	0	0
normalized size	1	1.	0.21	0.71	0.	0.	0.	0.
time (sec)	N/A	0.201	0.012	0.013	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	0	0	0
normalized size	1	1.	0.46	1.06	0.	0.	0.	0.
time (sec)	N/A	0.088	0.014	0.005	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	53	201	0	0	0	0
normalized size	1	1.	0.19	0.71	0.	0.	0.	0.
time (sec)	N/A	0.251	0.013	0.006	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	94	188	0	0	0	0
normalized size	1	1.	0.51	1.01	0.	0.	0.	0.
time (sec)	N/A	0.23	0.055	0.014	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	84	217	0	0	0	0
normalized size	1	1.	0.28	0.71	0.	0.	0.	0.
time (sec)	N/A	0.3	0.042	0.012	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	83	166	0	0	0	0
normalized size	1	1.	0.53	1.05	0.	0.	0.	0.
time (sec)	N/A	0.134	0.035	0.011	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	52	195	0	0	0	0
normalized size	1	1.	0.19	0.71	0.	0.	0.	0.
time (sec)	N/A	0.225	0.012	0.011	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	49	144	0	0	0	0
normalized size	1	1.	0.37	1.07	0.	0.	0.	0.
time (sec)	N/A	0.132	0.014	0.012	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	52	194	0	0	0	0
normalized size	1	1.	0.19	0.71	0.	0.	0.	0.
time (sec)	N/A	0.233	0.014	0.016	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	54	139	0	0	0	0
normalized size	1	1.	0.4	1.04	0.	0.	0.	0.
time (sec)	N/A	0.131	0.015	0.017	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	54	196	0	0	0	0
normalized size	1	1.	0.19	0.71	0.	0.	0.	0.
time (sec)	N/A	0.261	0.013	0.016	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	54	142	0	0	0	0
normalized size	1	1.	0.39	1.04	0.	0.	0.	0.
time (sec)	N/A	0.134	0.016	0.015	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	54	223	0	0	0	0
normalized size	1	1.	0.18	0.73	0.	0.	0.	0.
time (sec)	N/A	0.312	0.016	0.018	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	169	0	0	0	0
normalized size	1	1.	0.33	1.04	0.	0.	0.	0.
time (sec)	N/A	0.178	0.016	0.019	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	80	149	0	0	0	0
normalized size	1	1.	0.57	1.06	0.	0.	0.	0.
time (sec)	N/A	0.137	0.032	0.013	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	66	178	0	0	0	0
normalized size	1	1.	0.26	0.69	0.	0.	0.	0.
time (sec)	N/A	0.205	0.026	0.011	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	0	0	0
normalized size	1	1.	0.55	1.09	0.	0.	0.	0.
time (sec)	N/A	0.091	0.025	0.013	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	53	158	0	0	0	0
normalized size	1	1.	0.23	0.69	0.	0.	0.	0.
time (sec)	N/A	0.15	0.013	0.011	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	0	0	0
normalized size	1	1.	0.53	1.17	0.	0.	0.	0.
time (sec)	N/A	0.049	0.01	0.013	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	48	182	0	0	0	0
normalized size	1	1.	0.19	0.72	0.	0.	0.	0.
time (sec)	N/A	0.2	0.012	0.015	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	0	0	0
normalized size	1	1.	0.45	1.08	0.	0.	0.	0.
time (sec)	N/A	0.089	0.015	0.014	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	53	204	0	0	0	0
normalized size	1	1.	0.19	0.71	0.	0.	0.	0.
time (sec)	N/A	0.254	0.014	0.016	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	80	172	0	0	0	0
normalized size	1	1.	0.5	1.07	0.	0.	0.	0.
time (sec)	N/A	0.193	0.033	0.018	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	68	200	0	0	0	0
normalized size	1	1.	0.24	0.72	0.	0.	0.	0.
time (sec)	N/A	0.269	0.026	0.015	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	67	147	0	0	0	0
normalized size	1	1.	0.49	1.07	0.	0.	0.	0.
time (sec)	N/A	0.148	0.029	0.015	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	57	182	0	0	0	0
normalized size	1	1.	0.23	0.72	0.	0.	0.	0.
time (sec)	N/A	0.213	0.022	0.016	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	0	0	0
normalized size	1	1.	0.47	1.13	0.	0.	0.	0.
time (sec)	N/A	0.1	0.02	0.015	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	56	184	0	0	0	0
normalized size	1	1.	0.22	0.72	0.	0.	0.	0.
time (sec)	N/A	0.209	0.015	0.015	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	0	0	0
normalized size	1	1.	0.47	1.16	0.	0.	0.	0.
time (sec)	N/A	0.084	0.019	0.016	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	51	206	0	0	0	0
normalized size	1	1.	0.19	0.75	0.	0.	0.	0.
time (sec)	N/A	0.224	0.013	0.018	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	56	150	0	0	0	0
normalized size	1	1.	0.4	1.08	0.	0.	0.	0.
time (sec)	N/A	0.142	0.017	0.018	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	56	228	0	0	0	0
normalized size	1	1.	0.18	0.75	0.	0.	0.	0.
time (sec)	N/A	0.323	0.016	0.02	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	130	212	0	845	0	123
normalized size	1	1.	0.82	1.33	0.	5.31	0.	0.77
time (sec)	N/A	0.248	0.181	0.04	0.	1.589	0.	1.326

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	77	70	0	234	0	108
normalized size	1	1.	0.61	0.56	0.	1.86	0.	0.86
time (sec)	N/A	0.2	0.037	0.007	0.	1.466	0.	1.281

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	120	198	0	783	0	105
normalized size	1	1.	0.92	1.52	0.	6.02	0.	0.81
time (sec)	N/A	0.206	0.253	0.022	0.	1.472	0.	1.408

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	59	0	207	0	86
normalized size	1	1.	0.65	0.58	0.	2.05	0.	0.85
time (sec)	N/A	0.161	0.03	0.005	0.	1.413	0.	1.271

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	130	0	23
normalized size	1	1.	1.	1.08	0.	5.2	0.	0.92
time (sec)	N/A	0.037	0.017	0.004	0.	1.429	0.	1.236

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	55	48	0	184	0	68
normalized size	1	1.	0.72	0.63	0.	2.42	0.	0.89
time (sec)	N/A	0.119	0.026	0.005	0.	1.426	0.	1.249

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	37	0	159	0	39
normalized size	1	1.	0.86	0.73	0.	3.12	0.	0.76
time (sec)	N/A	0.075	0.021	0.004	0.	1.433	0.	1.337

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	37	0	158	0	45
normalized size	1	1.	0.86	0.73	0.	3.1	0.	0.88
time (sec)	N/A	0.074	0.02	0.006	0.	1.592	0.	1.263

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	55	48	0	185	0	58
normalized size	1	1.	0.72	0.63	0.	2.43	0.	0.76
time (sec)	N/A	0.113	0.021	0.007	0.	1.66	0.	1.274

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	132	0	31
normalized size	1	1.	1.	1.08	0.	5.28	0.	1.24
time (sec)	N/A	0.037	0.014	0.003	0.	1.709	0.	1.274

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	59	0	203	0	74
normalized size	1	1.	0.65	0.58	0.	2.01	0.	0.73
time (sec)	N/A	0.157	0.023	0.005	0.	1.501	0.	1.311

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	43	217	0	803	0	154
normalized size	1	1.	0.33	1.67	0.	6.18	0.	1.18
time (sec)	N/A	0.203	0.012	0.012	0.	1.295	0.	1.288

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	77	70	0	239	0	96
normalized size	1	1.	0.61	0.56	0.	1.9	0.	0.76
time (sec)	N/A	0.192	0.03	0.004	0.	1.671	0.	1.441

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	44	234	0	884	0	142
normalized size	1	1.	0.28	1.47	0.	5.56	0.	0.89
time (sec)	N/A	0.241	0.02	0.013	0.	1.377	0.	1.329

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	88	81	0	265	0	116
normalized size	1	1.	0.58	0.53	0.	1.74	0.	0.76
time (sec)	N/A	0.233	0.025	0.006	0.	1.851	0.	1.38

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	46	247	0	963	0	165
normalized size	1	1.	0.24	1.31	0.	5.1	0.	0.87
time (sec)	N/A	0.293	0.019	0.013	0.	1.426	0.	1.359

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	99	92	0	298	0	136
normalized size	1	1.	0.55	0.51	0.	1.66	0.	0.76
time (sec)	N/A	0.285	0.031	0.007	0.	2.285	0.	1.402

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	81	997	0	315	0	61
normalized size	1	1.	1.47	18.13	0.	5.73	0.	1.11
time (sec)	N/A	0.07	0.033	0.032	0.	1.98	0.	1.323

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	61	979	0	225	0	31
normalized size	1	1.	1.91	30.59	0.	7.03	0.	0.97
time (sec)	N/A	0.032	0.013	0.015	0.	1.896	0.	1.309

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	35	43	0	19
normalized size	1	1.	1.	1.17	1.52	1.87	0.	0.83
time (sec)	N/A	0.034	0.011	0.003	1.067	1.268	0.	1.197

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	31	35	51	63	0	36
normalized size	1	1.	0.65	0.73	1.06	1.31	0.	0.75
time (sec)	N/A	0.066	0.014	0.004	1.014	1.199	0.	1.2

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	48	68	90	0	58
normalized size	1	1.	0.59	0.65	0.92	1.22	0.	0.78
time (sec)	N/A	0.102	0.015	0.005	1.081	1.186	0.	1.235

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0
normalized size	1	1.	0.29	3.07	0.	0.	0.	0.
time (sec)	N/A	0.213	0.023	0.014	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	0	0	0
normalized size	1	1.	0.25	3.41	0.	0.	0.	0.
time (sec)	N/A	0.13	0.01	0.016	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	0	0	0
normalized size	1	1.	0.24	3.09	0.	0.	0.	0.
time (sec)	N/A	0.187	0.013	0.018	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	66	1079	0	0	0	0
normalized size	1	1.	0.13	2.15	0.	0.	0.	0.
time (sec)	N/A	0.514	0.024	0.016	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	53	1054	0	0	0	0
normalized size	1	1.	0.11	2.22	0.	0.	0.	0.
time (sec)	N/A	0.39	0.012	0.015	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	48	1083	0	0	0	0
normalized size	1	1.	0.1	2.18	0.	0.	0.	0.
time (sec)	N/A	0.472	0.011	0.015	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	151	223	0	0	0	150
normalized size	1	1.	0.87	1.28	0.	0.	0.	0.86
time (sec)	N/A	0.151	0.17	0.06	0.	0.	0.	1.398

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	129	181	0	0	0	112
normalized size	1	1.	1.11	1.56	0.	0.	0.	0.97
time (sec)	N/A	0.087	0.106	0.007	0.	0.	0.	1.366

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	88	83	0	0	0	73
normalized size	1	1.	1.57	1.48	0.	0.	0.	1.3
time (sec)	N/A	0.06	0.048	0.003	0.	0.	0.	1.334

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	160	0	51	0	34
normalized size	1	1.	1.	6.4	0.	2.04	0.	1.36
time (sec)	N/A	0.038	0.008	0.011	0.	2.431	0.	1.222

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	48	218	0	103	0	113
normalized size	1	1.	0.57	2.6	0.	1.23	0.	1.35
time (sec)	N/A	0.117	0.048	0.01	0.	2.315	0.	1.252

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	72	262	0	155	0	197
normalized size	1	1.	0.51	1.85	0.	1.09	0.	1.39
time (sec)	N/A	0.203	0.053	0.013	0.	2.336	0.	1.169

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	96	306	0	212	0	281
normalized size	1	1.	0.48	1.53	0.	1.06	0.	1.4
time (sec)	N/A	0.297	0.059	0.013	0.	2.414	0.	1.278

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	64	549	0	0	0	0
normalized size	1	1.	0.32	2.79	0.	0.	0.	0.
time (sec)	N/A	0.175	0.06	0.012	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	64	503	0	0	0	0
normalized size	1	1.	0.46	3.62	0.	0.	0.	0.
time (sec)	N/A	0.127	0.043	0.01	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	64	237	0	0	0	0
normalized size	1	1.	0.83	3.08	0.	0.	0.	0.
time (sec)	N/A	0.074	0.038	0.006	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	404	0	77	0	46
normalized size	1	1.	1.	16.16	0.	3.08	0.	1.84
time (sec)	N/A	0.005	0.016	0.008	0.	2.282	0.	1.147

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	48	524	0	132	0	0
normalized size	1	1.	0.61	6.63	0.	1.67	0.	0.
time (sec)	N/A	0.122	0.052	0.011	0.	1.871	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	72	570	0	190	0	0
normalized size	1	1.	0.53	4.16	0.	1.39	0.	0.
time (sec)	N/A	0.202	0.059	0.01	0.	1.789	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	96	614	0	246	0	0
normalized size	1	1.	0.49	3.15	0.	1.26	0.	0.
time (sec)	N/A	0.3	0.063	0.012	0.	1.9	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	164	245	0	0	0	169
normalized size	1	1.	0.8	1.2	0.	0.	0.	0.83
time (sec)	N/A	0.172	0.179	0.01	0.	0.	0.	1.325

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	142	203	0	0	0	131
normalized size	1	1.	0.97	1.39	0.	0.	0.	0.9
time (sec)	N/A	0.12	0.137	0.006	0.	0.	0.	1.311

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	102	160	0	0	0	93
normalized size	1	1.	1.17	1.84	0.	0.	0.	1.07
time (sec)	N/A	0.079	0.08	0.007	0.	0.	0.	1.346

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	65	133	0	0	0	50
normalized size	1	1.	1.91	3.91	0.	0.	0.	1.47
time (sec)	N/A	0.049	0.042	0.009	0.	0.	0.	1.291

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	37	194	0	72	0	72
normalized size	1	1.	0.69	3.59	0.	1.33	0.	1.33
time (sec)	N/A	0.074	0.047	0.009	0.	2.305	0.	1.278

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	59	240	0	123	0	155
normalized size	1	1.	0.53	2.14	0.	1.1	0.	1.38
time (sec)	N/A	0.154	0.05	0.008	0.	2.368	0.	1.228

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	83	284	0	178	0	239
normalized size	1	1.	0.49	1.67	0.	1.05	0.	1.41
time (sec)	N/A	0.245	0.055	0.01	0.	2.135	0.	1.189

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	62	527	0	0	0	0
normalized size	1	1.	0.36	3.08	0.	0.	0.	0.
time (sec)	N/A	0.147	0.052	0.009	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	62	440	0	0	0	0
normalized size	1	1.	0.55	3.89	0.	0.	0.	0.
time (sec)	N/A	0.103	0.049	0.008	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	79	238	0	0	0	0
normalized size	1	1.	1.32	3.97	0.	0.	0.	0.
time (sec)	N/A	0.067	0.105	0.006	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	45	111	0	109	0	35
normalized size	1	1.	1.5	3.7	0.	3.63	0.	1.17
time (sec)	N/A	0.047	0.037	0.01	0.	2.292	0.	1.217

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	548	0	163	0	0
normalized size	1	1.	0.53	5.12	0.	1.52	0.	0.
time (sec)	N/A	0.156	0.051	0.011	0.	2.343	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	83	592	0	220	0	0
normalized size	1	1.	0.5	3.59	0.	1.33	0.	0.
time (sec)	N/A	0.257	0.052	0.01	0.	2.399	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	107	636	0	278	0	0
normalized size	1	1.	0.48	2.85	0.	1.25	0.	0.
time (sec)	N/A	0.353	0.063	0.011	0.	2.363	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	155	264	0	0	0	0
normalized size	1	1.	0.51	0.88	0.	0.	0.	0.
time (sec)	N/A	0.509	0.161	0.033	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	136	273	0	0	0	0
normalized size	1	1.	0.33	0.66	0.	0.	0.	0.
time (sec)	N/A	0.578	0.114	0.011	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	118	198	0	0	0	0
normalized size	1	1.	0.55	0.93	0.	0.	0.	0.
time (sec)	N/A	0.299	0.099	0.013	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	94	207	0	0	0	0
normalized size	1	1.	0.29	0.64	0.	0.	0.	0.
time (sec)	N/A	0.326	0.054	0.012	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	54	132	0	0	0	0
normalized size	1	1.	0.44	1.07	0.	0.	0.	0.
time (sec)	N/A	0.146	0.044	0.012	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	59	213	0	0	0	0
normalized size	1	1.	0.18	0.66	0.	0.	0.	0.
time (sec)	N/A	0.353	0.046	0.018	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	59	179	0	0	0	0
normalized size	1	1.	0.31	0.95	0.	0.	0.	0.
time (sec)	N/A	0.244	0.05	0.017	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	59	281	0	0	0	0
normalized size	1	1.	0.14	0.68	0.	0.	0.	0.
time (sec)	N/A	0.544	0.047	0.018	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	59	245	0	0	0	0
normalized size	1	1.	0.21	0.89	0.	0.	0.	0.
time (sec)	N/A	0.413	0.055	0.019	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	142	197	0	0	0	0
normalized size	1	1.	0.48	0.66	0.	0.	0.	0.
time (sec)	N/A	0.501	0.138	0.027	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	123	263	0	0	0	0
normalized size	1	1.	0.3	0.64	0.	0.	0.	0.
time (sec)	N/A	0.554	0.1	0.018	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	106	164	0	0	0	0
normalized size	1	1.	0.51	0.79	0.	0.	0.	0.
time (sec)	N/A	0.271	0.081	0.017	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	60	230	0	0	0	0
normalized size	1	1.	0.19	0.72	0.	0.	0.	0.
time (sec)	N/A	0.329	0.046	0.017	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	130	0	0	0	0
normalized size	1	1.	0.42	0.9	0.	0.	0.	0.
time (sec)	N/A	0.202	0.058	0.02	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	62	339	0	0	0	0
normalized size	1	1.	0.18	0.97	0.	0.	0.	0.
time (sec)	N/A	0.431	0.053	0.022	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	62	168	0	0	0	0
normalized size	1	1.	0.29	0.79	0.	0.	0.	0.
time (sec)	N/A	0.309	0.066	0.023	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	62	411	0	0	0	0
normalized size	1	1.	0.14	0.94	0.	0.	0.	0.
time (sec)	N/A	0.64	0.055	0.026	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	62	201	0	0	0	0
normalized size	1	1.	0.21	0.67	0.	0.	0.	0.
time (sec)	N/A	0.479	0.07	0.026	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	161	196	0	0	0	0
normalized size	1	1.	0.53	0.64	0.	0.	0.	0.
time (sec)	N/A	0.506	0.092	0.032	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	143	261	0	0	0	0
normalized size	1	1.	0.35	0.63	0.	0.	0.	0.
time (sec)	N/A	0.566	0.071	0.024	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	124	164	0	0	0	0
normalized size	1	1.	0.57	0.76	0.	0.	0.	0.
time (sec)	N/A	0.319	0.067	0.007	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	106	230	0	0	0	0
normalized size	1	1.	0.33	0.71	0.	0.	0.	0.
time (sec)	N/A	0.349	0.057	0.008	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	80	128	0	0	0	0
normalized size	1	1.	0.63	1.02	0.	0.	0.	0.
time (sec)	N/A	0.118	0.036	0.005	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	54	253	0	0	0	0
normalized size	1	1.	0.18	0.86	0.	0.	0.	0.
time (sec)	N/A	0.294	0.049	0.018	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	59	142	0	0	0	0
normalized size	1	1.	0.36	0.87	0.	0.	0.	0.
time (sec)	N/A	0.201	0.05	0.022	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	59	363	0	0	0	0
normalized size	1	1.	0.15	0.94	0.	0.	0.	0.
time (sec)	N/A	0.476	0.05	0.025	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	59	179	0	0	0	0
normalized size	1	1.	0.24	0.71	0.	0.	0.	0.
time (sec)	N/A	0.345	0.057	0.026	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	131	384	0	0	0	0
normalized size	1	1.	0.3	0.88	0.	0.	0.	0.
time (sec)	N/A	0.674	0.093	0.026	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	124	261	0	0	0	0
normalized size	1	1.	0.52	1.09	0.	0.	0.	0.
time (sec)	N/A	0.377	0.083	0.024	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	94	314	0	0	0	0
normalized size	1	1.	0.27	0.9	0.	0.	0.	0.
time (sec)	N/A	0.426	0.067	0.011	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	82	185	0	0	0	0
normalized size	1	1.	0.55	1.24	0.	0.	0.	0.
time (sec)	N/A	0.209	0.063	0.01	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	62	245	0	0	0	0
normalized size	1	1.	0.21	0.83	0.	0.	0.	0.
time (sec)	N/A	0.26	0.029	0.005	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	62	181	0	0	0	0
normalized size	1	1.	0.39	1.15	0.	0.	0.	0.
time (sec)	N/A	0.205	0.06	0.012	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	64	341	0	0	0	0
normalized size	1	1.	0.17	0.89	0.	0.	0.	0.
time (sec)	N/A	0.479	0.056	0.011	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	64	262	0	0	0	0
normalized size	1	1.	0.26	1.07	0.	0.	0.	0.
time (sec)	N/A	0.365	0.056	0.014	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	64	413	0	0	0	0
normalized size	1	1.	0.14	0.88	0.	0.	0.	0.
time (sec)	N/A	0.694	0.055	0.013	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	181	156	0	0	0	259
normalized size	1	1.	0.49	0.42	0.	0.	0.	0.7
time (sec)	N/A	0.627	0.135	0.01	0.	0.	0.	1.133

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	144	123	0	0	0	203
normalized size	1	1.	0.51	0.43	0.	0.	0.	0.72
time (sec)	N/A	0.441	0.095	0.004	0.	0.	0.	1.126

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	107	90	0	0	0	146
normalized size	1	1.	0.55	0.46	0.	0.	0.	0.75
time (sec)	N/A	0.273	0.069	0.002	0.	0.	0.	1.131

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	70	57	0	0	0	89
normalized size	1	1.	0.64	0.52	0.	0.	0.	0.82
time (sec)	N/A	0.137	0.04	0.003	0.	0.	0.	1.119

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	0	0	31
normalized size	1	1.	1.	1.17	0.	0.	0.	1.35
time (sec)	N/A	0.04	0.011	0.003	0.	0.	0.	1.126

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	57	80	0	0	0	97
normalized size	1	1.	0.63	0.89	0.	0.	0.	1.08
time (sec)	N/A	0.139	0.042	0.01	0.	0.	0.	1.217

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	57	125	0	0	0	170
normalized size	1	1.	0.32	0.7	0.	0.	0.	0.96
time (sec)	N/A	0.296	0.041	0.011	0.	0.	0.	1.205

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	57	167	0	0	0	239
normalized size	1	1.	0.21	0.63	0.	0.	0.	0.9
time (sec)	N/A	0.475	0.046	0.013	0.	0.	0.	1.269

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	57	209	0	0	0	308
normalized size	1	1.	0.16	0.59	0.	0.	0.	0.87
time (sec)	N/A	0.66	0.049	0.015	0.	0.	0.	1.399

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	172	145	0	0	0	508
normalized size	1	1.	0.5	0.42	0.	0.	0.	1.48
time (sec)	N/A	0.616	0.131	0.004	0.	0.	0.	1.183

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	135	112	0	0	0	394
normalized size	1	1.	0.53	0.44	0.	0.	0.	1.55
time (sec)	N/A	0.422	0.086	0.004	0.	0.	0.	1.169

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	98	79	0	0	0	281
normalized size	1	1.	0.58	0.47	0.	0.	0.	1.66
time (sec)	N/A	0.249	0.058	0.003	0.	0.	0.	1.228

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	63	48	0	0	0	155
normalized size	1	1.	0.75	0.57	0.	0.	0.	1.85
time (sec)	N/A	0.139	0.058	0.004	0.	0.	0.	1.177

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	88	69	0	0	0	112
normalized size	1	1.	1.13	0.88	0.	0.	0.	1.44
time (sec)	N/A	0.137	0.069	0.004	0.	0.	0.	1.215

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	61	93	0	0	0	124
normalized size	1	1.	0.54	0.82	0.	0.	0.	1.1
time (sec)	N/A	0.184	0.053	0.01	0.	0.	0.	1.245

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	61	139	0	0	0	193
normalized size	1	1.	0.3	0.68	0.	0.	0.	0.95
time (sec)	N/A	0.34	0.046	0.012	0.	0.	0.	1.251

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	61	181	0	0	0	262
normalized size	1	1.	0.21	0.62	0.	0.	0.	0.9
time (sec)	N/A	0.522	0.05	0.014	0.	0.	0.	1.372

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	61	223	0	0	0	331
normalized size	1	1.	0.16	0.59	0.	0.	0.	0.87
time (sec)	N/A	0.718	0.061	0.016	0.	0.	0.	1.447

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	185	167	0	0	0	278
normalized size	1	1.	0.46	0.42	0.	0.	0.	0.69
time (sec)	N/A	0.728	0.153	0.004	0.	0.	0.	1.144

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	148	134	0	0	0	221
normalized size	1	1.	0.47	0.43	0.	0.	0.	0.71
time (sec)	N/A	0.531	0.115	0.003	0.	0.	0.	1.157

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	111	101	0	0	0	165
normalized size	1	1.	0.49	0.45	0.	0.	0.	0.73
time (sec)	N/A	0.346	0.088	0.005	0.	0.	0.	1.166

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	74	68	0	0	0	108
normalized size	1	1.	0.54	0.5	0.	0.	0.	0.79
time (sec)	N/A	0.18	0.064	0.003	0.	0.	0.	1.133

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	0	0	49
normalized size	1	1.	0.77	0.77	0.	0.	0.	1.04
time (sec)	N/A	0.05	0.027	0.003	0.	0.	0.	1.142

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	90	61	0	0	0	69
normalized size	1	1.	1.48	1.	0.	0.	0.	1.13
time (sec)	N/A	0.093	0.087	0.007	0.	0.	0.	1.156

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	48	126	0	0	0	147
normalized size	1	1.	0.31	0.82	0.	0.	0.	0.96
time (sec)	N/A	0.239	0.055	0.007	0.	0.	0.	1.19

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	48	188	0	0	0	216
normalized size	1	1.	0.2	0.78	0.	0.	0.	0.9
time (sec)	N/A	0.408	0.056	0.006	0.	0.	0.	1.269

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	48	248	0	0	0	285
normalized size	1	1.	0.15	0.75	0.	0.	0.	0.87
time (sec)	N/A	0.577	0.055	0.007	0.	0.	0.	1.354

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	161	143	0	0	0	289
normalized size	1	1.	0.48	0.43	0.	0.	0.	0.86
time (sec)	N/A	0.599	0.133	0.005	0.	0.	0.	1.234

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	122	110	0	0	0	220
normalized size	1	1.	0.49	0.44	0.	0.	0.	0.89
time (sec)	N/A	0.414	0.089	0.006	0.	0.	0.	1.215

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	85	77	0	0	0	151
normalized size	1	1.	0.53	0.48	0.	0.	0.	0.94
time (sec)	N/A	0.242	0.071	0.006	0.	0.	0.	1.145

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	45	0	0	0	81
normalized size	1	1.	0.88	0.66	0.	0.	0.	1.19
time (sec)	N/A	0.084	0.05	0.005	0.	0.	0.	1.179

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	55	0	0	0	96
normalized size	1	1.	0.75	0.92	0.	0.	0.	1.6
time (sec)	N/A	0.056	0.026	0.006	0.	0.	0.	1.16

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	48	88	0	0	0	142
normalized size	1	1.	0.33	0.6	0.	0.	0.	0.97
time (sec)	N/A	0.241	0.057	0.013	0.	0.	0.	1.232

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	48	126	0	0	0	211
normalized size	1	1.	0.2	0.53	0.	0.	0.	0.89
time (sec)	N/A	0.41	0.058	0.017	0.	0.	0.	1.298

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	48	159	0	0	0	279
normalized size	1	1.	0.15	0.49	0.	0.	0.	0.86
time (sec)	N/A	0.597	0.057	0.018	0.	0.	0.	1.366

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	48	192	0	0	0	348
normalized size	1	1.	0.12	0.47	0.	0.	0.	0.84
time (sec)	N/A	0.84	0.059	0.025	0.	0.	0.	1.519

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.009	0.001	0.001	0.994	1.061	0.117	1.148

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.008	0.001	0.001	0.989	1.024	0.142	1.104

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.003	0.	0.	0.99	1.09	0.218	1.154

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.006	0.001	0.	0.985	0.958	0.081	1.198

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.004	0.	0.001	0.975	0.801	0.101	1.133

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.031	0.002	0.	0.986	0.575	0.123	1.143

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.017	0.002	0.	0.98	0.617	0.242	1.133

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.013	0.002	0.	0.993	0.627	0.122	1.154

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.017	0.002	0.002	1.012	0.705	0.24	1.147

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.016	0.002	0.	0.993	0.711	0.22	1.206

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	70	117	49	72
normalized size	1	1.	1.	0.91	1.23	2.05	0.86	1.26
time (sec)	N/A	0.036	0.005	0.003	0.961	0.735	0.722	1.146

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	57	92	37	58
normalized size	1	1.	1.	0.93	1.3	2.09	0.84	1.32
time (sec)	N/A	0.027	0.004	0.002	0.971	0.872	0.487	1.129

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	39	68	26	41
normalized size	1	1.	1.	0.97	1.26	2.19	0.84	1.32
time (sec)	N/A	0.021	0.003	0.001	0.977	0.69	0.625	1.157

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	14	26
normalized size	1	1.	1.	1.06	1.33	2.11	0.78	1.44
time (sec)	N/A	0.016	0.003	0.002	0.989	0.831	0.648	1.152

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.007	0.001	0.001	1.097	0.762	0.23	1.153

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	10	27
normalized size	1	1.	1.	1.06	1.33	2.11	0.56	1.5
time (sec)	N/A	0.007	0.004	0.005	0.979	0.781	0.543	1.16

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	61	19	41
normalized size	1	1.	1.	1.04	1.36	2.18	0.68	1.46
time (sec)	N/A	0.015	0.004	0.006	1.017	0.796	0.715	1.141

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	54	103	31	61
normalized size	1	1.	1.	0.98	1.29	2.45	0.74	1.45
time (sec)	N/A	0.021	0.004	0.007	1.041	0.781	1.032	1.154

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	69	126	44	76
normalized size	1	1.	1.	0.95	1.23	2.25	0.79	1.36
time (sec)	N/A	0.027	0.005	0.006	0.972	0.726	1.21	1.209

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	80	155	54	84
normalized size	1	1.	0.93	0.98	1.38	2.67	0.93	1.45
time (sec)	N/A	0.04	0.02	0.006	0.976	0.679	1.195	1.153

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	63	132	44	65
normalized size	1	1.	0.93	0.98	1.37	2.87	0.96	1.41
time (sec)	N/A	0.03	0.014	0.006	0.993	0.757	0.97	1.236

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	49	97	31	46
normalized size	1	1.	0.88	1.03	1.48	2.94	0.94	1.39
time (sec)	N/A	0.024	0.012	0.006	0.992	0.78	0.558	1.167

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	35	62	20	32
normalized size	1	1.	0.87	1.04	1.52	2.7	0.87	1.39
time (sec)	N/A	0.018	0.006	0.005	0.983	0.765	0.528	1.198

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	24	10	16
normalized size	1	1.	1.	1.08	1.5	2.	0.83	1.33
time (sec)	N/A	0.007	0.003	0.001	0.989	0.789	0.921	1.149

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	38	89	22	42
normalized size	1	1.	0.83	1.03	1.31	3.07	0.76	1.45
time (sec)	N/A	0.019	0.01	0.006	1.017	0.73	1.039	1.145

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	61	138	36	61
normalized size	1	1.	0.83	1.02	1.45	3.29	0.86	1.45
time (sec)	N/A	0.024	0.037	0.009	1.001	0.725	1.241	1.168

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	86	177	54	86
normalized size	1	1.	0.91	0.98	1.48	3.05	0.93	1.48
time (sec)	N/A	0.032	0.05	0.01	1.017	0.791	0.759	1.184

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	99	204	66	99
normalized size	1	1.	0.96	0.99	1.43	2.96	0.96	1.43
time (sec)	N/A	0.039	0.053	0.01	0.987	0.81	1.767	1.163

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	116	231	80	116
normalized size	1	1.	0.94	0.94	1.38	2.75	0.95	1.38
time (sec)	N/A	0.052	0.043	0.01	1.109	0.677	1.703	1.141

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	53	57	72	134	0	84
normalized size	1	1.	0.5	0.54	0.69	1.28	0.	0.8
time (sec)	N/A	0.12	0.028	0.004	1.004	0.82	0.	1.144

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	46	57	111	0	68
normalized size	1	1.	0.52	0.57	0.71	1.39	0.	0.85
time (sec)	N/A	0.074	0.02	0.003	1.017	0.813	0.	1.151

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	84	0	51
normalized size	1	1.	0.6	0.67	0.79	1.62	0.	0.98
time (sec)	N/A	0.043	0.014	0.003	0.976	0.873	0.	1.186

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	16	55	0	34
normalized size	1	1.	0.92	1.08	0.64	2.2	0.	1.36
time (sec)	N/A	0.036	0.009	0.001	1.008	0.824	0.	1.174

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	52	0	247	0	88
normalized size	1	1.	1.04	1.02	0.	4.84	0.	1.73
time (sec)	N/A	0.049	0.028	0.006	0.	0.85	0.	1.213

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	56	0	282	0	58
normalized size	1	1.	0.92	1.08	0.	5.42	0.	1.12
time (sec)	N/A	0.052	0.036	0.01	0.	0.87	0.	1.206

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	42	73	0	338	0	92
normalized size	1	1.	0.5	0.87	0.	4.02	0.	1.1
time (sec)	N/A	0.092	0.012	0.01	0.	0.866	0.	1.214

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	42	89	0	393	0	116
normalized size	1	1.	0.38	0.79	0.	3.51	0.	1.04
time (sec)	N/A	0.138	0.012	0.012	0.	0.933	0.	1.188

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	80	79	116	217	0	242
normalized size	1	1.	0.5	0.49	0.72	1.35	0.	1.5
time (sec)	N/A	0.231	0.041	0.005	1.009	0.868	0.	1.135

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	68	101	193	0	211
normalized size	1	1.	0.51	0.5	0.74	1.42	0.	1.55
time (sec)	N/A	0.172	0.033	0.004	1.057	0.808	0.	1.171

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	58	57	86	161	0	177
normalized size	1	1.	0.54	0.53	0.8	1.49	0.	1.64
time (sec)	N/A	0.141	0.027	0.003	0.988	0.74	0.	1.264

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	47	46	72	134	0	144
normalized size	1	1.	0.59	0.57	0.9	1.68	0.	1.8
time (sec)	N/A	0.133	0.022	0.004	1.008	0.829	0.	1.289

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	35	55	105	0	112
normalized size	1	1.	0.69	0.67	1.06	2.02	0.	2.15
time (sec)	N/A	0.083	0.017	0.003	1.027	0.818	0.	1.275

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	38	77	0	70
normalized size	1	1.	0.92	1.08	1.52	3.08	0.	2.8
time (sec)	N/A	0.041	0.013	0.002	0.989	0.811	0.	1.242

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	63	0	296	0	115
normalized size	1	1.	0.92	0.85	0.	4.	0.	1.55
time (sec)	N/A	0.096	0.043	0.007	0.	0.81	0.	1.192

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	40	72	0	304	0	84
normalized size	1	1.	0.55	0.99	0.	4.16	0.	1.15
time (sec)	N/A	0.093	0.013	0.011	0.	0.899	0.	1.456

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	74	0	342	0	95
normalized size	1	1.	0.89	0.91	0.	4.22	0.	1.17
time (sec)	N/A	0.092	0.046	0.01	0.	0.928	0.	1.271

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	42	87	0	397	0	124
normalized size	1	1.	0.39	0.8	0.	3.64	0.	1.14
time (sec)	N/A	0.134	0.016	0.012	0.	0.786	0.	1.215

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	42	101	0	443	0	147
normalized size	1	1.	0.31	0.74	0.	3.23	0.	1.07
time (sec)	N/A	0.184	0.015	0.01	0.	0.893	0.	1.237

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	42	113	0	504	0	170
normalized size	1	1.	0.25	0.68	0.	3.05	0.	1.03
time (sec)	N/A	0.236	0.015	0.013	0.	0.909	0.	1.326

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	53	55	72	109	0	0
normalized size	1	1.	0.51	0.53	0.7	1.06	0.	0.
time (sec)	N/A	0.148	0.032	0.004	1.06	0.805	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	42	44	57	86	0	0
normalized size	1	1.	0.56	0.59	0.76	1.15	0.	0.
time (sec)	N/A	0.1	0.02	0.004	1.151	0.793	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	33	41	61	0	0
normalized size	1	1.	0.61	0.67	0.84	1.24	0.	0.
time (sec)	N/A	0.055	0.015	0.003	1.034	0.858	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	16	39	0	35
normalized size	1	1.	0.91	1.09	0.7	1.7	0.	1.52
time (sec)	N/A	0.01	0.007	0.001	1.169	0.798	0.	1.19

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	39	0	171	0	61
normalized size	1	1.	1.53	1.3	0.	5.7	0.	2.03
time (sec)	N/A	0.011	0.009	0.004	0.	0.804	0.	1.168

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	66	55	0	289	0	0
normalized size	1	1.	1.22	1.02	0.	5.35	0.	0.
time (sec)	N/A	0.049	0.045	0.007	0.	0.829	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	40	77	0	347	0	0
normalized size	1	1.	0.46	0.89	0.	3.99	0.	0.
time (sec)	N/A	0.091	0.01	0.007	0.	0.901	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	40	95	0	402	0	0
normalized size	1	1.	0.35	0.83	0.	3.5	0.	0.
time (sec)	N/A	0.135	0.01	0.006	0.	0.809	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	50	56	55	122	0	0
normalized size	1	1.	0.51	0.57	0.56	1.24	0.	0.
time (sec)	N/A	0.151	0.024	0.005	1.159	0.906	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	39	46	41	99	0	0
normalized size	1	1.	0.54	0.64	0.57	1.38	0.	0.
time (sec)	N/A	0.105	0.017	0.006	1.145	0.829	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	26	74	0	38
normalized size	1	1.	0.55	0.72	0.55	1.57	0.	0.81
time (sec)	N/A	0.057	0.012	0.003	1.113	0.829	0.	1.134

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	16	57	0	50
normalized size	1	1.	0.9	1.29	0.76	2.71	0.	2.38
time (sec)	N/A	0.018	0.006	0.002	1.078	0.838	0.	1.182

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	53	0	333	0	0
normalized size	1	1.	0.67	1.02	0.	6.4	0.	0.
time (sec)	N/A	0.058	0.009	0.007	0.	0.801	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	36	62	0	402	0	0
normalized size	1	1.	0.48	0.83	0.	5.36	0.	0.
time (sec)	N/A	0.086	0.009	0.012	0.	0.804	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	38	76	0	466	0	0
normalized size	1	1.	0.35	0.69	0.	4.24	0.	0.
time (sec)	N/A	0.105	0.007	0.012	0.	0.902	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	38	86	0	524	0	0
normalized size	1	1.	0.28	0.62	0.	3.8	0.	0.
time (sec)	N/A	0.186	0.009	0.013	0.	0.817	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	38	100	0	575	0	0
normalized size	1	1.	0.23	0.6	0.	3.46	0.	0.
time (sec)	N/A	0.232	0.01	0.014	0.	0.822	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	104	103	0	428	0	86
normalized size	1	1.	0.83	0.82	0.	3.42	0.	0.69
time (sec)	N/A	0.169	0.109	0.006	0.	0.793	0.	1.308

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	90	92	0	375	0	70
normalized size	1	1.	0.95	0.97	0.	3.95	0.	0.74
time (sec)	N/A	0.125	0.052	0.005	0.	0.83	0.	1.293

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	73	79	0	315	0	51
normalized size	1	1.	1.22	1.32	0.	5.25	0.	0.85
time (sec)	N/A	0.083	0.039	0.005	0.	0.885	0.	1.355

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	55	58	0	189	0	31
normalized size	1	1.	1.62	1.71	0.	5.56	0.	0.91
time (sec)	N/A	0.042	0.016	0.005	0.	0.84	0.	1.397

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	0	49	0	41
normalized size	1	1.	0.92	1.08	0.	1.96	0.	1.64
time (sec)	N/A	0.038	0.008	0.003	0.	0.713	0.	1.268

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	31	33	0	69	0	74
normalized size	1	1.	0.55	0.59	0.	1.23	0.	1.32
time (sec)	N/A	0.077	0.015	0.003	0.	0.891	0.	1.208

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	44	46	0	96	0	104
normalized size	1	1.	0.51	0.53	0.	1.12	0.	1.21
time (sec)	N/A	0.118	0.015	0.004	0.	0.789	0.	1.16

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	55	57	0	117	0	139
normalized size	1	1.	0.47	0.49	0.	1.01	0.	1.2
time (sec)	N/A	0.163	0.018	0.004	0.	0.757	0.	1.269

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.018	0.296	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0
normalized size	1	1.27	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.012	0.252	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.008	0.241	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	77	0	0
normalized size	1	1.	0.94	1.12	0.	2.41	0.	0.
time (sec)	N/A	0.025	0.014	0.003	0.	0.946	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	136	0	0
normalized size	1	1.	0.63	0.71	0.	1.94	0.	0.
time (sec)	N/A	0.054	0.021	0.005	0.	0.812	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	72	84	0	217	0	0
normalized size	1	1.	0.62	0.72	0.	1.87	0.	0.
time (sec)	N/A	0.091	0.029	0.005	0.	0.915	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	73	36	30
normalized size	1	1.	1.26	1.63	2.58	3.84	1.89	1.58
time (sec)	N/A	0.01	0.008	0.007	1.172	0.719	1.509	1.164

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	48	62	89	0	0
normalized size	1	1.	0.57	0.6	0.78	1.11	0.	0.
time (sec)	N/A	0.115	0.025	0.006	1.035	0.651	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	37	46	63	0	0
normalized size	1	1.	0.65	0.71	0.88	1.21	0.	0.
time (sec)	N/A	0.065	0.017	0.005	1.159	0.81	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	19	42	0	0
normalized size	1	1.	1.	1.08	0.76	1.68	0.	0.
time (sec)	N/A	0.017	0.008	0.004	1.106	0.796	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	180	0	63
normalized size	1	1.	1.69	1.34	0.	5.62	0.	1.97
time (sec)	N/A	0.011	0.01	0.005	0.	0.875	0.	1.164

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	66	0	294	0	77
normalized size	1	1.	1.2	1.12	0.	4.98	0.	1.31
time (sec)	N/A	0.055	0.061	0.006	0.	0.81	0.	1.166

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	0	0	0
normalized size	1	1.	0.29	1.04	0.	0.	0.	0.
time (sec)	N/A	0.142	0.024	0.297	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	0	0	0
normalized size	1	1.	0.25	1.09	0.	0.	0.	0.
time (sec)	N/A	0.067	0.009	0.005	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	0	0	0
normalized size	1	1.	0.23	1.02	0.	0.	0.	0.
time (sec)	N/A	0.129	0.013	0.005	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	68	676	0	0	0	0
normalized size	1	1.	0.13	1.32	0.	0.	0.	0.
time (sec)	N/A	0.308	0.023	0.008	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	55	394	0	0	0	0
normalized size	1	1.	0.11	0.81	0.	0.	0.	0.
time (sec)	N/A	0.19	0.011	0.005	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	50	673	0	0	0	0
normalized size	1	1.	0.1	1.32	0.	0.	0.	0.
time (sec)	N/A	0.273	0.011	0.009	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	86	2017	0	0	0	0
normalized size	1	1.	0.32	7.61	0.	0.	0.	0.
time (sec)	N/A	0.282	0.032	0.533	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	70	2586	0	0	0	0
normalized size	1	1.	0.13	4.93	0.	0.	0.	0.
time (sec)	N/A	0.5	0.026	0.052	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	3347	0	358	0	59
normalized size	1	1.	1.25	51.49	0.	5.51	0.	0.91
time (sec)	N/A	0.09	0.037	0.051	0.	1.133	0.	1.398

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	70	1793	0	0	0	0
normalized size	1	1.	0.3	7.57	0.	0.	0.	0.
time (sec)	N/A	0.225	0.025	0.032	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	57	2374	0	0	0	0
normalized size	1	1.	0.12	4.83	0.	0.	0.	0.
time (sec)	N/A	0.398	0.013	0.034	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	59	480	0	247	0	55
normalized size	1	1.	1.64	13.33	0.	6.86	0.	1.53
time (sec)	N/A	0.048	0.012	0.118	0.	1.331	0.	1.289

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	0	0	0
normalized size	1	1.	0.27	2.15	0.	0.	0.	0.
time (sec)	N/A	0.166	0.014	0.126	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	55	2860	0	0	0	0
normalized size	1	1.	0.11	5.51	0.	0.	0.	0.
time (sec)	N/A	0.487	0.015	0.041	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	35	51	0	31
normalized size	1	1.	1.	1.07	1.3	1.89	0.	1.15
time (sec)	N/A	0.04	0.011	0.003	1.12	0.921	0.	1.449

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	57	1795	0	0	0	0
normalized size	1	1.	0.24	7.64	0.	0.	0.	0.
time (sec)	N/A	0.221	0.013	0.037	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	57	3048	0	0	0	0
normalized size	1	1.	0.1	5.49	0.	0.	0.	0.
time (sec)	N/A	0.573	0.014	0.043	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	51	73	0	49
normalized size	1	1.	0.62	0.66	0.91	1.3	0.	0.88
time (sec)	N/A	0.083	0.017	0.006	1.104	0.823	0.	1.201

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	57	2009	0	0	0	0
normalized size	1	1.	0.22	7.58	0.	0.	0.	0.
time (sec)	N/A	0.284	0.014	0.041	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	61	19	41
normalized size	1	1.	1.	1.04	1.36	2.18	0.68	1.46
time (sec)	N/A	0.017	0.004	0.007	1.104	0.736	0.787	1.203

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	54	103	31	61
normalized size	1	1.	1.	0.98	1.29	2.45	0.74	1.45
time (sec)	N/A	0.019	0.005	0.006	1.069	0.793	0.529	1.305

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	94	120	0	390	0	0
normalized size	1	1.	0.84	1.07	0.	3.48	0.	0.
time (sec)	N/A	0.179	0.145	0.004	0.	0.915	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	92	98	0	338	0	0
normalized size	1	1.	1.07	1.14	0.	3.93	0.	0.
time (sec)	N/A	0.127	0.052	0.004	0.	0.811	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	78	0	277	0	55
normalized size	1	1.	1.34	1.39	0.	4.95	0.	0.98
time (sec)	N/A	0.082	0.041	0.005	0.	0.81	0.	1.212

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	56	0	173	0	31
normalized size	1	1.	1.84	1.75	0.	5.41	0.	0.97
time (sec)	N/A	0.034	0.017	0.003	0.	0.821	0.	1.399

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	43	0	36
normalized size	1	1.	0.91	1.09	0.	1.87	0.	1.57
time (sec)	N/A	0.005	0.007	0.002	0.	0.623	0.	1.214

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	29	30	0	63	0	36
normalized size	1	1.	0.56	0.58	0.	1.21	0.	0.69
time (sec)	N/A	0.045	0.011	0.004	0.	0.86	0.	1.206

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	46	0	90	0	58
normalized size	1	1.	0.52	0.57	0.	1.12	0.	0.72
time (sec)	N/A	0.086	0.012	0.003	0.	0.764	0.	1.216

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	57	0	112	0	77
normalized size	1	1.	0.49	0.53	0.	1.04	0.	0.71
time (sec)	N/A	0.134	0.017	0.005	0.	0.79	0.	1.234

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	68	0	143	0	96
normalized size	1	1.	0.47	0.5	0.	1.05	0.	0.71
time (sec)	N/A	0.174	0.019	0.003	0.	0.821	0.	1.278

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	30	72	22	43
normalized size	1	1.	1.	0.88	1.15	2.77	0.85	1.65
time (sec)	N/A	0.018	0.005	0.005	1.152	0.847	0.522	1.235

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	30	72	22	43
normalized size	1	1.	1.	0.85	1.11	2.67	0.81	1.59
time (sec)	N/A	0.018	0.005	0.004	1.038	0.764	0.263	1.282

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	19	22	5	20
normalized size	1	1.	1.75	1.12	2.38	2.75	0.62	2.5
time (sec)	N/A	0.003	0.003	0.	1.026	0.842	0.093	1.189

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	22	38	15	22
normalized size	1	1.	1.	1.1	2.2	3.8	1.5	2.2
time (sec)	N/A	0.003	0.003	0.	1.11	0.801	0.112	1.13

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	22	59	27	22
normalized size	1	1.	1.	0.92	1.83	4.92	2.25	1.83
time (sec)	N/A	0.003	0.003	0.001	0.995	0.821	0.108	1.152

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	14
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.4
time (sec)	N/A	0.002	0.001	0.	1.087	0.64	0.081	1.137

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	28	41	32	0
normalized size	1	1.	1.	0.95	1.4	2.05	1.6	0.
time (sec)	N/A	0.006	0.004	0.002	1.071	0.873	0.839	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	54	80	82	0
normalized size	1	1.	1.	1.05	2.7	4.	4.1	0.
time (sec)	N/A	0.008	0.003	0.001	1.053	0.917	1.384	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	72	112	119	0
normalized size	1	1.	1.	1.05	3.6	5.6	5.95	0.
time (sec)	N/A	0.009	0.003	0.002	1.014	0.919	1.806	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	365	160	181
normalized size	1	1.	10.	8.44	11.31	22.81	10.	11.31
time (sec)	N/A	0.004	0.004	0.003	1.024	0.598	0.122	1.156

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	367	160	181
normalized size	1	1.	10.	8.44	11.31	22.94	10.	11.31
time (sec)	N/A	0.011	0.007	0.003	1.116	0.682	0.24	1.155

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	371	160	181
normalized size	1	1.	10.	8.44	11.31	23.19	10.	11.31
time (sec)	N/A	0.012	0.008	0.001	1.056	0.761	0.234	1.119

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	339	0	578	0	0
normalized size	1	1.	0.89	12.56	0.	21.41	0.	0.
time (sec)	N/A	0.015	0.013	0.106	0.	0.724	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	365	160	181
normalized size	1	1.	10.	8.44	11.31	22.81	10.	11.31
time (sec)	N/A	0.004	0.003	0.	1.104	0.597	0.222	1.158

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	367	160	181
normalized size	1	1.	10.	8.44	11.31	22.94	10.	11.31
time (sec)	N/A	0.005	0.005	0.003	1.029	0.659	0.172	1.128

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	371	160	181
normalized size	1	1.	10.	8.44	11.31	23.19	10.	11.31
time (sec)	N/A	0.004	0.006	0.003	1.064	0.665	0.164	1.147

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	287	0	493	0	277
normalized size	1	1.	0.89	10.63	0.	18.26	0.	10.26
time (sec)	N/A	0.008	0.004	0.041	0.	0.863	0.	1.255

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	126	0	204	0	126
normalized size	1	1.	1.	4.67	0.	7.56	0.	4.67
time (sec)	N/A	0.01	0.013	0.018	0.	0.851	0.	1.123

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	89	162	0	0
normalized size	1	1.	1.	1.44	3.3	6.	0.	0.
time (sec)	N/A	0.014	0.019	0.02	1.104	0.893	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.005	0.003	0.002	1.266	0.651	0.114	1.121

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.006	0.004	0.001	1.081	0.729	0.188	1.157

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.005	0.005	0.001	1.12	0.706	0.174	1.1

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	73	36	30
normalized size	1	1.	1.26	1.63	2.58	3.84	1.89	1.58
time (sec)	N/A	0.005	0.008	0.007	1.159	0.784	0.482	1.151

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	76	36	30
normalized size	1	1.	1.26	1.63	2.58	4.	1.89	1.58
time (sec)	N/A	0.006	0.009	0.004	1.057	0.631	5.422	1.158

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	76	0	30
normalized size	1	1.	1.26	1.63	2.58	4.	0.	1.58
time (sec)	N/A	0.006	0.011	0.008	1.109	0.82	0.	1.173

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	50	19	30
normalized size	1	1.	1.	0.96	1.25	2.08	0.79	1.25
time (sec)	N/A	0.011	0.001	0.003	1.106	0.726	0.317	1.174

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	46	85	37	62
normalized size	1	1.	1.	0.88	1.15	2.12	0.92	1.55
time (sec)	N/A	0.023	0.007	0.007	1.154	0.77	0.419	1.112

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	59	104	48	61
normalized size	1	1.	1.	0.9	1.18	2.08	0.96	1.22
time (sec)	N/A	0.021	0.006	0.006	1.172	0.758	0.35	1.114

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	144	156	167	2210	36	151
normalized size	1	1.	0.78	0.84	0.9	11.95	0.19	0.82
time (sec)	N/A	0.346	0.125	0.019	1.546	5.635	1.655	1.159

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	363	0	753	0	363
normalized size	1	1.	1.	12.52	0.	25.97	0.	12.52
time (sec)	N/A	0.017	0.016	0.168	0.	0.854	0.	4.317

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	19	365	160	19
normalized size	1	1.	10.	8.44	1.19	22.81	10.	1.19
time (sec)	N/A	0.003	0.005	0.	1.026	0.612	0.122	1.212

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	367	160	181
normalized size	1	1.	10.	8.44	11.31	22.94	10.	11.31
time (sec)	N/A	0.009	0.006	0.	1.054	0.62	0.152	1.179

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	371	160	181
normalized size	1	1.	10.	8.44	11.31	23.19	10.	11.31
time (sec)	N/A	0.012	0.007	0.001	1.064	0.581	0.127	1.193

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	19	367	160	19
normalized size	1	1.	10.	8.44	1.19	22.94	10.	1.19
time (sec)	N/A	0.003	0.004	0.001	1.102	0.68	0.121	1.204

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	181	371	160	181
normalized size	1	1.	10.	8.44	11.31	23.19	10.	11.31
time (sec)	N/A	0.008	0.006	0.002	1.016	0.644	0.145	1.177

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	19	371	160	19
normalized size	1	1.	10.	8.44	1.19	23.19	10.	1.19
time (sec)	N/A	0.003	0.006	0.001	0.981	0.589	0.133	1.187

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	36	50	55	48	0
normalized size	1	1.	1.	1.57	2.17	2.39	2.09	0.
time (sec)	N/A	0.011	0.008	0.01	1.006	0.931	0.693	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	39	36	66	41	0
normalized size	1	1.	0.96	1.7	1.57	2.87	1.78	0.
time (sec)	N/A	0.013	0.006	0.008	1.038	0.877	1.712	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	26	68	39	0
normalized size	1	1.	1.	2.73	1.73	4.53	2.6	0.
time (sec)	N/A	0.008	0.004	0.01	1.024	0.907	1.994	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	32	22	66	20	0
normalized size	1	1.	1.	1.45	1.	3.	0.91	0.
time (sec)	N/A	0.01	0.006	0.028	1.111	0.832	1.989	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	15	68	22	0
normalized size	1	1.	1.	2.27	1.	4.53	1.47	0.
time (sec)	N/A	0.006	0.003	0.009	1.048	0.722	1.642	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	27	8	12
normalized size	1	1.	1.	1.	0.92	2.25	0.67	1.
time (sec)	N/A	0.004	0.002	0.003	1.041	0.823	0.152	1.172

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	116	23	30	22	24
normalized size	1	1.	1.	8.29	1.64	2.14	1.57	1.71
time (sec)	N/A	0.005	0.002	0.096	1.035	0.788	0.385	1.177

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	30	10	43
normalized size	1	1.	1.	0.75	0.92	2.5	0.83	3.58
time (sec)	N/A	0.004	0.002	0.	1.683	0.847	0.261	1.178

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	42	78	76	0
normalized size	1	1.	1.	1.62	1.75	3.25	3.17	0.
time (sec)	N/A	0.016	0.021	0.02	1.613	0.774	0.681	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.2	0.532	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	109	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.064	0.584	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.247	0.379	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	99	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.144	0.341	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.251	0.383	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	39	0	223	78	0
normalized size	1	1.	0.9	0.76	0.	4.37	1.53	0.
time (sec)	N/A	0.029	0.013	0.002	0.	0.904	2.231	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	85	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.062	0.368	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	78	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.053	0.34	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.068	0.346	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	131	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.233	0.61	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.293	0.335	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	117	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.158	0.324	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	120	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	0.251	0.323	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	58	51	0	277	88	0
normalized size	1	1.	0.79	0.7	0.	3.79	1.21	0.
time (sec)	N/A	0.041	0.026	0.002	0.	0.968	4.757	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.071	0.323	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	0.068	0.322	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.084	0.322	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.072	0.327	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	58	47	0	231	0	88
normalized size	1	1.	1.14	0.92	0.	4.53	0.	1.73
time (sec)	N/A	0.077	0.026	0.007	0.	0.917	0.	1.125

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	61	0	259	0	92
normalized size	1	1.	1.48	1.45	0.	6.17	0.	2.19
time (sec)	N/A	0.023	0.028	0.005	0.	0.86	0.	1.248

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	55	0	261	0	93
normalized size	1	1.	1.29	1.08	0.	5.12	0.	1.82
time (sec)	N/A	0.065	0.037	0.007	0.	0.847	0.	1.138

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	70	74	0	255	0	0
normalized size	1	1.	1.15	1.21	0.	4.18	0.	0.
time (sec)	N/A	0.08	0.021	1.267	0.	0.776	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	56	0	228	0	82
normalized size	1	1.	1.25	1.06	0.	4.3	0.	1.55
time (sec)	N/A	0.079	0.03	0.01	0.	0.858	0.	1.286

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	0	259	0	86
normalized size	1	1.	1.58	1.88	0.	6.02	0.	2.
time (sec)	N/A	0.023	0.031	0.008	0.	0.757	0.	1.227

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	258	0	88
normalized size	1	1.	1.38	1.38	0.	4.87	0.	1.66
time (sec)	N/A	0.068	0.04	0.019	0.	0.822	0.	1.302

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	78	105	0	252	0	0
normalized size	1	1.	1.24	1.67	0.	4.	0.	0.
time (sec)	N/A	0.08	0.029	0.721	0.	0.929	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.14	0.374	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.154	0.331	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.068	0.326	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.119	0.338	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	169	27	0
normalized size	1	1.	1.	0.84	0.	5.45	0.87	0.
time (sec)	N/A	0.02	0.006	0.004	0.	0.79	1.894	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.048	0.339	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.047	0.319	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.05	0.329	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	117	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	0.176	0.37	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.187	0.322	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	91	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.139	0.319	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.187	0.335	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	40	42	0	335	185	0
normalized size	1	1.	0.74	0.78	0.	6.2	3.43	0.
time (sec)	N/A	0.033	0.014	0.003	0.	0.847	3.496	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	55	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.046	0.328	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.041	0.326	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	55	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	0.052	0.327	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.042	0.336	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	63	477	0	236	0	0
normalized size	1	1.	1.97	14.91	0.	7.38	0.	0.
time (sec)	N/A	0.014	0.024	0.014	0.	1.165	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	49	0	196	0	0
normalized size	1	1.	1.84	1.53	0.	6.12	0.	0.
time (sec)	N/A	0.016	0.021	0.051	0.	0.836	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	63	0	0	244	0	0
normalized size	1	1.	1.97	0.	0.	7.62	0.	0.
time (sec)	N/A	0.016	0.024	0.026	0.	2.591	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	216	0	0
normalized size	1	1.	2.05	0.	0.	5.84	0.	0.
time (sec)	N/A	0.024	0.039	0.358	0.	0.912	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	66	471	0	240	0	0
normalized size	1	1.	2.	14.27	0.	7.27	0.	0.
time (sec)	N/A	0.014	0.026	0.566	0.	1.541	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	62	53	0	198	0	0
normalized size	1	1.	1.88	1.61	0.	6.	0.	0.
time (sec)	N/A	0.015	0.022	0.013	0.	1.026	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	248	0	0
normalized size	1	1.	2.	0.	0.	7.52	0.	0.
time (sec)	N/A	0.016	0.025	0.031	0.	3.252	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	219	0	0
normalized size	1	1.	2.05	0.	0.	5.76	0.	0.
time (sec)	N/A	0.023	0.043	0.586	0.	1.004	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.075	0.349	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	252	0	0
normalized size	1	1.	2.11	0.	0.	6.81	0.	0.
time (sec)	N/A	0.019	0.022	2.16	0.	1.025	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.022	0.334	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.08	0.346	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	251	0	0
normalized size	1	1.	2.11	0.	0.	6.61	0.	0.
time (sec)	N/A	0.02	0.022	0.813	0.	1.011	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.023	0.345	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0
normalized size	1	1.	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.355	0.692	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.191	0.512	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	106	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.081	0.37	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	116	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.128	0.372	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	166	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.377	0.357	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.194	0.398	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.143	0.387	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.051	0.357	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.083	0.36	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0
normalized size	1	1.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.243	0.348	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	0	46	0	68
normalized size	1	1.	1.06	0.89	0.	2.56	0.	3.78
time (sec)	N/A	0.007	0.005	0.024	0.	0.921	0.	1.288

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	55	0	15
normalized size	1	1.	1.	0.9	0.	2.75	0.	0.75
time (sec)	N/A	0.003	0.01	0.009	0.	1.192	0.	1.276

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	26	7	8
normalized size	1	1.	1.	0.88	1.	3.25	0.88	1.
time (sec)	N/A	0.004	0.002	0.004	1.718	0.959	0.231	1.327

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	58	0	36
normalized size	1	1.	1.	1.16	0.76	2.32	0.	1.44
time (sec)	N/A	0.01	0.012	0.005	1.249	0.909	0.	1.458

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	58	0	36
normalized size	1	1.	1.	1.16	0.76	2.32	0.	1.44
time (sec)	N/A	0.011	0.002	0.002	1.133	0.938	0.	1.171

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	61	0	19
normalized size	1	1.	1.	1.16	0.76	2.44	0.	0.76
time (sec)	N/A	0.008	0.011	0.004	1.165	0.918	0.	1.242

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	988	988	61	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	2.245	0.019	0.203	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	61	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.974	0.016	0.202	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	92	0	0	0	0	0
normalized size	1	1.03	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.104	0.582	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.117	0.632	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	74	74	0	0	0	0	0
normalized size	1	1.12	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.096	0.511	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.159	0.51	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.132	0.528	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	147	0	0
normalized size	1	1.	0.98	0.	0.	3.34	0.	0.
time (sec)	N/A	0.015	0.035	0.547	0.	0.755	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	131	0	0
normalized size	1	1.	0.98	0.	0.	2.98	0.	0.
time (sec)	N/A	0.015	0.003	0.388	0.	0.759	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	174	0	0
normalized size	1	1.	0.98	0.	0.	3.78	0.	0.
time (sec)	N/A	0.075	0.042	0.542	0.	0.768	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	190	0	0
normalized size	1	1.	0.98	0.	0.	4.13	0.	0.
time (sec)	N/A	0.045	0.004	0.542	0.	0.997	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	23	88	0	0
normalized size	1	1.	0.82	0.91	0.52	2.	0.	0.
time (sec)	N/A	0.018	0.031	0.028	1.179	0.775	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	23	90	0	0
normalized size	1	1.	0.82	0.91	0.52	2.05	0.	0.
time (sec)	N/A	0.017	0.03	0.019	1.19	0.679	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	23	90	0	0
normalized size	1	1.	0.82	0.91	0.52	2.05	0.	0.
time (sec)	N/A	0.018	0.03	0.018	1.069	0.934	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	101	0	0
normalized size	1	1.	0.82	0.	0.	1.77	0.	0.
time (sec)	N/A	0.028	0.021	0.435	0.	0.919	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	107	0	0
normalized size	1	1.	0.74	0.	0.	1.75	0.	0.
time (sec)	N/A	0.025	0.025	0.395	0.	0.908	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	155	0	0
normalized size	1	1.	1.03	0.	0.	3.97	0.	0.
time (sec)	N/A	0.05	0.024	0.53	0.	0.879	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	38	0	0	135	0	0
normalized size	1	1.	0.95	0.	0.	3.38	0.	0.
time (sec)	N/A	0.074	0.04	0.47	0.	0.858	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [347] had the largest ratio of [0.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	13	0.077
2	A	2	1	1.	11	0.091
3	A	1	0	1.	9	0.
4	A	2	1	1.	13	0.077
5	A	2	1	1.	13	0.077
6	A	3	2	1.	15	0.133
7	A	4	3	1.	13	0.231
8	A	3	2	1.	11	0.182
9	A	2	2	1.	15	0.133
10	A	3	2	1.	15	0.133
11	A	3	2	1.	11	0.182
12	A	4	3	1.	15	0.2
13	A	3	3	1.	15	0.2
14	A	2	2	1.	15	0.133
15	A	2	2	1.	13	0.154
16	A	5	5	1.	11	0.454
17	A	3	3	1.	15	0.2
18	A	4	3	1.	15	0.2
19	A	4	3	1.	15	0.2
20	A	4	3	1.	15	0.2
21	A	3	3	1.	15	0.2
22	A	4	3	1.	13	0.231
23	A	4	4	1.	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	4	3	1.	15	0.2
25	A	5	4	1.	15	0.267
26	A	4	3	1.	13	0.231
27	A	4	3	1.	13	0.231
28	A	3	3	1.	13	0.231
29	A	2	2	1.	13	0.154
30	A	2	2	1.	11	0.182
31	A	5	5	1.	9	0.556
32	A	3	3	1.	13	0.231
33	A	4	3	1.	13	0.231
34	A	4	3	1.	13	0.231
35	A	4	3	1.	13	0.231
36	A	5	5	1.	9	0.556
37	A	5	5	1.	11	0.454
38	A	6	5	1.	17	0.294
39	A	7	7	1.	17	0.412
40	A	5	5	1.	15	0.333
41	A	6	6	1.	13	0.462
42	A	4	4	1.	17	0.235
43	A	6	6	1.	17	0.353
44	A	4	4	1.	17	0.235
45	A	7	7	1.	17	0.412
46	A	7	5	1.	17	0.294
47	A	8	7	1.	15	0.467
48	A	6	6	1.	13	0.462
49	A	7	7	1.	17	0.412
50	A	5	4	1.	17	0.235
51	A	7	7	1.	17	0.412
52	A	5	5	1.	17	0.294
53	A	7	6	1.	17	0.353
54	A	5	4	1.	17	0.235
55	A	8	7	1.	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	6	5	1.	17	0.294
57	A	5	4	1.	17	0.235
58	A	6	6	1.	17	0.353
59	A	4	4	1.	17	0.235
60	A	5	5	1.	15	0.333
61	A	3	3	1.	13	0.231
62	A	6	6	1.	17	0.353
63	A	4	4	1.	17	0.235
64	A	7	6	1.	17	0.353
65	A	6	5	1.	17	0.294
66	A	7	7	1.	17	0.412
67	A	5	5	1.	17	0.294
68	A	6	6	1.	17	0.353
69	A	4	4	1.	17	0.235
70	A	6	6	1.	17	0.353
71	A	4	4	1.	15	0.267
72	A	7	7	1.	13	0.538
73	A	5	5	1.	17	0.294
74	A	8	7	1.	17	0.412
75	A	7	4	1.	19	0.21
76	A	5	2	1.	19	0.105
77	A	6	3	1.	19	0.158
78	A	4	2	1.	19	0.105
79	A	1	1	1.	19	0.053
80	A	3	2	1.	19	0.105
81	A	2	2	1.	19	0.105
82	A	2	2	1.	19	0.105
83	A	3	2	1.	19	0.105
84	A	1	1	1.	19	0.053
85	A	4	2	1.	19	0.105
86	A	6	3	1.	19	0.158
87	A	5	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	7	4	1.	19	0.21
89	A	6	3	1.	19	0.158
90	A	8	4	1.	19	0.21
91	A	7	3	1.	19	0.158
92	A	3	3	1.	17	0.176
93	A	2	2	1.	15	0.133
94	A	1	1	1.	17	0.059
95	A	2	2	1.	17	0.118
96	A	3	2	1.	17	0.118
97	A	4	4	1.	17	0.235
98	A	3	3	1.	13	0.231
99	A	4	4	1.	17	0.235
100	A	6	6	1.	17	0.353
101	A	5	5	1.	17	0.294
102	A	6	6	1.	17	0.353
103	A	8	5	1.	19	0.263
104	A	6	5	1.	17	0.294
105	A	4	4	1.	15	0.267
106	A	1	1	1.	19	0.053
107	A	3	2	1.	19	0.105
108	A	5	2	1.	19	0.105
109	A	7	2	1.	19	0.105
110	A	9	6	1.	19	0.316
111	A	7	6	1.	19	0.316
112	A	5	5	1.	17	0.294
113	A	1	1	1.	15	0.067
114	A	3	3	1.	19	0.158
115	A	5	3	1.	19	0.158
116	A	7	3	1.	19	0.158
117	A	9	5	1.	21	0.238
118	A	7	5	1.	21	0.238
119	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.	21	0.143
121	A	2	2	1.	21	0.095
122	A	4	2	1.	21	0.095
123	A	6	2	1.	21	0.095
124	A	8	6	1.	21	0.286
125	A	6	6	1.	21	0.286
126	A	4	4	1.	21	0.19
127	A	2	2	1.	21	0.095
128	A	4	3	1.	21	0.143
129	A	6	3	1.	21	0.143
130	A	8	3	1.	21	0.143
131	A	11	6	1.	19	0.316
132	A	11	8	1.	19	0.421
133	A	8	6	1.	17	0.353
134	A	8	8	1.	15	0.533
135	A	5	5	1.	19	0.263
136	A	8	8	1.	19	0.421
137	A	7	6	1.	19	0.316
138	A	11	8	1.	19	0.421
139	A	10	6	1.	19	0.316
140	A	11	6	1.	19	0.316
141	A	11	8	1.	17	0.471
142	A	8	7	1.	15	0.467
143	A	8	8	1.	19	0.421
144	A	6	6	1.	19	0.316
145	A	9	8	1.	19	0.421
146	A	8	6	1.	19	0.316
147	A	12	8	1.	19	0.421
148	A	11	6	1.	19	0.316
149	A	11	5	1.	19	0.263
150	A	11	7	1.	19	0.368
151	A	8	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	8	7	1.	17	0.412
153	A	5	5	1.	15	0.333
154	A	7	7	1.	19	0.368
155	A	6	5	1.	19	0.263
156	A	10	7	1.	19	0.368
157	A	9	5	1.	19	0.263
158	A	12	8	1.	19	0.421
159	A	9	6	1.	19	0.316
160	A	9	8	1.	19	0.421
161	A	6	6	1.	17	0.353
162	A	7	7	1.	15	0.467
163	A	6	6	1.	19	0.316
164	A	10	8	1.	19	0.421
165	A	9	6	1.	19	0.316
166	A	13	8	1.	19	0.421
167	A	13	3	1.	19	0.158
168	A	10	3	1.	19	0.158
169	A	7	3	1.	17	0.176
170	A	4	3	1.	15	0.2
171	A	1	1	1.	19	0.053
172	A	4	4	1.	19	0.21
173	A	7	4	1.	19	0.21
174	A	10	4	1.	19	0.21
175	A	13	4	1.	19	0.21
176	A	12	3	1.	19	0.158
177	A	9	3	1.	17	0.176
178	A	6	3	1.	15	0.2
179	A	3	2	1.	19	0.105
180	A	4	3	1.	19	0.158
181	A	5	4	1.	19	0.21
182	A	8	4	1.	19	0.21
183	A	11	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	14	4	1.	19	0.21
185	A	14	3	1.	19	0.158
186	A	11	3	1.	19	0.158
187	A	8	3	1.	19	0.158
188	A	5	3	1.	17	0.176
189	A	2	2	1.	15	0.133
190	A	3	3	1.	19	0.158
191	A	6	3	1.	19	0.158
192	A	9	3	1.	19	0.158
193	A	12	3	1.	19	0.158
194	A	12	4	1.	19	0.21
195	A	9	4	1.	19	0.21
196	A	6	4	1.	19	0.21
197	A	3	3	1.	17	0.176
198	A	3	3	1.	15	0.2
199	A	6	4	1.	19	0.21
200	A	9	4	1.	19	0.21
201	A	12	4	1.	19	0.21
202	A	15	4	1.	19	0.21
203	A	2	1	1.	15	0.067
204	A	2	1	1.	13	0.077
205	A	1	0	1.	11	0.
206	A	2	1	1.	15	0.067
207	A	2	1	1.	15	0.067
208	A	3	2	1.	17	0.118
209	A	3	2	1.	15	0.133
210	A	3	2	1.	13	0.154
211	A	3	2	1.	17	0.118
212	A	3	2	1.	17	0.118
213	A	3	2	1.	17	0.118
214	A	3	2	1.	17	0.118
215	A	3	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	3	2	1.	17	0.118
217	A	2	2	1.	17	0.118
218	A	4	4	1.	15	0.267
219	A	3	2	1.	13	0.154
220	A	3	2	1.	17	0.118
221	A	3	2	1.	17	0.118
222	A	3	2	1.	17	0.118
223	A	3	2	1.	17	0.118
224	A	3	2	1.	17	0.118
225	A	3	2	1.	17	0.118
226	A	2	2	1.	17	0.118
227	A	3	2	1.	17	0.118
228	A	3	2	1.	17	0.118
229	A	3	2	1.	15	0.133
230	A	3	2	1.	13	0.154
231	A	3	2	1.	17	0.118
232	A	4	3	1.	19	0.158
233	A	3	3	1.	17	0.176
234	A	2	2	1.	15	0.133
235	A	1	1	1.	19	0.053
236	A	3	3	1.	19	0.158
237	A	3	3	1.	19	0.158
238	A	4	4	1.	19	0.21
239	A	5	4	1.	19	0.21
240	A	6	3	1.	19	0.158
241	A	5	3	1.	17	0.176
242	A	4	3	1.	15	0.2
243	A	3	2	1.	19	0.105
244	A	2	2	1.	19	0.105
245	A	1	1	1.	19	0.053
246	A	4	3	1.	19	0.158
247	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	4	3	1.	19	0.158
249	A	5	4	1.	19	0.21
250	A	6	4	1.	19	0.21
251	A	7	4	1.	19	0.21
252	A	4	2	1.	19	0.105
253	A	3	2	1.	19	0.105
254	A	2	2	1.	19	0.105
255	A	1	1	1.	17	0.059
256	A	2	2	1.	15	0.133
257	A	3	3	1.	19	0.158
258	A	4	3	1.	19	0.158
259	A	5	3	1.	19	0.158
260	A	4	3	1.	19	0.158
261	A	3	3	1.	19	0.158
262	A	2	2	1.	19	0.105
263	A	1	1	1.	19	0.053
264	A	3	3	1.	19	0.158
265	A	4	4	1.	17	0.235
266	A	5	4	1.	15	0.267
267	A	6	4	1.	19	0.21
268	A	7	4	1.	19	0.21
269	A	5	3	1.	21	0.143
270	A	4	3	1.	21	0.143
271	A	3	3	1.	21	0.143
272	A	2	2	1.	21	0.095
273	A	1	1	1.	21	0.048
274	A	2	2	1.	21	0.095
275	A	3	2	1.	21	0.095
276	A	4	2	1.	21	0.095
277	A	3	3	1.	21	0.143
278	A	3	3	1.27	19	0.158
279	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	1	1	1.	21	0.048
281	A	2	2	1.	21	0.095
282	A	3	2	1.	21	0.095
283	A	2	2	1.	17	0.118
284	A	3	2	1.	19	0.105
285	A	2	2	1.	19	0.105
286	A	1	1	1.	19	0.053
287	A	2	2	1.	15	0.133
288	A	3	3	1.	19	0.158
289	A	3	3	1.	19	0.158
290	A	2	2	1.	17	0.118
291	A	3	3	1.	19	0.158
292	A	5	5	1.	19	0.263
293	A	4	4	1.	19	0.21
294	A	5	5	1.	19	0.263
295	A	5	4	1.	21	0.19
296	A	6	6	1.	21	0.286
297	A	3	3	1.	21	0.143
298	A	4	4	1.	21	0.19
299	A	5	5	1.	21	0.238
300	A	2	2	1.	21	0.095
301	A	3	3	1.	21	0.143
302	A	6	6	1.	21	0.286
303	A	1	1	1.	21	0.048
304	A	4	4	1.	21	0.19
305	A	7	6	1.	21	0.286
306	A	2	2	1.	21	0.095
307	A	5	4	1.	21	0.19
308	A	3	2	1.	15	0.133
309	A	3	2	1.	13	0.154
310	A	5	3	1.	19	0.158
311	A	4	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	3	3	1.	19	0.158
313	A	2	2	1.	17	0.118
314	A	1	1	1.	15	0.067
315	A	2	2	1.	19	0.105
316	A	3	2	1.	19	0.105
317	A	4	2	1.	19	0.105
318	A	5	2	1.	19	0.105
319	A	4	3	1.	11	0.273
320	A	4	3	1.	13	0.231
321	A	3	3	1.	9	0.333
322	A	3	3	1.	9	0.333
323	A	3	3	1.	9	0.333
324	A	3	3	1.	13	0.231
325	A	3	3	1.	13	0.231
326	A	3	3	1.	13	0.231
327	A	3	3	1.	13	0.231
328	A	2	2	1.	11	0.182
329	A	2	2	1.	15	0.133
330	A	2	2	1.	15	0.133
331	A	2	2	1.	23	0.087
332	A	2	2	1.	11	0.182
333	A	2	2	1.	13	0.154
334	A	2	2	1.	13	0.154
335	A	2	2	1.	17	0.118
336	A	2	2	1.	17	0.118
337	A	2	2	1.	17	0.118
338	A	2	2	1.	11	0.182
339	A	2	2	1.	11	0.182
340	A	2	2	1.	11	0.182
341	A	2	2	1.	11	0.182
342	A	2	2	1.	13	0.154
343	A	2	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
344	A	3	2	1.	11	0.182
345	A	4	3	1.	11	0.273
346	A	3	2	1.	11	0.182
347	A	7	7	1.	9	0.778
348	A	2	2	1.	22	0.091
349	A	1	1	1.	13	0.077
350	A	2	2	1.	15	0.133
351	A	2	2	1.	17	0.118
352	A	1	1	1.	13	0.077
353	A	2	2	1.	15	0.133
354	A	1	1	1.	13	0.077
355	A	2	2	1.	11	0.182
356	A	5	5	1.	13	0.385
357	A	2	2	1.	15	0.133
358	A	5	5	1.	13	0.385
359	A	2	2	1.	15	0.133
360	A	2	2	1.	11	0.182
361	A	2	2	1.	11	0.182
362	A	2	2	1.	9	0.222
363	A	5	5	1.	11	0.454
364	A	3	3	1.	25	0.12
365	A	4	4	1.	27	0.148
366	A	4	4	1.	23	0.174
367	A	4	4	1.	22	0.182
368	A	4	4	1.	21	0.19
369	A	5	5	1.	18	0.278
370	A	4	4	1.	23	0.174
371	A	3	3	1.	15	0.2
372	A	4	4	1.	23	0.174
373	A	5	4	1.	27	0.148
374	A	5	4	1.	23	0.174
375	A	5	4	1.	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	4	1.	21	0.19
377	A	6	5	1.	18	0.278
378	A	5	4	1.	23	0.174
379	A	5	5	1.	22	0.227
380	A	5	4	1.	23	0.174
381	A	5	4	1.	22	0.182
382	A	5	5	1.	13	0.385
383	A	5	5	1.	15	0.333
384	A	4	4	1.	15	0.267
385	A	4	4	1.	15	0.267
386	A	5	5	1.	15	0.333
387	A	5	5	1.	17	0.294
388	A	4	4	1.	17	0.235
389	A	4	4	1.	17	0.235
390	A	3	3	1.	27	0.111
391	A	3	3	1.	23	0.13
392	A	2	2	1.	15	0.133
393	A	3	3	1.	21	0.143
394	A	4	4	1.	18	0.222
395	A	3	3	1.	23	0.13
396	A	3	3	1.	22	0.136
397	A	3	3	1.	23	0.13
398	A	4	4	1.	27	0.148
399	A	4	4	1.	23	0.174
400	A	4	4	1.	22	0.182
401	A	4	4	1.	21	0.19
402	A	5	5	1.	18	0.278
403	A	4	4	1.	23	0.174
404	A	4	4	1.	22	0.182
405	A	4	4	1.	23	0.174
406	A	4	4	1.	22	0.182
407	A	3	3	1.	15	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	3	3	1.	15	0.2
409	A	3	3	1.	15	0.2
410	A	3	3	1.	19	0.158
411	A	3	3	1.	16	0.188
412	A	3	3	1.	16	0.188
413	A	3	3	1.	16	0.188
414	A	3	3	1.	20	0.15
415	A	3	3	1.	19	0.158
416	A	3	3	1.	17	0.176
417	A	3	3	1.	17	0.176
418	A	3	3	1.	20	0.15
419	A	3	3	1.	19	0.158
420	A	3	3	1.	18	0.167
421	A	3	3	1.	21	0.143
422	A	3	3	1.	21	0.143
423	A	3	3	1.	21	0.143
424	A	3	3	1.	21	0.143
425	A	3	3	1.	21	0.143
426	A	3	3	1.	15	0.2
427	A	3	3	1.	15	0.2
428	A	3	3	1.	15	0.2
429	A	3	3	1.	15	0.2
430	A	3	3	1.	15	0.2
431	A	2	2	1.	11	0.182
432	A	1	1	1.	11	0.091
433	A	3	3	1.	13	0.231
434	A	1	1	1.	17	0.059
435	A	1	1	1.	17	0.059
436	A	2	2	1.	15	0.133
437	A	11	10	1.	19	0.526
438	A	9	8	1.	19	0.421
439	A	3	3	1.03	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	3	3	1.	22	0.136
441	A	3	3	1.12	22	0.136
442	A	3	3	1.	27	0.111
443	A	3	3	1.	27	0.111
444	A	1	1	1.	18	0.056
445	A	2	2	1.	17	0.118
446	A	2	2	1.	22	0.091
447	A	1	1	1.	23	0.043
448	A	2	2	1.	19	0.105
449	A	2	2	1.	19	0.105
450	A	2	2	1.	19	0.105
451	A	2	2	1.	19	0.105
452	A	2	2	1.	19	0.105
453	A	1	1	1.	25	0.04
454	A	2	2	1.	28	0.071

Chapter 3

Listing of integrals

3.1 $\int x^2 (ax + bx^3) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

[Out] (a*x^4)/4 + (b*x^6)/6

Rubi [A] time = 0.0047563, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^6)/6

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)

+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned}\int x^2 (ax + bx^3) dx &= \int (ax^3 + bx^5) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0014992, size = 17, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3), x]

[Out] (a*x^4)/4 + (b*x^6)/6

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a*x), x)

[Out] 1/4*a*x^4+1/6*b*x^6

Maxima [A] time = 1.0604, size = 18, normalized size = 1.06

$$\frac{1}{6} bx^6 + \frac{1}{4} ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")

[Out] 1/6*b*x^6 + 1/4*a*x^4

Fricas [A] time = 1.18363, size = 31, normalized size = 1.82

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")

[Out] 1/6*x^6*b + 1/4*x^4*a

Sympy [A] time = 0.05323, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a*x),x)

[Out] a*x**4/4 + b*x**6/6

Giac [A] time = 1.12549, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")

[Out] 1/6*b*x^6 + 1/4*a*x^4

3.2 $\int x(ax + bx^3) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

[Out] (a*x^3)/3 + (b*x^5)/5

Rubi [A] time = 0.0044615, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3), x]

[Out] (a*x^3)/3 + (b*x^5)/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x(ax + bx^3) dx &= \int (ax^2 + bx^4) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.0014376, size = 17, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3),x]

[Out] (a*x^3)/3 + (b*x^5)/5

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x),x)

[Out] 1/3*a*x^3+1/5*b*x^5

Maxima [A] time = 1.07896, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")

[Out] 1/5*b*x^5 + 1/3*a*x^3

Fricas [A] time = 1.21982, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")

[Out] $1/5*x^5*b + 1/3*x^3*a$

Sympy [A] time = 0.053318, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**3+a*x),x)`

[Out] $a*x**3/3 + b*x**5/5$

Giac [A] time = 1.18127, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x),x, algorithm="giac")`

[Out] $1/5*b*x^5 + 1/3*a*x^3$

3.3 $\int (ax + bx^3) dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

[Out] (a*x^2)/2 + (b*x^4)/4

Rubi [A] time = 0.0027001, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[a*x + b*x^3, x]

[Out] (a*x^2)/2 + (b*x^4)/4

Rubi steps

$$\int (ax + bx^3) dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

Mathematica [A] time = 0.0000298, size = 17, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[a*x + b*x^3, x]

[Out] (a*x^2)/2 + (b*x^4)/4

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^3+a*x,x)

[Out] 1/2*a*x^2+1/4*b*x^4

Maxima [A] time = 1.06282, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^3+a*x,x, algorithm="maxima")

[Out] 1/4*b*x^4 + 1/2*a*x^2

Fricas [A] time = 1.27065, size = 31, normalized size = 1.82

$$\frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^3+a*x,x, algorithm="fricas")

[Out] 1/4*x^4*b + 1/2*x^2*a

Sympy [A] time = 0.051718, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A] time = 1.17737, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

3.4 $\int \frac{ax+bx^3}{x} dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

[Out] a*x + (b*x^3)/3

Rubi [A] time = 0.0033423, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3}{x} dx &= \int (a + bx^2) dx \\ &= ax + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0004057, size = 12, normalized size = 1.

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

Maple [A] time = 0., size = 11, normalized size = 0.9

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)/x,x)

[Out] a*x+1/3*b*x^3

Maxima [A] time = 1.04063, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")

[Out] 1/3*b*x^3 + a*x

Fricas [A] time = 1.3455, size = 23, normalized size = 1.92

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")

[Out] $\frac{1}{3}bx^3 + ax$

Sympy [A] time = 0.051531, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)/x,x)`

[Out] $a*x + b*x**3/3$

Giac [A] time = 1.23919, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)/x,x, algorithm="giac")`

[Out] $\frac{1}{3}bx^3 + ax$

3.5 $\int \frac{ax+bx^3}{x^2} dx$

Optimal. Leaf size=13

$$a \log(x) + \frac{bx^2}{2}$$

[Out] (b*x^2)/2 + a*Log[x]

Rubi [A] time = 0.0047781, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$a \log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)/x^2,x]

[Out] (b*x^2)/2 + a*Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3}{x^2} dx &= \int \left(\frac{a}{x} + bx \right) dx \\ &= \frac{bx^2}{2} + a \log(x) \end{aligned}$$

Mathematica [A] time = 0.0009369, size = 13, normalized size = 1.

$$a \log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)/x^2,x]

[Out] (b*x^2)/2 + a*Log[x]

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$\frac{bx^2}{2} + a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)/x^2,x)

[Out] 1/2*b*x^2+a*ln(x)

Maxima [A] time = 1.03596, size = 15, normalized size = 1.15

$$\frac{1}{2}bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*log(x)

Fricas [A] time = 1.33364, size = 30, normalized size = 2.31

$$\frac{1}{2}bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")

[Out] $1/2*b*x^2 + a*\log(x)$

Sympy [A] time = 0.077242, size = 10, normalized size = 0.77

$$a \log(x) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)/x**2,x)`

[Out] $a*\log(x) + b*x**2/2$

Giac [A] time = 1.18372, size = 19, normalized size = 1.46

$$\frac{1}{2}bx^2 + \frac{1}{2}a \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)/x^2,x, algorithm="giac")`

[Out] $1/2*b*x^2 + 1/2*a*\log(x^2)$

3.6 $\int x^2 (ax + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9

Rubi [A] time = 0.0150718, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 270}

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 (ax + bx^3)^2 dx &= \int x^4 (a + bx^2)^2 dx \\ &= \int (a^2x^4 + 2abx^6 + b^2x^8) dx \\ &= \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0007609, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a*x)^2,x)

[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9

Maxima [A] time = 1.0588, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$

Fricas [A] time = 1.20442, size = 55, normalized size = 1.83

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9b^2 + \frac{2}{7}x^7b*a + \frac{1}{5}x^5a^2$

Sympy [A] time = 0.060723, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a*x)**2,x)`

[Out] $a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9$

Giac [A] time = 1.13094, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")`

[Out] $\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$

3.7 $\int x(ax + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8

Rubi [A] time = 0.0195837, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 43}

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3)^2,x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int x(ax + bx^3)^2 dx &= \int x^3(a + bx^2)^2 dx \\
&= \frac{1}{2} \text{Subst}\left(\int x(a + bx)^2 dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int (a^2x + 2abx^2 + b^2x^3) dx, x, x^2\right) \\
&= \frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}
\end{aligned}$$

Mathematica [A] time = 0.0009825, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3)^2,x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x)^2,x)

[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8

Maxima [A] time = 1.02882, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4

Fricas [A] time = 1.1766, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2

Sympy [A] time = 0.059262, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a*x)**2,x)

[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8

Giac [A] time = 1.16584, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4

3.8 $\int (ax + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7

Rubi [A] time = 0.0106576, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 270}

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 270

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}\int (ax + bx^3)^2 dx &= \int x^2 (a + bx^2)^2 dx \\ &= \int (a^2x^2 + 2abx^4 + b^2x^6) dx \\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.001557, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{x^3a^2}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^2,x)

[Out] 1/3*x^3*a^2+2/5*a*b*x^5+1/7*b^2*x^7

Maxima [A] time = 1.08874, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$

Fricas [A] time = 1.20089, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2,x, algorithm="fricas")`

[Out] $\frac{1}{7}x^7b^2 + \frac{2}{5}x^5b*a + \frac{1}{3}x^3a^2$

Sympy [A] time = 0.059754, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**2,x)`

[Out] $a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7$

Giac [A] time = 1.20869, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2,x, algorithm="giac")`

[Out] $\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$

$$3.9 \quad \int \frac{(ax+bx^3)^2}{x} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^2)^3}{6b}$$

[Out] (a + b*x^2)^3/(6*b)

Rubi [A] time = 0.0070228, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{(a + bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^2/x,x]

[Out] (a + b*x^2)^3/(6*b)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
  ^ (p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
  NeQ[p, -1]
```

Rubi steps

$$\int \frac{(ax + bx^3)^2}{x} dx = \int x(a + bx^2)^2 dx$$

$$= \frac{(a + bx^2)^3}{6b}$$

Mathematica [A] time = 0.0016032, size = 16, normalized size = 1.

$$\frac{(a + bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^2/x,x]

[Out] (a + b*x^2)^3/(6*b)

Maple [A] time = 0.002, size = 25, normalized size = 1.6

$$\frac{b^2x^6}{6} + \frac{abx^4}{2} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^2/x,x)

[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2

Maxima [A] time = 1.10286, size = 32, normalized size = 2.

$$\frac{1}{6}b^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")

[Out] $1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2$

Fricas [A] time = 1.34173, size = 55, normalized size = 3.44

$$\frac{1}{6}b^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")`

[Out] $1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2$

Sympy [B] time = 0.067713, size = 24, normalized size = 1.5

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**2/x,x)`

[Out] $a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6$

Giac [A] time = 1.17046, size = 32, normalized size = 2.

$$\frac{1}{6}b^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2/x,x, algorithm="giac")`

[Out] $1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2$

3.10

$$\int \frac{(ax+bx^3)^2}{x^2} dx$$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

[Out] $a^2x + (2abx^3)/3 + (b^2x^5)/5$

Rubi [A] time = 0.011589, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 194}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^2/x^2,x]

[Out] $a^2x + (2abx^3)/3 + (b^2x^5)/5$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax + bx^3)^2}{x^2} dx &= \int (a + bx^2)^2 dx \\ &= \int (a^2 + 2abx^2 + b^2x^4) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0009442, size = 25, normalized size = 1.

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^2/x^2,x]

[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5

Maple [A] time = 0.002, size = 22, normalized size = 0.9

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^2/x^2,x)

[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5

Maxima [A] time = 0.995564, size = 28, normalized size = 1.12

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")

[Out] $\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$

Fricas [A] time = 1.31853, size = 47, normalized size = 1.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")`

[Out] $\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$

Sympy [A] time = 0.059626, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**2/x**2,x)`

[Out] $a**2*x + 2*a*b*x**3/3 + b**2*x**5/5$

Giac [A] time = 1.22034, size = 28, normalized size = 1.12

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")`

[Out] $\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$

3.11 $\int (-4x + 3x^3)^6 dx$

Optimal. Leaf size=46

$$\frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (5832*x^17)/17 + (729*x^19)/19

Rubi [A] time = 0.0157418, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 270}

$$\frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(-4*x + 3*x^3)^6, x]

[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (5832*x^17)/17 + (729*x^19)/19

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (-4x + 3x^3)^6 dx &= \int x^6 (-4 + 3x^2)^6 dx \\ &= \int (4096x^6 - 18432x^8 + 34560x^{10} - 34560x^{12} + 19440x^{14} - 5832x^{16} + 729x^{18}) dx \\ &= \frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19} \end{aligned}$$

Mathematica [A] time = 0.0018519, size = 46, normalized size = 1.

$$\frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(-4*x + 3*x^3)^6,x]

[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (5832*x^17)/17 + (729*x^19)/19

Maple [A] time = 0.002, size = 37, normalized size = 0.8

$$\frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-4*x)^6,x)

[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/19*x^19

Maxima [A] time = 1.04227, size = 49, normalized size = 1.07

$$\frac{729}{19}x^{19} - \frac{5832}{17}x^{17} + 1296x^{15} - \frac{34560}{13}x^{13} + \frac{34560}{11}x^{11} - 2048x^9 + \frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")

[Out] $729/19*x^{19} - 5832/17*x^{17} + 1296*x^{15} - 34560/13*x^{13} + 34560/11*x^{11} - 2048*x^9 + 4096/7*x^7$

Fricas [A] time = 1.20751, size = 130, normalized size = 2.83

$$\frac{729}{19}x^{19} - \frac{5832}{17}x^{17} + 1296x^{15} - \frac{34560}{13}x^{13} + \frac{34560}{11}x^{11} - 2048x^9 + \frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")

[Out] $729/19*x^{19} - 5832/17*x^{17} + 1296*x^{15} - 34560/13*x^{13} + 34560/11*x^{11} - 2048*x^9 + 4096/7*x^7$

Sympy [A] time = 0.062123, size = 42, normalized size = 0.91

$$\frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**3-4*x)**6,x)

[Out] $729*x^{19}/19 - 5832*x^{17}/17 + 1296*x^{15} - 34560*x^{13}/13 + 34560*x^{11}/11 - 2048*x^9 + 4096*x^7/7$

Giac [A] time = 1.25294, size = 49, normalized size = 1.07

$$\frac{729}{19}x^{19} - \frac{5832}{17}x^{17} + 1296x^{15} - \frac{34560}{13}x^{13} + \frac{34560}{11}x^{11} - 2048x^9 + \frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")

[Out] $729/19*x^{19} - 5832/17*x^{17} + 1296*x^{15} - 34560/13*x^{13} + 34560/11*x^{11} - 2048*x^9 + 4096/7*x^7$

3.12

$$\int \frac{x^4}{ax+bx^3} dx$$

Optimal. Leaf size=27

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

[Out] $x^2/(2*b) - (a*\text{Log}[a + b*x^2])/(2*b^2)$

Rubi [A] time = 0.0227478, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 266, 43}

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(a*x + b*x^3), x]$

[Out] $x^2/(2*b) - (a*\text{Log}[a + b*x^2])/(2*b^2)$

Rule 1584

$\text{Int}[(u_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol]$
 $:\> \text{Int}[u*x^{(m + n*p)}*(a + b*x^{(q - p)})^n, x] /;$ $\text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

Rule 266

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] :\> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /;$ $\text{FreeQ}\{a, b, m, n, p\}, x]$ $\&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] :\> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x]$ $\&\& \text{NeQ}[b*c - a*d, 0]$ $\&\& \text{IGtQ}[m, 0]$ $\&\& (!\text{IntegerQ}[n] \|\ (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \|\ \text{LtQ}[9*m + 5*(n + 1), 0] \|\ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax + bx^3} dx &= \int \frac{x^3}{a + bx^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{a + bx} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{b} - \frac{a}{b(a + bx)} \right) dx, x, x^2 \right) \\
&= \frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0039764, size = 27, normalized size = 1.

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x + b*x^3), x]

[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)

Maple [A] time = 0.001, size = 24, normalized size = 0.9

$$\frac{x^2}{2b} - \frac{a \ln(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x), x)

[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2

Maxima [A] time = 1.06432, size = 31, normalized size = 1.15

$$\frac{x^2}{2b} - \frac{a \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")`

[Out] $1/2*x^2/b - 1/2*a*\log(b*x^2 + a)/b^2$

Fricas [A] time = 1.38497, size = 49, normalized size = 1.81

$$\frac{bx^2 - a \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")`

[Out] $1/2*(b*x^2 - a*\log(b*x^2 + a))/b^2$

Sympy [A] time = 0.289509, size = 20, normalized size = 0.74

$$-\frac{a \log(a + bx^2)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**3+a*x),x)`

[Out] $-a*\log(a + b*x**2)/(2*b**2) + x**2/(2*b)$

Giac [A] time = 1.20129, size = 32, normalized size = 1.19

$$\frac{x^2}{2b} - \frac{a \log(|bx^2 + a|)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x),x, algorithm="giac")`

[Out] $1/2*x^2/b - 1/2*a*\log(\text{abs}(b*x^2 + a))/b^2$

3.13 $\int \frac{x^3}{ax+bx^3} dx$

Optimal. Leaf size=31

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)

Rubi [A] time = 0.0161081, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 321, 205}

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x + b*x^3),x]

[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax + bx^3} dx &= \int \frac{x^2}{a + bx^2} dx \\ &= \frac{x}{b} - \frac{a \int \frac{1}{a+bx^2} dx}{b} \\ &= \frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0085384, size = 31, normalized size = 1.

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3),x]

[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$\frac{x}{b} - \frac{a}{b} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x),x)

[Out] x/b-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.41214, size = 165, normalized size = 5.32

$$\left[\frac{\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 2x}{2b}, -\frac{\sqrt{\frac{a}{b}} \arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - x}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b, - (sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]

Sympy [B] time = 0.287938, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}} \log\left(-b\sqrt{-\frac{a}{b^3}} + x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}} \log\left(b\sqrt{-\frac{a}{b^3}} + x\right)}{2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**3+a*x),x)

[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**3) + x)/2 + x/b

Giac [A] time = 1.24738, size = 35, normalized size = 1.13

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb}} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

3.14

$$\int \frac{x^2}{ax+bx^3} dx$$

Optimal. Leaf size=15

$$\frac{\log(a + bx^2)}{2b}$$

[Out] Log[a + b*x^2]/(2*b)

Rubi [A] time = 0.0074074, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 260}

$$\frac{\log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x + b*x^3), x]

[Out] Log[a + b*x^2]/(2*b)

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{ax + bx^3} dx = \int \frac{x}{a + bx^2} dx = \frac{\log(a + bx^2)}{2b}$$

Mathematica [A] time = 0.0020339, size = 15, normalized size = 1.

$$\frac{\log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x + b*x^3),x]

[Out] Log[a + b*x^2]/(2*b)

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x),x)

[Out] 1/2*ln(b*x^2+a)/b

Maxima [A] time = 1.05242, size = 18, normalized size = 1.2

$$\frac{\log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")

[Out] 1/2*log(b*x^2 + a)/b

Fricas [A] time = 1.36721, size = 30, normalized size = 2.

$$\frac{\log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")`

[Out] $1/2*\log(b*x^2 + a)/b$

Sympy [A] time = 0.101106, size = 10, normalized size = 0.67

$$\frac{\log(a + bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a*x),x)`

[Out] $\log(a + b*x**2)/(2*b)$

Giac [A] time = 1.27121, size = 19, normalized size = 1.27

$$\frac{\log(|bx^2 + a|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x),x, algorithm="giac")`

[Out] $1/2*\log(\text{abs}(b*x^2 + a))/b$

3.15

$$\int \frac{x}{ax+bx^3} dx$$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Rubi [A] time = 0.0089429, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{ax + bx^3} dx = \int \frac{1}{a + bx^2} dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0038165, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Maple [A] time = 0.003, size = 16, normalized size = 0.7

$$\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x), x)

[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.41005, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2ab}, \frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")

[Out] $[-1/2*\sqrt{-a*b}*\log((b*x^2 - 2*\sqrt{-a*b}*x - a)/(b*x^2 + a))/(a*b), \sqrt{a*b}*\arctan(\sqrt{a*b}*x/a)/(a*b)]$

Sympy [B] time = 0.122746, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}} \log\left(-a\sqrt{-\frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(a\sqrt{-\frac{1}{ab}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a*x),x)

[Out] $-\sqrt{-1/(a*b)}*\log(-a*\sqrt{-1/(a*b)} + x)/2 + \sqrt{-1/(a*b)}*\log(a*\sqrt{-1/(a*b)} + x)/2$

Giac [A] time = 1.16161, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")

[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)

$$3.16 \quad \int \frac{1}{ax+bx^3} dx$$

Optimal. Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

[Out] Log[x]/a - Log[a + b*x^2]/(2*a)

Rubi [A] time = 0.0120126, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(-1),x]

[Out] Log[x]/a - Log[a + b*x^2]/(2*a)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{ax + bx^3} dx &= \int \frac{1}{x(a + bx^2)} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx)} dx, x, x^2 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2a} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx} dx, x, x^2 \right)}{2a} \\
 &= \frac{\log(x)}{a} - \frac{\log(a + bx^2)}{2a}
 \end{aligned}$$

Mathematica [A] time = 0.0042559, size = 22, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log(a + bx^2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-1), x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Maple [A] time = 0.005, size = 21, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln(bx^2 + a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x), x)
```

[Out] $\ln(x)/a - 1/2 * \ln(b*x^2+a)/a$

Maxima [A] time = 1.06132, size = 27, normalized size = 1.23

$$-\frac{\log(bx^2 + a)}{2a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x),x, algorithm="maxima")`

[Out] $-1/2 * \log(b*x^2 + a)/a + \log(x)/a$

Fricas [A] time = 1.40941, size = 49, normalized size = 2.23

$$-\frac{\log(bx^2 + a) - 2 \log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x),x, algorithm="fricas")`

[Out] $-1/2 * (\log(b*x^2 + a) - 2 * \log(x))/a$

Sympy [A] time = 0.178339, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a*x),x)`

[Out] $\log(x)/a - \log(a/b + x**2)/(2*a)$

Giac [A] time = 1.18213, size = 32, normalized size = 1.45

$$\frac{\log(x^2)}{2a} - \frac{\log(|bx^2 + a|)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

$$3.17 \quad \int \frac{1}{x(ax+bx^3)} dx$$

Optimal. Leaf size=34

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

[Out] $-(1/(a*x)) - (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

Rubi [A] time = 0.0167518, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 325, 205}

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*(a*x + b*x^3)), x]$

[Out] $-(1/(a*x)) - (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

Rule 1584

$\text{Int}[(u_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(p_*)} + (b_*)*(x_)^{(q_*)})^{(n_*)}, x_Symbol]$
 $:\> \text{Int}[u*x^{(m+n*p)}*(a+b*x^{(q-p)})^n, x] /; \text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \ \&\& \text{PosQ}[q-p]$

Rule 325

$\text{Int}[((c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] :\> \text{Simp}[((c*x)^{(m+1)}*(a+b*x^n)^{(p+1)})/(a*c*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), \text{Int}[(c*x)^{(m+n)}*(a+b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{LtQ}[m, -1] \ \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 205

$\text{Int}[((a_*) + (b_*)*(x_)^2)^{(-1)}, x_Symbol] :\> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x(ax + bx^3)} dx &= \int \frac{1}{x^2(a + bx^2)} dx \\ &= -\frac{1}{ax} - \frac{b \int \frac{1}{a+bx^2} dx}{a} \\ &= -\frac{1}{ax} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0120363, size = 34, normalized size = 1.

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3)),x]

[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)

Maple [A] time = 0.005, size = 30, normalized size = 0.9

$$-\frac{1}{ax} - \frac{b}{a} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x),x)

[Out] -1/a/x-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.39829, size = 173, normalized size = 5.09

$$\left[\frac{x\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 2}{2ax}, -\frac{x\sqrt{\frac{b}{a}} \arctan\left(x\sqrt{\frac{b}{a}}\right) + 1}{ax} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")

[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]

Sympy [B] time = 0.312298, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}} \log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}} \log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a*x),x)

[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqrt(-b/a**3)/b + x)/2 - 1/(a*x)

Giac [A] time = 1.27111, size = 39, normalized size = 1.15

$$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{aba}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

$$3.18 \quad \int \frac{1}{x^2(ax+bx^3)} dx$$

Optimal. Leaf size=35

$$\frac{b \log(a + bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

[Out] $-1/(2*a*x^2) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x^2])/(2*a^2)$

Rubi [A] time = 0.0259816, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 266, 44}

$$\frac{b \log(a + bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*(a*x + b*x^3)), x]$

[Out] $-1/(2*a*x^2) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x^2])/(2*a^2)$

Rule 1584

$\text{Int}[(u_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol]$
 $:= \text{Int}[u*x^{(m + n*p)}*(a + b*x^{(q - p)})^n, x] /; \text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

Rule 266

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 44

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& !(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax+bx^3)} dx &= \int \frac{1}{x^3(a+bx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a+bx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0062902, size = 35, normalized size = 1.

$$\frac{b \log(a+bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3)),x]

[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$-\frac{1}{2ax^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx^2+a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a*x),x)

[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2

Maxima [A] time = 1.04789, size = 42, normalized size = 1.2

$$\frac{b \log(bx^2+a)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")

[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)

Fricas [A] time = 1.42273, size = 80, normalized size = 2.29

$$\frac{bx^2 \log(bx^2 + a) - 2bx^2 \log(x) - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")

[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)

Sympy [A] time = 0.39978, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a*x),x)

[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)

Giac [A] time = 1.30532, size = 58, normalized size = 1.66

$$-\frac{b \log(x^2)}{2a^2} + \frac{b \log(|bx^2 + a|)}{2a^2} + \frac{bx^2 - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*x^2)
```

$$3.19 \quad \int \frac{1}{x^3(ax+bx^3)} dx$$

Optimal. Leaf size=43

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

[Out] $-1/(3*a*x^3) + b/(a^2*x) + (b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^{(5/2)}$

Rubi [A] time = 0.0259111, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 325, 205}

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a*x + b*x^3)),x]

[Out] $-1/(3*a*x^3) + b/(a^2*x) + (b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^{(5/2)}$

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p +
1 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```


Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(ax + bx^3)} dx &= \int \frac{1}{x^4(a + bx^2)} dx \\
&= -\frac{1}{3ax^3} - \frac{b \int \frac{1}{x^2(a+bx^2)} dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2 \int \frac{1}{a+bx^2} dx}{a^2} \\
&= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.019149, size = 43, normalized size = 1.

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a*x + b*x^3)),x]

[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)

Maple [A] time = 0.005, size = 39, normalized size = 0.9

$$-\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2}{a^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^3+a*x),x)

[Out] -1/3/a/x^3+b/a^2/x+1/a^2*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.39757, size = 234, normalized size = 5.44

$$\left[\frac{3bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6bx^2-2a}{6a^2x^3}, \frac{3bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3bx^2-a}{3a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")

[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*x^2 - a)/(a^2*x^3)]

Sympy [B] time = 0.369346, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**3+a*x),x)

[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)

Giac [A] time = 1.25553, size = 54, normalized size = 1.26

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3bx^2 - a}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")

[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)

3.20 $\int \frac{1}{x^4(ax+bx^3)} dx$

Optimal. Leaf size=49

$$-\frac{b^2 \log(a+bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

[Out] $-1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*\text{Log}[x])/a^3 - (b^2*\text{Log}[a + b*x^2])/(2*a^3)$

Rubi [A] time = 0.0329381, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 266, 44}

$$-\frac{b^2 \log(a+bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a*x + b*x^3)), x]

[Out] $-1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*\text{Log}[x])/a^3 - (b^2*\text{Log}[a + b*x^2])/(2*a^3)$

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m

+ n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4(ax + bx^3)} dx &= \int \frac{1}{x^5(a + bx^2)} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3(a + bx)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a + bx)} \right) dx, x, x^2 \right) \\
 &= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a + bx^2)}{2a^3}
 \end{aligned}$$

Mathematica [A] time = 0.0066015, size = 49, normalized size = 1.

$$-\frac{b^2 \log(a + bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a*x + b*x^3)),x]

[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a^3)

Maple [A] time = 0.006, size = 44, normalized size = 0.9

$$-\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(bx^2 + a)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a*x),x)

[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3

Maxima [A] time = 1.07039, size = 59, normalized size = 1.2

$$-\frac{b^2 \log(bx^2 + a)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{2bx^2 - a}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")

[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)

Fricas [A] time = 1.43787, size = 108, normalized size = 2.2

$$\frac{2b^2x^4 \log(bx^2 + a) - 4b^2x^4 \log(x) - 2abx^2 + a^2}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")

[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x^4)

Sympy [A] time = 0.453392, size = 42, normalized size = 0.86

$$\frac{-a + 2bx^2}{4a^2x^4} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**3+a*x),x)

[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*a**3)

Giac [A] time = 1.21454, size = 77, normalized size = 1.57

$$\frac{b^2 \log(x^2)}{2a^3} - \frac{b^2 \log(|bx^2 + a|)}{2a^3} - \frac{3b^2x^4 - 2abx^2 + a^2}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2  
*a*b*x^2 + a^2)/(a^3*x^4)
```

$$3.21 \quad \int \frac{x^2}{(ax+bx^3)^2} dx$$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])

Rubi [A] time = 0.015014, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x + b*x^3)^2,x]

[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1)
  )/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
  p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
  Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
  ator[p + 1/n] < Denominator[p])
```

Rule 205


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax + bx^3)^2} dx &= \int \frac{1}{(a + bx^2)^2} dx \\ &= \frac{x}{2a(a + bx^2)} + \frac{\int \frac{1}{a+bx^2} dx}{2a} \\ &= \frac{x}{2a(a + bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0237474, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a + bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{x}{2a(bx^2 + a)} + \frac{1}{2a} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.42494, size = 261, normalized size = 5.8

$$\left[\frac{2abx - (bx^2 + a)\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{4(a^2b^2x^2 + a^3b)}, \frac{abx + (bx^2 + a)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2(a^2b^2x^2 + a^3b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]

Sympy [B] time = 0.344998, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}} \log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a*x)**2,x)

[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4

Giac [A] time = 1.26264, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{aba}} + \frac{x}{2(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)

$$3.22 \quad \int \frac{x}{(ax+bx^3)^2} dx$$

Optimal. Leaf size=38

$$-\frac{\log(a+bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a+bx^2)}$$

[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)

Rubi [A] time = 0.0283458, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 44}

$$-\frac{\log(a+bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3)^2,x]

[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_], x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m

+ n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(ax + bx^3)^2} dx &= \int \frac{1}{x(a + bx^2)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx)^2} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a^2 x} - \frac{b}{a(a + bx)^2} - \frac{b}{a^2(a + bx)} \right) dx, x, x^2 \right) \\
 &= \frac{1}{2a(a + bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx^2)}{2a^2}
 \end{aligned}$$

Mathematica [A] time = 0.0131571, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} - \log(a + bx^2) + 2 \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3)^2,x]

[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)

Maple [A] time = 0.009, size = 35, normalized size = 0.9

$$\frac{1}{2a(bx^2 + a)} + \frac{\ln(x)}{a^2} - \frac{\ln(bx^2 + a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x)^2,x)

[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2

Maxima [A] time = 1.0081, size = 46, normalized size = 1.21

$$\frac{1}{2(abx^2 + a^2)} - \frac{\log(bx^2 + a)}{2a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2

Fricas [A] time = 1.4085, size = 108, normalized size = 2.84

$$\frac{(bx^2 + a)\log(bx^2 + a) - 2(bx^2 + a)\log(x) - a}{2(a^2bx^2 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")

[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a^3)

Sympy [A] time = 0.414961, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a*x)**2,x)

[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)

Giac [A] time = 1.20168, size = 63, normalized size = 1.66

$$\frac{\log(x^2)}{2a^2} - \frac{\log(|bx^2 + a|)}{2a^2} + \frac{bx^2 + 2a}{2(bx^2 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2 + a)*a^2)

3.23

$$\int \frac{1}{(ax+bx^3)^2} dx$$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)}$$

[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2))

Rubi [A] time = 0.0178089, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1593, 290, 325, 205}

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(-2), x]

[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 290

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax + bx^3)^2} dx &= \int \frac{1}{x^2(a + bx^2)^2} dx \\ &= \frac{1}{2ax(a + bx^2)} + \frac{3 \int \frac{1}{x^2(a + bx^2)} dx}{2a} \\ &= -\frac{3}{2a^2x} + \frac{1}{2ax(a + bx^2)} - \frac{(3b) \int \frac{1}{a + bx^2} dx}{2a^2} \\ &= -\frac{3}{2a^2x} + \frac{1}{2ax(a + bx^2)} - \frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0339675, size = 54, normalized size = 0.95

$$-\frac{bx}{2a^2(a + bx^2)} - \frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(-2), x]

[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*sqrt[b]*ArcTan[(sqrt[b]*x)/sqrt[a]])/(2*a^(5/2))

Maple [A] time = 0.008, size = 46, normalized size = 0.8

$$-\frac{1}{a^2x} - \frac{bx}{2a^2(bx^2+a)} - \frac{3b}{2a^2} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a*x)^2,x)

[Out] -1/a^2/x-1/2*b/a^2*x/(b*x^2+a)-3/2*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.46032, size = 288, normalized size = 5.05

$$\left[\frac{6bx^2 - 3(bx^3 + ax)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 4a}{4(a^2bx^3 + a^3x)}, -\frac{3bx^2 + 3(bx^3 + ax)\sqrt{\frac{b}{a}} \arctan\left(x\sqrt{\frac{b}{a}}\right) + 2a}{2(a^2bx^3 + a^3x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]

Sympy [A] time = 0.436365, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}} \log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}} \log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{2a + 3bx^2}{2a^3x + 2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**3+a*x)**2,x)

[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)

Giac [A] time = 1.1826, size = 63, normalized size = 1.11

$$-\frac{3b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{aba^2}} - \frac{3bx^2 + 2a}{2(bx^3 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")

[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3 + a*x)*a^2)

3.24

$$\int \frac{1}{x(ax+bx^3)^2} dx$$

Optimal. Leaf size=49

$$-\frac{b}{2a^2(a+bx^2)} + \frac{b \log(a+bx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{1}{2a^2x^2}$$

[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2])/a^3

Rubi [A] time = 0.0390713, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1584, 266, 44}

$$-\frac{b}{2a^2(a+bx^2)} + \frac{b \log(a+bx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3)^2), x]

[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2])/a^3

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
  m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(ax+bx^3)^2} dx &= \int \frac{1}{x^3(a+bx^2)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a+bx)^2} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)} \right) dx, x, x^2 \right) \\
 &= -\frac{1}{2a^2x^2} - \frac{b}{2a^2(a+bx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \log(a+bx^2)}{a^3}
 \end{aligned}$$

Mathematica [A] time = 0.0345072, size = 41, normalized size = 0.84

$$\frac{a \left(\frac{b}{a+bx^2} + \frac{1}{x^2} \right) - 2b \log(a+bx^2) + 4b \log(x)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3)^2), x]

[Out] -(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(2*a^3)

Maple [A] time = 0.012, size = 46, normalized size = 0.9

$$-\frac{1}{2a^2x^2} - \frac{b}{2a^2(bx^2+a)} - 2\frac{b \ln(x)}{a^3} + \frac{b \ln(bx^2+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x)^2, x)

[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3

Maxima [A] time = 1.1098, size = 68, normalized size = 1.39

$$-\frac{2bx^2 + a}{2(a^2bx^4 + a^3x^2)} + \frac{b \log(bx^2 + a)}{a^3} - \frac{2b \log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")

[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x)/a^3

Fricas [A] time = 1.42221, size = 157, normalized size = 3.2

$$\frac{2abx^2 + a^2 - 2(b^2x^4 + abx^2) \log(bx^2 + a) + 4(b^2x^4 + abx^2) \log(x)}{2(a^3bx^4 + a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")

[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 + a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)

Sympy [A] time = 0.51499, size = 49, normalized size = 1.

$$-\frac{a + 2bx^2}{2a^3x^2 + 2a^2bx^4} - \frac{2b \log(x)}{a^3} + \frac{b \log\left(\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a*x)**2,x)

[Out] -(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b + x**2)/a**3

Giac [A] time = 1.19464, size = 69, normalized size = 1.41

$$-\frac{b \log(x^2)}{a^3} + \frac{b \log(|bx^2 + a|)}{a^3} - \frac{2bx^2 + a}{2(bx^4 + ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")

[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a*x^2)*a^2)

$$3.25 \quad \int \frac{1}{x^2(ax+bx^3)^2} dx$$

Optimal. Leaf size=68

$$\frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{5b}{2a^3x} - \frac{5}{6a^2x^3} + \frac{1}{2ax^3(a+bx^2)}$$

[Out] $-5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(7/2)})$

Rubi [A] time = 0.0293549, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1584, 290, 325, 205}

$$\frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{5b}{2a^3x} - \frac{5}{6a^2x^3} + \frac{1}{2ax^3(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3)^2), x]

[Out] $-5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(7/2)})$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```


Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax + bx^3)^2} dx &= \int \frac{1}{x^4(a + bx^2)^2} dx \\
&= \frac{1}{2ax^3(a + bx^2)} + \frac{5 \int \frac{1}{x^4(a + bx^2)} dx}{2a} \\
&= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3(a + bx^2)} - \frac{(5b) \int \frac{1}{x^2(a + bx^2)} dx}{2a^2} \\
&= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3(a + bx^2)} + \frac{(5b^2) \int \frac{1}{a + bx^2} dx}{2a^3} \\
&= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3(a + bx^2)} + \frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0368775, size = 67, normalized size = 0.99

$$\frac{b^2x}{2a^3(a + bx^2)} + \frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3)^2), x]

[Out] $-1/(3a^2x^3) + (2b)/(a^3x) + (b^2x)/(2a^3(a + bx^2)) + (5b^{3/2})\text{ArcTan}[(\text{Sqrt}[b]x)/\text{Sqrt}[a]]/(2a^{7/2})$

Maple [A] time = 0.01, size = 59, normalized size = 0.9

$$-\frac{1}{3x^3a^2} + 2\frac{b}{a^3x} + \frac{b^2x}{2a^3(bx^2 + a)} + \frac{5b^2}{2a^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^3+a*x)^2,x)`

[Out] $-1/3/x^3/a^2+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^{(1/2)*\arctan(b*x/(a*b)^{(1/2)})}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.46605, size = 359, normalized size = 5.28

$$\left[\frac{30b^2x^4 + 20abx^2 + 15(b^2x^5 + abx^3)\sqrt{\frac{b}{a}} \log\left(\frac{bx^2 + 2ax\sqrt{\frac{b}{a}} - a}{bx^2 + a}\right) - 4a^2}{12(a^3bx^5 + a^4x^3)}, \frac{15b^2x^4 + 10abx^2 + 15(b^2x^5 + abx^3)\sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{6(a^3bx^5 + a^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")`

[Out] $[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*\sqrt{-b/a}*\log((b*x^2 + 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*\sqrt{b/a}*\arctan(x*\sqrt{b/a}) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]$

Sympy [A] time = 0.516177, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a*x)**2,x)

[Out] $-5*\sqrt{-b**3/a**7}*\log(-a**4*\sqrt{-b**3/a**7}/b**2 + x)/4 + 5*\sqrt{-b**3/a**7}*\log(a**4*\sqrt{-b**3/a**7}/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)$

Giac [A] time = 1.23994, size = 80, normalized size = 1.18

$$\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{aba^3}} + \frac{b^2x}{2(bx^2+a)a^3} + \frac{6bx^2-a}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")

[Out] $5/2*b^2*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3) + 1/3*(6*b*x^2 - a)/(a^3*x^3)$

3.26

$$\int \frac{x^5}{x-x^3} dx$$

Optimal. Leaf size=13

$$-\frac{x^3}{3} - x + \tanh^{-1}(x)$$

[Out] -x - x^3/3 + ArcTanh[x]

Rubi [A] time = 0.010937, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 302, 206}

$$-\frac{x^3}{3} - x + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^5/(x - x^3), x]

[Out] -x - x^3/3 + ArcTanh[x]

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{x-x^3} dx &= \int \frac{x^4}{1-x^2} dx \\
&= \int \left(-1 - x^2 + \frac{1}{1-x^2} \right) dx \\
&= -x - \frac{x^3}{3} + \int \frac{1}{1-x^2} dx \\
&= -x - \frac{x^3}{3} + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.004208, size = 29, normalized size = 2.23

$$-\frac{x^3}{3} - x - \frac{1}{2} \log(1-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(x - x^3), x]

[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2

Maple [A] time = 0.003, size = 22, normalized size = 1.7

$$-\frac{x^3}{3} - x - \frac{\ln(-1+x)}{2} + \frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-x^3+x), x)

[Out] -1/3*x^3-x-1/2*ln(-1+x)+1/2*ln(1+x)

Maxima [A] time = 1.06704, size = 28, normalized size = 2.15

$$-\frac{1}{3} x^3 - x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")

[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)

Fricas [A] time = 1.50523, size = 65, normalized size = 5.

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")

[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)

Sympy [B] time = 0.093696, size = 19, normalized size = 1.46

$$-\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-x**3+x),x)

[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2

Giac [B] time = 1.19669, size = 31, normalized size = 2.38

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-x^3+x),x, algorithm="giac")

[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))

$$3.27 \quad \int \frac{x^4}{x-x^3} dx$$

Optimal. Leaf size=20

$$-\frac{x^2}{2} - \frac{1}{2} \log(1-x^2)$$

[Out] $-x^2/2 - \text{Log}[1 - x^2]/2$

Rubi [A] time = 0.0151658, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 43}

$$-\frac{x^2}{2} - \frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(x - x^3), x]$

[Out] $-x^2/2 - \text{Log}[1 - x^2]/2$

Rule 1584

$\text{Int}[(u_.)*(x_)^{(m_.)}*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol]$
 $\rightarrow \text{Int}[u*x^{(m+n*p)}*(a+b*x^{(q-p)})^n, x] /;$ $\text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q-p]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n)-1}*(a+b*x)^p, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, m, n, p\}, x]$ $\&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 43

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a+b*x)^m*(c+d*x)^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x]$ $\&\& \text{NeQ}[b*c - a*d, 0]$ $\&\& \text{IGtQ}[m, 0]$ $\&\& (!\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n+1), 0] \parallel \text{GtQ}[m+n+2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{x-x^3} dx &= \int \frac{x^3}{1-x^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{1-x} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-1 + \frac{1}{1-x} \right) dx, x, x^2 \right) \\
&= -\frac{x^2}{2} - \frac{1}{2} \log(1-x^2)
\end{aligned}$$

Mathematica [A] time = 0.0027918, size = 18, normalized size = 0.9

$$-\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(x - x^3), x]

[Out] -x^2/2 - Log[-1 + x^2]/2

Maple [A] time = 0.002, size = 19, normalized size = 1.

$$-\frac{x^2}{2} - \frac{\ln(-1+x)}{2} - \frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^3+x), x)

[Out] -1/2*x^2-1/2*ln(-1+x)-1/2*ln(1+x)

Maxima [A] time = 1.03579, size = 24, normalized size = 1.2

$$-\frac{1}{2}x^2 - \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^3+x),x, algorithm="maxima")`

[Out] $-1/2*x^2 - 1/2*\log(x + 1) - 1/2*\log(x - 1)$

Fricas [A] time = 1.56051, size = 39, normalized size = 1.95

$$-\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^3+x),x, algorithm="fricas")`

[Out] $-1/2*x^2 - 1/2*\log(x^2 - 1)$

Sympy [A] time = 0.075149, size = 14, normalized size = 0.7

$$-\frac{x^2}{2} - \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-x**3+x),x)`

[Out] $-x**2/2 - \log(x**2 - 1)/2$

Giac [A] time = 1.32408, size = 20, normalized size = 1.

$$-\frac{1}{2}x^2 - \frac{1}{2}\log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^3+x),x, algorithm="giac")`

[Out] $-1/2*x^2 - 1/2*\log(\text{abs}(x^2 - 1))$

3.28

$$\int \frac{x^3}{x-x^3} dx$$

Optimal. Leaf size=6

$$\tanh^{-1}(x) - x$$

[Out] -x + ArcTanh[x]

Rubi [A] time = 0.0080615, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 321, 206}

$$\tanh^{-1}(x) - x$$

Antiderivative was successfully verified.

[In] Int[x^3/(x - x^3), x]

[Out] -x + ArcTanh[x]

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{x-x^3} dx &= \int \frac{x^2}{1-x^2} dx \\ &= -x + \int \frac{1}{1-x^2} dx \\ &= -x + \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0030891, size = 22, normalized size = 3.67

$$-x - \frac{1}{2} \log(1-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(x - x^3),x]

[Out] -x - Log[1 - x]/2 + Log[1 + x]/2

Maple [B] time = 0.001, size = 17, normalized size = 2.8

$$-x - \frac{\ln(-1+x)}{2} + \frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-x^3+x),x)

[Out] -x-1/2*ln(-1+x)+1/2*ln(1+x)

Maxima [B] time = 1.0418, size = 22, normalized size = 3.67

$$-x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")

[Out] $-x + 1/2*\log(x + 1) - 1/2*\log(x - 1)$

Fricas [B] time = 1.68595, size = 51, normalized size = 8.5

$$-x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(-x^3+x),x, algorithm="fricas")`

[Out] $-x + 1/2*\log(x + 1) - 1/2*\log(x - 1)$

Sympy [B] time = 0.083079, size = 14, normalized size = 2.33

$$-x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(-x**3+x),x)`

[Out] $-x - \log(x - 1)/2 + \log(x + 1)/2$

Giac [B] time = 1.23107, size = 24, normalized size = 4.

$$-x + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(-x^3+x),x, algorithm="giac")`

[Out] $-x + 1/2*\log(\text{abs}(x + 1)) - 1/2*\log(\text{abs}(x - 1))$

$$3.29 \quad \int \frac{x^2}{x-x^3} dx$$

Optimal. Leaf size=12

$$-\frac{1}{2} \log(1-x^2)$$

[Out] -Log[1 - x^2]/2

Rubi [A] time = 0.0063917, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1584, 260}

$$-\frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Int[x^2/(x - x^3), x]

[Out] -Log[1 - x^2]/2

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{x-x^3} dx &= \int \frac{x}{1-x^2} dx \\ &= -\frac{1}{2} \log(1-x^2) \end{aligned}$$

Mathematica [A] time = 0.0018832, size = 12, normalized size = 1.

$$-\frac{1}{2} \log(1 - x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(x - x^3), x]

[Out] -Log[1 - x^2]/2

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$-\frac{\ln(-1 + x)}{2} - \frac{\ln(1 + x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^3+x), x)

[Out] -1/2*ln(-1+x)-1/2*ln(1+x)

Maxima [A] time = 1.0234, size = 18, normalized size = 1.5

$$-\frac{1}{2} \log(x + 1) - \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^3+x), x, algorithm="maxima")

[Out] -1/2*log(x + 1) - 1/2*log(x - 1)

Fricas [A] time = 1.60973, size = 26, normalized size = 2.17

$$-\frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 1)
```

Sympy [A] time = 0.074027, size = 8, normalized size = 0.67

$$\frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+x),x)
```

```
[Out] -log(x**2 - 1)/2
```

Giac [A] time = 1.27095, size = 20, normalized size = 1.67

$$-\frac{1}{2} \log(|x + 1|) - \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.30

$$\int \frac{x}{x-x^3} dx$$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

[Out] ArcTanh[x]

Rubi [A] time = 0.0039866, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1584, 206}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/(x - x^3), x]

[Out] ArcTanh[x]

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{x-x^3} dx &= \int \frac{1}{1-x^2} dx \\ &= \tanh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.0020419, size = 19, normalized size = 9.5

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[x/(x - x^3), x]

[Out] -Log[1 - x]/2 + Log[1 + x]/2

Maple [A] time = 0.001, size = 3, normalized size = 1.5

Artanh(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+x), x)

[Out] arctanh(x)

Maxima [B] time = 1.07402, size = 18, normalized size = 9.

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^3+x), x, algorithm="maxima")

[Out] 1/2*log(x + 1) - 1/2*log(x - 1)

Fricas [B] time = 1.60764, size = 45, normalized size = 22.5

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] time = 0.082663, size = 12, normalized size = 6.

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Giac [B] time = 1.23626, size = 20, normalized size = 10.

$$\frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

$$3.31 \quad \int \frac{1}{x-x^3} dx$$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2} \log(1-x^2)$$

[Out] Log[x] - Log[1 - x^2]/2

Rubi [A] time = 0.0082264, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1593, 266, 36, 31, 29}

$$\log(x) - \frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Int[(x - x^3)^(-1), x]

[Out] Log[x] - Log[1 - x^2]/2

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^( -1), x_Symbol] := Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x-x^3} dx &= \int \frac{1}{x(1-x^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right) \\ &= \log(x) - \frac{1}{2} \log(1-x^2) \end{aligned}$$

Mathematica [A] time = 0.0024746, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Maple [A] time = 0.006, size = 16, normalized size = 1.1

$$\ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x),x)
```

[Out] $\ln(x) - 1/2 \ln(1+x) - 1/2 \ln(-1+x)$

Maxima [A] time = 1.02813, size = 20, normalized size = 1.33

$$-\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^3+x),x, algorithm="maxima")`

[Out] $-1/2 \log(x+1) - 1/2 \log(x-1) + \log(x)$

Fricas [A] time = 1.44473, size = 38, normalized size = 2.53

$$-\frac{1}{2} \log(x^2-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^3+x),x, algorithm="fricas")`

[Out] $-1/2 \log(x^2-1) + \log(x)$

Sympy [A] time = 0.087841, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log(x^2-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**3+x),x)`

[Out] $\log(x) - \log(x**2-1)/2$

Giac [A] time = 1.22125, size = 22, normalized size = 1.47

$$\frac{1}{2} \log(x^2) - \frac{1}{2} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

$$3.32 \quad \int \frac{1}{x(x-x^3)} dx$$

Optimal. Leaf size=8

$$\tanh^{-1}(x) - \frac{1}{x}$$

[Out] $-x^{-1} + \text{ArcTanh}[x]$

Rubi [A] time = 0.007941, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 325, 206}

$$\tanh^{-1}(x) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*(x - x^3)), x]$

[Out] $-x^{-1} + \text{ArcTanh}[x]$

Rule 1584

$\text{Int}[(u_.)*(x_)^{(m_.)}*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol]$
 $:\> \text{Int}[u*x^{(m + n*p)}*(a + b*x^{(q - p)})^n, x] /;$ $\text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

Rule 325

$\text{Int}[((c_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] :\> \text{Simp}[((c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*c*(m + 1)), x] - \text{Dist}[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), \text{Int}[(c*x)^{(m + n)}*(a + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 206

$\text{Int}[((a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] :\> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\| \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}\int \frac{1}{x(x-x^3)} dx &= \int \frac{1}{x^2(1-x^2)} dx \\ &= -\frac{1}{x} + \int \frac{1}{1-x^2} dx \\ &= -\frac{1}{x} + \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0033792, size = 24, normalized size = 3.

$$-\frac{1}{x} - \frac{1}{2} \log(1-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(x - x^3)),x]

[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2

Maple [B] time = 0.005, size = 19, normalized size = 2.4

$$\frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2} - x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+x),x)

[Out] 1/2*ln(1+x)-1/2*ln(-1+x)-1/x

Maxima [B] time = 1.0997, size = 24, normalized size = 3.

$$-\frac{1}{x} + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")

[Out] $-1/x + 1/2*\log(x + 1) - 1/2*\log(x - 1)$

Fricas [B] time = 1.43392, size = 55, normalized size = 6.88

$$\frac{x \log(x + 1) - x \log(x - 1) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")

[Out] $1/2*(x*\log(x + 1) - x*\log(x - 1) - 2)/x$

Sympy [B] time = 0.093039, size = 15, normalized size = 1.88

$$-\frac{\log(x - 1)}{2} + \frac{\log(x + 1)}{2} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**3+x),x)

[Out] $-\log(x - 1)/2 + \log(x + 1)/2 - 1/x$

Giac [B] time = 1.19293, size = 27, normalized size = 3.38

$$-\frac{1}{x} + \frac{1}{2} \log(|x + 1|) - \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^3+x),x, algorithm="giac")

[Out] $-1/x + 1/2*\log(\text{abs}(x + 1)) - 1/2*\log(\text{abs}(x - 1))$

3.33

$$\int \frac{1}{x^2(x-x^3)} dx$$

Optimal. Leaf size=22

$$-\frac{1}{2x^2} - \frac{1}{2} \log(1-x^2) + \log(x)$$

[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2

Rubi [A] time = 0.0146911, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2} - \frac{1}{2} \log(1-x^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(x - x^3)),x]

[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
  & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
  + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(x-x^3)} dx &= \int \frac{1}{x^3(1-x^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{1-x} + \frac{1}{x^2} + \frac{1}{x} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \log(x) - \frac{1}{2} \log(1-x^2)
\end{aligned}$$

Mathematica [A] time = 0.0034872, size = 22, normalized size = 1.

$$-\frac{1}{2x^2} - \frac{1}{2} \log(1-x^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(x - x^3)),x]

[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2

Maple [A] time = 0.007, size = 21, normalized size = 1.

$$-\frac{1}{2x^2} + \ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-x^3+x),x)

[Out] -1/2/x^2+ln(x)-1/2*ln(1+x)-1/2*ln(-1+x)

Maxima [A] time = 1.04656, size = 27, normalized size = 1.23

$$-\frac{1}{2x^2} - \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")

[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)

Fricas [A] time = 1.385, size = 65, normalized size = 2.95

$$-\frac{x^2 \log(x^2 - 1) - 2x^2 \log(x) + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")

[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2

Sympy [A] time = 0.096854, size = 17, normalized size = 0.77

$$\log(x) - \frac{\log(x^2 - 1)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(-x**3+x),x)

[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)

Giac [A] time = 1.27046, size = 35, normalized size = 1.59

$$-\frac{x^2 + 1}{2x^2} + \frac{1}{2} \log(x^2) - \frac{1}{2} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")

[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))

$$3.34 \quad \int \frac{1}{x^3(x-x^3)} dx$$

Optimal. Leaf size=15

$$-\frac{1}{3x^3} - \frac{1}{x} + \tanh^{-1}(x)$$

[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]

Rubi [A] time = 0.0097331, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 325, 206}

$$-\frac{1}{3x^3} - \frac{1}{x} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(x - x^3)), x]

[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 325

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(x-x^3)} dx &= \int \frac{1}{x^4(1-x^2)} dx \\
&= -\frac{1}{3x^3} + \int \frac{1}{x^2(1-x^2)} dx \\
&= -\frac{1}{3x^3} - \frac{1}{x} + \int \frac{1}{1-x^2} dx \\
&= -\frac{1}{3x^3} - \frac{1}{x} + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.0036898, size = 31, normalized size = 2.07

$$-\frac{1}{3x^3} - \frac{1}{x} - \frac{1}{2} \log(1-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(x - x^3)),x]

[Out] -1/(3*x^3) - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2

Maple [A] time = 0.005, size = 24, normalized size = 1.6

$$-\frac{1}{3x^3} - x^{-1} + \frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-x^3+x),x)

[Out] -1/3/x^3-1/x+1/2*ln(1+x)-1/2*ln(-1+x)

Maxima [A] time = 1.09703, size = 34, normalized size = 2.27

$$-\frac{3x^2+1}{3x^3} + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(-x^3+x),x, algorithm="maxima")`

[Out] $-1/3*(3*x^2 + 1)/x^3 + 1/2*\log(x + 1) - 1/2*\log(x - 1)$

Fricas [B] time = 1.40575, size = 80, normalized size = 5.33

$$\frac{3x^3 \log(x+1) - 3x^3 \log(x-1) - 6x^2 - 2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(-x^3+x),x, algorithm="fricas")`

[Out] $1/6*(3*x^3*\log(x + 1) - 3*x^3*\log(x - 1) - 6*x^2 - 2)/x^3$

Sympy [A] time = 0.108128, size = 24, normalized size = 1.6

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{3x^2+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-x**3+x),x)`

[Out] $-\log(x - 1)/2 + \log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)$

Giac [A] time = 1.23086, size = 36, normalized size = 2.4

$$-\frac{3x^2+1}{3x^3} + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(-x^3+x),x, algorithm="giac")`

[Out] $-1/3*(3*x^2 + 1)/x^3 + 1/2*\log(\text{abs}(x + 1)) - 1/2*\log(\text{abs}(x - 1))$

$$3.35 \quad \int \frac{1}{x^4(x-x^3)} dx$$

Optimal. Leaf size=29

$$-\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^2) + \log(x)$$

[Out] $-1/(4*x^4) - 1/(2*x^2) + \text{Log}[x] - \text{Log}[1 - x^2]/2$

Rubi [A] time = 0.0160911, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^2) + \log(x)$$

Antiderivative was successfully verified.

[In] `Int[1/(x^4*(x - x^3)),x]`

[Out] $-1/(4*x^4) - 1/(2*x^2) + \text{Log}[x] - \text{Log}[1 - x^2]/2$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  > Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] > Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] > Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
  & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
  + n + 2, 0])
```


Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(x-x^3)} dx &= \int \frac{1}{x^5(1-x^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x^3} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{1-x} + \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} \right) dx, x, x^2 \right) \\
&= -\frac{1}{4x^4} - \frac{1}{2x^2} + \log(x) - \frac{1}{2} \log(1-x^2)
\end{aligned}$$

Mathematica [A] time = 0.0034672, size = 29, normalized size = 1.

$$-\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(x - x^3)),x]

[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2

Maple [A] time = 0.007, size = 26, normalized size = 0.9

$$-\frac{1}{4x^4} - \frac{1}{2x^2} + \ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-x^3+x),x)

[Out] -1/4/x^4-1/2/x^2+ln(x)-1/2*ln(1+x)-1/2*ln(-1+x)

Maxima [A] time = 1.14142, size = 36, normalized size = 1.24

$$-\frac{2x^2+1}{4x^4} - \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")

[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)

Fricas [A] time = 1.40528, size = 78, normalized size = 2.69

$$-\frac{2x^4 \log(x^2 - 1) - 4x^4 \log(x) + 2x^2 + 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")

[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4

Sympy [A] time = 0.111545, size = 22, normalized size = 0.76

$$\log(x) - \frac{\log(x^2 - 1)}{2} - \frac{2x^2 + 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-x**3+x),x)

[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)

Giac [A] time = 1.23591, size = 45, normalized size = 1.55

$$-\frac{3x^4 + 2x^2 + 1}{4x^4} + \frac{1}{2} \log(x^2) - \frac{1}{2} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")

[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))

$$3.36 \quad \int \frac{1}{x+bx^3} dx$$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

[Out] Log[x] - Log[1 + b*x^2]/2

Rubi [A] time = 0.0094817, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1593, 266, 36, 29, 31}

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(x + b*x^3)^(-1), x]

[Out] Log[x] - Log[1 + b*x^2]/2

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x+bx^3} dx &= \int \frac{1}{x(1+bx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(1+bx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right) - \frac{1}{2} b \text{Subst} \left(\int \frac{1}{1+bx} dx, x, x^2 \right) \\ &= \log(x) - \frac{1}{2} \log(1+bx^2) \end{aligned}$$

Mathematica [A] time = 0.0031799, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + b*x^3)^(-1), x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\ln(x) - \frac{\ln(bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+x), x)
```

[Out] $\ln(x) - \frac{1}{2} \ln(bx^2 + 1)$

Maxima [A] time = 1.03523, size = 18, normalized size = 1.2

$$-\frac{1}{2} \log(bx^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+x),x, algorithm="maxima")`

[Out] $-\frac{1}{2} \log(bx^2 + 1) + \log(x)$

Fricas [A] time = 1.38484, size = 41, normalized size = 2.73

$$-\frac{1}{2} \log(bx^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+x),x, algorithm="fricas")`

[Out] $-\frac{1}{2} \log(bx^2 + 1) + \log(x)$

Sympy [A] time = 0.123091, size = 12, normalized size = 0.8

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+x),x)`

[Out] $\log(x) - \log(x^2 + 1/b)/2$

Giac [A] time = 1.21529, size = 24, normalized size = 1.6

$$\frac{1}{2} \log(x^2) - \frac{1}{2} \log(|bx^2 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

$$3.37 \quad \int \frac{1}{-x+bx^3} dx$$

Optimal. Leaf size=18

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

[Out] -Log[x] + Log[1 - b*x^2]/2

Rubi [A] time = 0.0102575, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-x + b*x^3)^(-1), x]

[Out] -Log[x] + Log[1 - b*x^2]/2

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-x + bx^3} dx &= \int \frac{1}{x(-1 + bx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(-1 + bx)} dx, x, x^2 \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right) \right) + \frac{1}{2} b \text{Subst} \left(\int \frac{1}{-1 + bx} dx, x, x^2 \right) \\ &= -\log(x) + \frac{1}{2} \log(1 - bx^2) \end{aligned}$$

Mathematica [A] time = 0.0034299, size = 18, normalized size = 1.

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + b*x^3)^(-1), x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.9

$$-\ln(x) + \frac{\ln(bx^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3-x), x)
```


[Out] $-\ln(x) + 1/2 * \ln(b * x^2 - 1)$

Maxima [A] time = 1.05053, size = 20, normalized size = 1.11

$$\frac{1}{2} \log(bx^2 - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3-x),x, algorithm="maxima")`

[Out] $1/2 * \log(b * x^2 - 1) - \log(x)$

Fricas [A] time = 1.40847, size = 39, normalized size = 2.17

$$\frac{1}{2} \log(bx^2 - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3-x),x, algorithm="fricas")`

[Out] $1/2 * \log(b * x^2 - 1) - \log(x)$

Sympy [A] time = 0.122316, size = 12, normalized size = 0.67

$$-\log(x) + \frac{\log\left(x^2 - \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3-x),x)`

[Out] $-\log(x) + \log(x**2 - 1/b)/2$

Giac [A] time = 1.23484, size = 24, normalized size = 1.33

$$-\frac{1}{2} \log(x^2) + \frac{1}{2} \log(|bx^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

3.38 $\int x^3 \sqrt{ax + bx^3} dx$

Optimal. Leaf size=163

$$\frac{10a^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{231b^{9/4}\sqrt{ax + bx^3}} - \frac{20a^2\sqrt{ax + bx^3}}{231b^2} + \frac{2}{11}x^4\sqrt{ax + bx^3} + \frac{4ax^2\sqrt{ax + bx^3}}{77b}$$

[Out] $(-20*a^2*\text{Sqrt}[a*x + b*x^3])/(231*b^2) + (4*a*x^2*\text{Sqrt}[a*x + b*x^3])/(77*b) + (2*x^4*\text{Sqrt}[a*x + b*x^3])/11 + (10*a^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(231*b^{(9/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.188378, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{20a^2\sqrt{ax + bx^3}}{231b^2} + \frac{10a^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{ax + bx^3}} + \frac{2}{11}x^4\sqrt{ax + bx^3} + \frac{4ax^2\sqrt{ax + bx^3}}{77b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{Sqrt}[a*x + b*x^3], x]$

[Out] $(-20*a^2*\text{Sqrt}[a*x + b*x^3])/(231*b^2) + (4*a*x^2*\text{Sqrt}[a*x + b*x^3])/(77*b) + (2*x^4*\text{Sqrt}[a*x + b*x^3])/11 + (10*a^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(231*b^{(9/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2021

$\text{Int}[(c*x)^m*(a*x^j + b*x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c*x)^{m+1}*(a*x^j + b*x^n)^p/(c*(m+n*p+1)), x] + \text{Dist}[(a*(n-j)*p)/(c^j*(m+n*p+1)), \text{Int}[(c*x)^{m+j}*(a*x^j + b*x^n)^{p-1}, x], x] /;$ FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2024

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{ax + bx^3} dx &= \frac{2}{11} x^4 \sqrt{ax + bx^3} + \frac{1}{11} (2a) \int \frac{x^4}{\sqrt{ax + bx^3}} dx \\
&= \frac{4ax^2 \sqrt{ax + bx^3}}{77b} + \frac{2}{11} x^4 \sqrt{ax + bx^3} - \frac{(10a^2) \int \frac{x^2}{\sqrt{ax + bx^3}} dx}{77b} \\
&= -\frac{20a^2 \sqrt{ax + bx^3}}{231b^2} + \frac{4ax^2 \sqrt{ax + bx^3}}{77b} + \frac{2}{11} x^4 \sqrt{ax + bx^3} + \frac{(10a^3) \int \frac{1}{\sqrt{ax + bx^3}} dx}{231b^2} \\
&= -\frac{20a^2 \sqrt{ax + bx^3}}{231b^2} + \frac{4ax^2 \sqrt{ax + bx^3}}{77b} + \frac{2}{11} x^4 \sqrt{ax + bx^3} + \frac{(10a^3 \sqrt{x} \sqrt{a + bx^2}) \int \frac{1}{\sqrt{x} \sqrt{a + bx^2}} dx}{231b^2 \sqrt{ax + bx^3}} \\
&= -\frac{20a^2 \sqrt{ax + bx^3}}{231b^2} + \frac{4ax^2 \sqrt{ax + bx^3}}{77b} + \frac{2}{11} x^4 \sqrt{ax + bx^3} + \frac{(20a^3 \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^4}} dx, \right)}{231b^2 \sqrt{ax + bx^3}} \\
&= -\frac{20a^2 \sqrt{ax + bx^3}}{231b^2} + \frac{4ax^2 \sqrt{ax + bx^3}}{77b} + \frac{2}{11} x^4 \sqrt{ax + bx^3} + \frac{10a^{11/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F \left(2t \right)}{231b^{9/4} \sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0504627, size = 95, normalized size = 0.58

$$\frac{2\sqrt{x(a+bx^2)} \left(\sqrt{\frac{bx^2}{a}} + 1 (-5a^2 + 2abx^2 + 7b^2x^4) + 5a^2 {}_2F_1 \left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a} \right) \right)}{77b^2 \sqrt{\frac{bx^2}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*sqrt[a*x + b*x^3],x]

[Out] (2*sqrt[x*(a + b*x^2)]*(sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.018, size = 168, normalized size = 1.

$$\frac{2x^4}{11} \sqrt{bx^3 + ax} + \frac{4ax^2}{77b} \sqrt{bx^3 + ax} - \frac{20a^2}{231b^2} \sqrt{bx^3 + ax} + \frac{10a^3}{231b^3} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^3+a*x)^(1/2),x)`

[Out] $2/11*x^4*(b*x^3+a*x)^{(1/2)}+4/77*a*x^2*(b*x^3+a*x)^{(1/2)}/b-20/231*a^2*(b*x^3+a*x)^{(1/2)}/b^2+10/231*a^3/b^3*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^3 + a*x)*x^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{x(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**3+a*x)**(1/2),x)`

```
[Out] Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

3.39 $\int x^2 \sqrt{ax + bx^3} dx$

Optimal. Leaf size=281

$$\frac{2a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{7/4}\sqrt{ax + bx^3}} - \frac{4a^2x(a + bx^2)}{15b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{4a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{15b^{7/4}\sqrt{ax + bx^3}}$$

[Out] $(-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])$
 $+ (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]$
 $*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x + b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x + b*x^3])$

Rubi [A] time = 0.26363, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4a^2x(a + bx^2)}{15b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{2a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15b^{7/4}\sqrt{ax + bx^3}} + \frac{4a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}}{15b^{7/4}\sqrt{ax + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a*x + b*x^3], x]

[Out] $(-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])$
 $+ (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]$
 $*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x + b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x + b*x^3])$

Rule 2021

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a

$(n - j)p / (c^j(m + np + 1))$, $\text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x]$, x /; $\text{FreeQ}\{a, b, c, m\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \mid\mid \text{GtQ}[c, 0])$ && $\text{GtQ}[p, 0]$ && $\text{NeQ}[m + np + 1, 0]$

Rule 2024

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$ $\rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)})/(b*(m+np+1)), x] - \text{Dist}[(a*c^{(n-j)}*(m+j*p-n+j+1))/(b*(m+np+1)), \text{Int}[(c*x)^{(m-(n-j))}*(a*x^j + b*x^n)^p, x], x]$ /; $\text{FreeQ}\{a, b, c, m, p\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \mid\mid \text{GtQ}[c, 0])$ && $\text{GtQ}[m + j*p + 1 - n + j, 0]$ && $\text{NeQ}[m + np + 1, 0]$

Rule 2032

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$ $\rightarrow \text{Dist}[c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]}/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n-j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m+j*p)}*(a + b*x^{(n-j)})^p, x], x]$ /; $\text{FreeQ}\{a, b, c, j, m, n, p\}, x$ && $\text{!IntegerQ}[p]$ && $\text{NeQ}[n, j]$ && $\text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$ $\rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]$ /; $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{FractractionQ}[m]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

$\text{Int}[x^2/\text{Sqrt}[a_*) + (b_*)(x_*)^4], x_Symbol]$ $\rightarrow \text{With}\{q = \text{Rt}[b/a, 2]\}, \text{Dist}[1/q, \text{Int}[1/\text{Sqrt}[a + b*x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^4], x], x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[b/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[a_*) + (b_*)(x_*)^4], x_Symbol]$ $\rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^4]/(a*(1 + q^2*x^2)^2)*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\text{Sqrt}[a + b*x^4]), x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_*) + (e_*)(x_*)^2/\text{Sqrt}[a_*) + (c_*)(x_*)^4], x_Symbol]$ $\rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[a + c*x^4]/(a*(1 + q^2*x^2)^2)*\text{EllipticE}[2*\text{ArcTan}[q*x],$

1/2]]/(q*sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{ax + bx^3} dx &= \frac{2}{9} x^3 \sqrt{ax + bx^3} + \frac{1}{9} (2a) \int \frac{x^3}{\sqrt{ax + bx^3}} dx \\
 &= \frac{4ax \sqrt{ax + bx^3}}{45b} + \frac{2}{9} x^3 \sqrt{ax + bx^3} - \frac{(2a^2) \int \frac{x}{\sqrt{ax + bx^3}} dx}{15b} \\
 &= \frac{4ax \sqrt{ax + bx^3}}{45b} + \frac{2}{9} x^3 \sqrt{ax + bx^3} - \frac{(2a^2 \sqrt{x} \sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{15b \sqrt{ax + bx^3}} \\
 &= \frac{4ax \sqrt{ax + bx^3}}{45b} + \frac{2}{9} x^3 \sqrt{ax + bx^3} - \frac{(4a^2 \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{15b \sqrt{ax + bx^3}} \\
 &= \frac{4ax \sqrt{ax + bx^3}}{45b} + \frac{2}{9} x^3 \sqrt{ax + bx^3} - \frac{(4a^{5/2} \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{15b^{3/2} \sqrt{ax + bx^3}} + \frac{(4a^{5/2} \sqrt{x} \sqrt{a + bx^2})}{15b^{3/2} \sqrt{ax + bx^3}} \\
 &= -\frac{4a^2 x (a + bx^2)}{15b^{3/2} (\sqrt{a} + \sqrt{bx}) \sqrt{ax + bx^3}} + \frac{4ax \sqrt{ax + bx^3}}{45b} + \frac{2}{9} x^3 \sqrt{ax + bx^3} + \frac{4a^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})}}}{15b^{7/4} \sqrt{ax}}
 \end{aligned}$$

Mathematica [C] time = 0.0315518, size = 80, normalized size = 0.28

$$\frac{2x \sqrt{x(a + bx^2)} \left((a + bx^2) \sqrt{\frac{bx^2}{a} + 1} - a {}_2F_1 \left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a} \right) \right)}{9b \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*sqrt[a*x + b*x^3], x]

[Out] (2*x*sqrt[x*(a + b*x^2)]*((a + b*x^2)*sqrt[1 + (b*x^2)/a] - a*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.011, size = 197, normalized size = 0.7

$$\frac{2x^3}{9}\sqrt{bx^3+ax} + \frac{4ax}{45b}\sqrt{bx^3+ax} - \frac{2a^2}{15b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-ab}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a*x)^(1/2),x)

[Out] $\frac{2}{9}x^3(bx^3+ax)^{1/2} + \frac{4}{45}ax(bx^3+ax)^{1/2}/b - \frac{2}{15}a^2(-ab)^{1/2}(1/2)*((x+1/b*(-ab)^{1/2})*b/(-ab)^{1/2})^{1/2}*(-2*(x-1/b*(-ab)^{1/2})*b/(-ab)^{1/2})^{1/2}*(-x*b/(-ab)^{1/2})^{1/2}/(bx^3+ax)^{1/2}*(-2/b*(-ab)^{1/2})*\text{EllipticE}(((x+1/b*(-ab)^{1/2})*b/(-ab)^{1/2})^{1/2}, 1/2*2^{1/2}) + 1/b*(-ab)^{1/2}*\text{EllipticF}(((x+1/b*(-ab)^{1/2})*b/(-ab)^{1/2})^{1/2}, 1/2*2^{1/2}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a*x)*x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a*x)**(1/2),x)

[Out] Integral(x**2*sqrt(x*(a + b*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)

3.40 $\int x\sqrt{ax + bx^3} dx$

Optimal. Leaf size=137

$$\frac{2a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{21b^{5/4}\sqrt{ax + bx^3}} + \frac{2}{7}x^2\sqrt{ax + bx^3} + \frac{4a\sqrt{ax + bx^3}}{21b}$$

[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.121844, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2021, 2024, 2011, 329, 220}

$$\frac{2a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax + bx^3}} + \frac{2}{7}x^2\sqrt{ax + bx^3} + \frac{4a\sqrt{ax + bx^3}}{21b}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a*x + b*x^3], x]

[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x^3])

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2024

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax+bx^3} dx &= \frac{2}{7}x^2\sqrt{ax+bx^3} + \frac{1}{7}(2a) \int \frac{x^2}{\sqrt{ax+bx^3}} dx \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{(2a^2) \int \frac{1}{\sqrt{ax+bx^3}} dx}{21b} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{(2a^2\sqrt{x}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{21b\sqrt{ax+bx^3}} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{(4a^2\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{21b\sqrt{ax+bx^3}} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{2a^{7/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0268107, size = 79, normalized size = 0.58

$$\frac{2\sqrt{x(a+bx^2)}\left((a+bx^2)\sqrt{\frac{bx^2}{a}+1} - a {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)\right)}{7b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a*x + b*x^3], x]

[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric2F1[-1/2, 1/4, 5/4, -(b*x^2)/a]))/(7*b*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.012, size = 146, normalized size = 1.1

$$\frac{2x^2}{7}\sqrt{bx^3+ax} + \frac{4a}{21b}\sqrt{bx^3+ax} - \frac{2a^2}{21b^2}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x)^(1/2), x)

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21/b^2*a^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^3 + a*x)*x, x)

3.41 $\int \sqrt{ax + bx^3} dx$

Optimal. Leaf size=255

$$\frac{2a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5b^{3/4}\sqrt{ax + bx^3}} - \frac{4a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax + bx^3}}$$

[Out] $(4*a*x*(a + b*x^2))/(5*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) + (2*x*\text{Sqrt}[a*x + b*x^3])/5 - (4*a^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (2*a^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.172987, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2004, 2032, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax + bx^3}} - \frac{4a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax + bx^3}} + \frac{1}{5\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3], x]

[Out] $(4*a*x*(a + b*x^2))/(5*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) + (2*x*\text{Sqrt}[a*x + b*x^3])/5 - (4*a^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (2*a^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2004

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n

] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]], Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \sqrt{ax + bx^3} dx &= \frac{2}{5}x\sqrt{ax + bx^3} + \frac{1}{5}(2a) \int \frac{x}{\sqrt{ax + bx^3}} dx \\
&= \frac{2}{5}x\sqrt{ax + bx^3} + \frac{(2a\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{5\sqrt{ax + bx^3}} \\
&= \frac{2}{5}x\sqrt{ax + bx^3} + \frac{(4a\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5\sqrt{ax + bx^3}} \\
&= \frac{2}{5}x\sqrt{ax + bx^3} + \frac{(4a^{3/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{ax + bx^3}} - \frac{(4a^{3/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1-\frac{\sqrt{b}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{ax + bx^3}} \\
&= \frac{4ax(a + bx^2)}{5\sqrt{b}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{2}{5}x\sqrt{ax + bx^3} - \frac{4a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0106898, size = 51, normalized size = 0.2

$$\frac{2x\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3], x]

[Out] (2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/(3*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.011, size = 175, normalized size = 0.7

$$\frac{2x}{5}\sqrt{bx^3 + ax} + \frac{2a}{5b}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-ab}}{b}\text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}}\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(1/2), x)

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)+2/5*a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^3 + a*x), x)

$$3.42 \quad \int \frac{\sqrt{ax+bx^3}}{x} dx$$

Optimal. Leaf size=113

$$\frac{2a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{2}{3}\sqrt{ax+bx^3}$$

[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.0857818, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2021, 2011, 329, 220}

$$\frac{2a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{2}{3}\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3]/x, x]

[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x

$(j*p)*(a + b*x^(n - j))^p, x]$, x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax + bx^3}}{x} dx &= \frac{2}{3} \sqrt{ax + bx^3} + \frac{1}{3} (2a) \int \frac{1}{\sqrt{ax + bx^3}} dx \\ &= \frac{2}{3} \sqrt{ax + bx^3} + \frac{(2a\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{3\sqrt{ax + bx^3}} \\ &= \frac{2}{3} \sqrt{ax + bx^3} + \frac{(4a\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{3\sqrt{ax + bx^3}} \\ &= \frac{2}{3} \sqrt{ax + bx^3} + \frac{2a^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0113656, size = 48, normalized size = 0.42

$$\frac{2\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3]/x, x]

[Out] $(2\sqrt{x(a + bx^2)} \text{Hypergeometric2F1}[-1/2, 1/4, 5/4, -(bx^2/a)]) / \sqrt{1 + (bx^2/a)}$

Maple [A] time = 0.012, size = 124, normalized size = 1.1

$$\frac{2}{3}\sqrt{bx^3 + ax} + \frac{2a}{3b}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((bx^3+ax)^{(1/2)}/x, x)$

[Out] $2/3*(bx^3+ax)^{(1/2)}+2/3*a/b*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(bx^3+ax)^{(1/2)}*\text{EllipticF}(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((bx^3+ax)^{(1/2)}/x, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\text{sqrt}(bx^3 + a*x)/x, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((bx^3+ax)^{(1/2)}/x, x, \text{algorithm}="fricas")$

[Out] `integral(sqrt(b*x^3 + a*x)/x, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a + bx^2)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**(1/2)/x,x)`

[Out] `Integral(sqrt(x*(a + b*x**2))/x, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^3 + a*x)/x, x)`

$$3.43 \quad \int \frac{\sqrt{ax+bx^3}}{x^2} dx$$

Optimal. Leaf size=248

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{\sqrt{ax+bx^3}} + \frac{4\sqrt{bx}(a+bx^2)}{(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{\sqrt{ax+bx^3}}$$

[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sqrt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]

Rubi [A] time = 0.201338, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{4\sqrt{bx}(a+bx^2)}{(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} + \frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{\sqrt{ax+bx^3}} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3]/x^2, x]

[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sqrt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*x*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),

$x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m + j*p + 1, 0]$

Rule 2032

$\text{Int}[(c \cdot x^m) \cdot ((a \cdot x^j) + (b \cdot x^n)^p), x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]} \cdot (c \cdot x)^{\text{FracPart}[m]} \cdot (a \cdot x^j + b \cdot x^n)^{\text{FracPart}[p]}) / (x^{\text{FracPart}[m] + j \cdot \text{FracPart}[p]} \cdot (a + b \cdot x^{n-j})^{\text{FracPart}[p]})], \text{Int}[x^{m+j*p} \cdot (a + b \cdot x^{n-j})^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n-j]$

Rule 329

$\text{Int}[(c \cdot x^m) \cdot ((a) + (b \cdot x^n)^p), x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k(m+1)-1} \cdot (a + (b \cdot x^{k \cdot n})) / c^n]^p, x], x, (c \cdot x)^{1/k}], x]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

$\text{Int}[x^2 / \text{Sqrt}[a + (b \cdot x)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 2]\}, \text{Dist}[1/q, \text{Int}[1 / \text{Sqrt}[a + b \cdot x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q \cdot x^2) / \text{Sqrt}[a + b \cdot x^4], x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rule 220

$\text{Int}[1 / \text{Sqrt}[a + (b \cdot x)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2 \cdot x^2) \cdot \text{Sqrt}[a + b \cdot x^4] / (a \cdot (1 + q^2 \cdot x^2)^2)] \cdot \text{EllipticF}[2 \cdot \text{ArcTan}[q \cdot x], 1/2]] / (2 \cdot q \cdot \text{Sqrt}[a + b \cdot x^4]), x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d) + (e \cdot x^2) / \text{Sqrt}[a + (c \cdot x)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d \cdot x \cdot \text{Sqrt}[a + c \cdot x^4]) / (a \cdot (1 + q^2 \cdot x^2)), x] + \text{Simp}[(d \cdot (1 + q^2 \cdot x^2) \cdot \text{Sqrt}[a + c \cdot x^4]) / (a \cdot (1 + q^2 \cdot x^2)^2)] \cdot \text{EllipticE}[2 \cdot \text{ArcTan}[q \cdot x], 1/2]] / (q \cdot \text{Sqrt}[a + c \cdot x^4]), x] /; \text{EqQ}[e + d \cdot q^2, 0]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^2} dx &= -\frac{2\sqrt{ax+bx^3}}{x} + (2b) \int \frac{x}{\sqrt{ax+bx^3}} dx \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{(2b\sqrt{x}\sqrt{a+bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{(4b\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} - \frac{(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= \frac{4\sqrt{bx}(a+bx^2)}{(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} - \frac{4\sqrt{a}\sqrt{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0122915, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{x(a+bx^2)} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3]/x^2,x]

[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/(x*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.013, size = 177, normalized size = 0.7

$$-2 \frac{bx^2+a}{\sqrt{x(bx^2+a)}} + 2 \frac{\sqrt{-ab}}{\sqrt{bx^3+ax}} \sqrt{\frac{b}{\sqrt{-ab}} \left(x + \frac{\sqrt{-ab}}{b}\right)} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-\frac{bx}{\sqrt{-ab}}} \left(-2 \frac{\sqrt{-ab}}{b} \text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^2,x)
```

```
[Out] -2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a + bx^2)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3} dx$

Optimal. Leaf size=116

$$\frac{2b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3x^2}$$

[Out] $(-2\sqrt{ax+bx^3})/(3x^2) + (2b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(3a^{1/4}\sqrt{ax+bx^3})$

Rubi [A] time = 0.0877961, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2020, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\sqrt{ax+bx^3}/x^3, x]$

[Out] $(-2\sqrt{ax+bx^3})/(3x^2) + (2b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(3a^{1/4}\sqrt{ax+bx^3})$

Rule 2020

$\text{Int}[(c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot (a \cdot x^j + b \cdot x^n)^p / (c \cdot (m+j \cdot p + 1)), x] - \text{Dist}[(b \cdot p \cdot (n-j)) / (c^n \cdot (m+j \cdot p + 1)), \text{Int}[(c \cdot x)^{m+n} \cdot (a \cdot x^j + b \cdot x^n)^{p-1}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j \cdot p + 1, 0]

Rule 2011

$\text{Int}[(a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Dist}[(a \cdot x^j + b \cdot x^n)^{\text{FracPart}[p]} / (x^{(j \cdot \text{FracPart}[p])} \cdot (a + b \cdot x^{(n-j)})^{\text{FracPart}[p]}), \text{Int}[x$

$(j*p)*(a + b*x^{(n - j)})^p, x], x] /; \text{FreeQ}\{a, b, j, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_) + (b_*)*(x_)^{(n_*)})^{(p_)}, x_Symbol] := \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_*)*(x_)^4], x_Symbol] := \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\text{Sqrt}[a + b*x^4]), x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax + bx^3}}{x^3} dx &= -\frac{2\sqrt{ax + bx^3}}{3x^2} + \frac{1}{3}(2b) \int \frac{1}{\sqrt{ax + bx^3}} dx \\ &= -\frac{2\sqrt{ax + bx^3}}{3x^2} + \frac{(2b\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{3\sqrt{ax + bx^3}} \\ &= -\frac{2\sqrt{ax + bx^3}}{3x^2} + \frac{(4b\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{3\sqrt{ax + bx^3}} \\ &= -\frac{2\sqrt{ax + bx^3}}{3x^2} + \frac{2b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0137491, size = 53, normalized size = 0.46

$$-\frac{2\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3]/x^3,x]

[Out] $(-2\sqrt{x(a+bx^2)}\text{Hypergeometric2F1}[-3/4, -1/2, 1/4, -(bx^2)/a])/(3x^2\sqrt{1+(bx^2)/a})$

Maple [A] time = 0.005, size = 123, normalized size = 1.1

$$-\frac{2}{3x^2}\sqrt{bx^3+ax} + \frac{2}{3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x^3+a*x)^{(1/2)}/x^3,x)$

[Out] $-2/3*(b*x^3+a*x)^{(1/2)}/x^2+2/3*(-a*b)^{(1/2)*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)*(-x*b/(-a*b)^{(1/2)})^{(1/2)}}/(b*x^3+a*x)^{(1/2)*\text{EllipticF}(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a*x)^{(1/2)}/x^3,x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\text{sqrt}(b*x^3 + a*x)/x^3, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*x^3+a*x)^{(1/2)}/x^3,x, \text{algorithm}="fricas")$

[Out] `integral(sqrt(b*x^3 + a*x)/x^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a + bx^2)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**(1/2)/x**3,x)`

[Out] `Integral(sqrt(x*(a + b*x**2))/x**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^3 + a*x)/x^3, x)`

3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4} dx$

Optimal. Leaf size=283

$$\frac{2b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} - \frac{4b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}$$

[Out] $(4*b^{(3/2)}*x*(a + b*x^2))/(5*a*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (2*\text{Sqrt}[a*x + b*x^3])/(5*x^3) - (4*b*\text{Sqrt}[a*x + b*x^3])/(5*a*x) - (4*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (2*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.250931, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} - \frac{4b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} + \frac{1}{5a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3]/x^4, x]

[Out] $(4*b^{(3/2)}*x*(a + b*x^2))/(5*a*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (2*\text{Sqrt}[a*x + b*x^3])/(5*x^3) - (4*b*\text{Sqrt}[a*x + b*x^3])/(5*a*x) - (4*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (2*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*

$p*(n - j)/(c^n*(m + j*p + 1))$, Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2025

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],

1/2))/(q*sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{ax + bx^3}}{x^4} dx &= -\frac{2\sqrt{ax + bx^3}}{5x^3} + \frac{1}{5}(2b) \int \frac{1}{x\sqrt{ax + bx^3}} dx \\
 &= -\frac{2\sqrt{ax + bx^3}}{5x^3} - \frac{4b\sqrt{ax + bx^3}}{5ax} + \frac{(2b^2) \int \frac{x}{\sqrt{ax + bx^3}} dx}{5a} \\
 &= -\frac{2\sqrt{ax + bx^3}}{5x^3} - \frac{4b\sqrt{ax + bx^3}}{5ax} + \frac{(2b^2 \sqrt{x} \sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{5a\sqrt{ax + bx^3}} \\
 &= -\frac{2\sqrt{ax + bx^3}}{5x^3} - \frac{4b\sqrt{ax + bx^3}}{5ax} + \frac{(4b^2 \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{5a\sqrt{ax + bx^3}} \\
 &= -\frac{2\sqrt{ax + bx^3}}{5x^3} - \frac{4b\sqrt{ax + bx^3}}{5ax} + \frac{(4b^{3/2} \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{5\sqrt{a}\sqrt{ax + bx^3}} - \frac{(4b^{3/2} \sqrt{x} \sqrt{a + bx^2})}{5\sqrt{a}\sqrt{ax + bx^3}} \\
 &= \frac{4b^{3/2} x (a + bx^2)}{5a(\sqrt{a} + \sqrt{bx}) \sqrt{ax + bx^3}} - \frac{2\sqrt{ax + bx^3}}{5x^3} - \frac{4b\sqrt{ax + bx^3}}{5ax} - \frac{4b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt{a + bx^2}}{\sqrt{a} + \sqrt{bx}} \right) \right)}{5a^{3/4} \sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0125866, size = 53, normalized size = 0.19

$$\frac{2\sqrt{x(a + bx^2)} {}_2F_1 \left(-\frac{5}{4}, -\frac{1}{2}; -\frac{1}{4}; -\frac{bx^2}{a} \right)}{5x^3 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3]/x^4, x]

[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -(b*x^2)/a])/(5*x^3*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.006, size = 201, normalized size = 0.7

$$-\frac{2}{5x^3}\sqrt{bx^3+ax}-\frac{(4bx^2+4a)b}{5a}\frac{1}{\sqrt{x(bx^2+a)}}+\frac{2b}{5a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(1/2)/x^4,x)

[Out] $-2/5*(b*x^3+a*x)^{(1/2)}/x^3-4/5*(b*x^2+a)*b/a/(x*(b*x^2+a))^{(1/2)}+2/5/a*b*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a*x)/x^4, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a+bx^2)}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x)**(1/2)/x**4,x)

[Out] Integral(sqrt(x*(a + b*x**2))/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)

3.46 $\int x^2 (ax + bx^3)^{3/2} dx$

Optimal. Leaf size=186

$$\frac{4a^{15/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}} - \frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3}$$

[Out] $(-8*a^3*\text{Sqrt}[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*\text{Sqrt}[a*x + b*x^3])/(385*b) + (4*a*x^4*\text{Sqrt}[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^{(3/2)})/15 + (4*a^{15/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[x])/a^{1/4}], 1/2])/(231*b^{9/4}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.229545, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{4a^{15/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a*x + b*x^3)^{(3/2)}, x]$

[Out] $(-8*a^3*\text{Sqrt}[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*\text{Sqrt}[a*x + b*x^3])/(385*b) + (4*a*x^4*\text{Sqrt}[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^{(3/2)})/15 + (4*a^{15/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[x])/a^{1/4}], 1/2])/(231*b^{9/4}*\text{Sqrt}[a*x + b*x^3])$

Rule 2021

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+n*p+1)), x] + \text{Dist}[(a*(n-j)*p)/(c^j*(m+n*p+1)), \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x^2 (ax + bx^3)^{3/2} dx &= \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{1}{5} (2a) \int x^3 \sqrt{ax + bx^3} dx \\
&= \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{1}{55} (4a^2) \int \frac{x^4}{\sqrt{ax + bx^3}} dx \\
&= \frac{8a^2 x^2 \sqrt{ax + bx^3}}{385b} + \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} - \frac{(4a^3) \int \frac{x^2}{\sqrt{ax + bx^3}} dx}{77b} \\
&= -\frac{8a^3 \sqrt{ax + bx^3}}{231b^2} + \frac{8a^2 x^2 \sqrt{ax + bx^3}}{385b} + \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{(4a^4) \int \frac{1}{\sqrt{ax + bx^3}} dx}{231b^2} \\
&= -\frac{8a^3 \sqrt{ax + bx^3}}{231b^2} + \frac{8a^2 x^2 \sqrt{ax + bx^3}}{385b} + \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{(4a^4 \sqrt{x} \sqrt{ax + bx^3})}{231b^2} \\
&= -\frac{8a^3 \sqrt{ax + bx^3}}{231b^2} + \frac{8a^2 x^2 \sqrt{ax + bx^3}}{385b} + \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{(8a^4 \sqrt{x} \sqrt{ax + bx^3})}{231b^2} \\
&= -\frac{8a^3 \sqrt{ax + bx^3}}{231b^2} + \frac{8a^2 x^2 \sqrt{ax + bx^3}}{385b} + \frac{4}{55} ax^4 \sqrt{ax + bx^3} + \frac{2}{15} x^3 (ax + bx^3)^{3/2} + \frac{4a^{15/4} \sqrt{x} (\sqrt{ax + bx^3})}{231b^2}
\end{aligned}$$

Mathematica [C] time = 0.0545894, size = 94, normalized size = 0.51

$$\frac{2\sqrt{x(a+bx^2)} \left(5a^3 {}_2F_1 \left(-\frac{3}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a} \right) - (5a - 11bx^2)(a + bx^2)^2 \sqrt{\frac{bx^2}{a} + 1} \right)}{165b^2 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3)^(3/2),x]

[Out] (2*Sqrt[x*(a + b*x^2)]*((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^2)/a])/(165*b^2*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.014, size = 188, normalized size = 1.

$$\frac{2bx^6}{15} \sqrt{bx^3 + ax} + \frac{34ax^4}{165} \sqrt{bx^3 + ax} + \frac{8a^2x^2}{385b} \sqrt{bx^3 + ax} - \frac{8a^3}{231b^2} \sqrt{bx^3 + ax} + \frac{4a^4}{231b^3} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a*x)^(3/2),x)`

[Out] $2/15*b*x^6*(b*x^3+a*x)^{(1/2)}+34/165*a*x^4*(b*x^3+a*x)^{(1/2)}+8/385*a^2*x^2*(b*x^3+a*x)^{(1/2)}/b-8/231*a^3*(b*x^3+a*x)^{(1/2)}/b^2+4/231*a^4/b^3*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x)^(3/2)*x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^5 + ax^3\right)\sqrt{bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^5 + a*x^3)*sqrt(b*x^3 + a*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (x(a + bx^2))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

3.47 $\int x(ax + bx^3)^{3/2} dx$

Optimal. Leaf size=304

$$\frac{4a^{13/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} - \frac{8a^3x(a+bx^2)}{65b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} + \frac{8a^{13/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticE}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

[Out] $(-8a^3x(a + bx^2))/(65b^{3/2}(\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[ax + bx^3]) + (8a^2x\text{Sqrt}[ax + bx^3])/(195b) + (4a^3x\text{Sqrt}[ax + bx^3])/39 + (2x^2(a + bx^3)^{3/2})/13 + (8a^{13/4}\text{Sqrt}[x](\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[(a + bx^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]x)^2]\text{EllipticE}[2\text{ArcTan}[(b^{1/4}\text{Sqrt}[x])/a^{1/4}], 1/2])/(65b^{7/4}\text{Sqrt}[ax + bx^3]) - (4a^{13/4}\text{Sqrt}[x](\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[(a + bx^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]x)^2]\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\text{Sqrt}[x])/a^{1/4}], 1/2])/(65b^{7/4}\text{Sqrt}[ax + bx^3])$

Rubi [A] time = 0.299659, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{8a^3x(a+bx^2)}{65b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{4a^{13/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} + \frac{8a^{13/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticE}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(ax + bx^3)^{3/2}, x]$

[Out] $(-8a^3x(a + bx^2))/(65b^{3/2}(\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[ax + bx^3]) + (8a^2x\text{Sqrt}[ax + bx^3])/(195b) + (4a^3x\text{Sqrt}[ax + bx^3])/39 + (2x^2(a + bx^3)^{3/2})/13 + (8a^{13/4}\text{Sqrt}[x](\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[(a + bx^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]x)^2]\text{EllipticE}[2\text{ArcTan}[(b^{1/4}\text{Sqrt}[x])/a^{1/4}], 1/2])/(65b^{7/4}\text{Sqrt}[ax + bx^3]) - (4a^{13/4}\text{Sqrt}[x](\text{Sqrt}[a] + \text{Sqrt}[b]x)\text{Sqrt}[(a + bx^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]x)^2]\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\text{Sqrt}[x])/a^{1/4}], 1/2])/(65b^{7/4}\text{Sqrt}[ax + bx^3])$

Rule 2021

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+n*p+1)), x] + \text{Dist}[(a$

$(n - j)p / (c^j(m + np + 1))$, $\text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x]$, x /; $\text{FreeQ}\{a, b, c, m\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \mid\mid \text{GtQ}[c, 0])$ && $\text{GtQ}[p, 0]$ && $\text{NeQ}[m + np + 1, 0]$

Rule 2024

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, x_Symbol] $\rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)})/(b*(m+np+1)), x]$ - $\text{Dist}[(a*c^{(n-j)}*(m+j*p-n+j+1))/(b*(m+np+1)), \text{Int}[(c*x)^{(m-(n-j))}*(a*x^j + b*x^n)^p, x]$, x /; $\text{FreeQ}\{a, b, c, m, p\}, x$] && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \mid\mid \text{GtQ}[c, 0])$ && $\text{GtQ}[m + j*p + 1 - n + j, 0]$ && $\text{NeQ}[m + np + 1, 0]$

Rule 2032

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, x_Symbol] $\rightarrow \text{Dist}[c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]}/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n-j)})^{\text{FracPart}[p]})$, $\text{Int}[x^{(m+j*p)}*(a + b*x^{(n-j)})^p, x]$, x /; $\text{FreeQ}\{a, b, c, j, m, n, p\}, x$] && $\text{!IntegerQ}[p]$ && $\text{NeQ}[n, j]$ && $\text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, x_Symbol] $\rightarrow \text{With}\{k = \text{Denominator}[m]\}$, $\text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x]$, $x, (c*x)^{(1/k)}$], x] /; $\text{FreeQ}\{a, b, c, p\}, x$] && $\text{IGtQ}[n, 0]$ && $\text{FractractionQ}[m]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

$\text{Int}[x^2/\text{Sqrt}[a_*) + (b_*)*(x_*)^4]$, x_Symbol] $\rightarrow \text{With}\{q = \text{Rt}[b/a, 2]\}$, $\text{Dist}[1/q, \text{Int}[1/\text{Sqrt}[a + b*x^4], x]$, $x]$ - $\text{Dist}[1/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^4], x]$, $x]$ /; $\text{FreeQ}\{a, b\}, x$] && $\text{PosQ}[b/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[a_*) + (b_*)*(x_*)^4]$, x_Symbol] $\rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}$, $\text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^4]/(a*(1 + q^2*x^2)^2)*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\text{Sqrt}[a + b*x^4])$, x] /; $\text{FreeQ}\{a, b\}, x$] && $\text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_*) + (e_*)*(x_*)^2/\text{Sqrt}[a_*) + (c_*)*(x_*)^4]$, x_Symbol] $\rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}$, $-\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2))$, $x]$ + $\text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)^2)*\text{EllipticE}[2*\text{ArcTan}[q*x]$,

1/2]]/(q*sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int x(ax + bx^3)^{3/2} dx &= \frac{2}{13}x^2(ax + bx^3)^{3/2} + \frac{1}{13}(6a) \int x^2\sqrt{ax + bx^3} dx \\
 &= \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} + \frac{1}{39}(4a^2) \int \frac{x^3}{\sqrt{ax + bx^3}} dx \\
 &= \frac{8a^2x\sqrt{ax + bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} - \frac{(4a^3) \int \frac{x}{\sqrt{ax + bx^3}} dx}{65b} \\
 &= \frac{8a^2x\sqrt{ax + bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} - \frac{(4a^3\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{65b\sqrt{ax + bx^3}} \\
 &= \frac{8a^2x\sqrt{ax + bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} - \frac{(8a^3\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a + bx^4}} dx\right)}{65b\sqrt{ax + bx^3}} \\
 &= \frac{8a^2x\sqrt{ax + bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} - \frac{(8a^{7/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx\right)}{65b^{3/2}\sqrt{ax + bx^3}} \\
 &= -\frac{8a^3x(a + bx^2)}{65b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{8a^2x\sqrt{ax + bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax + bx^3} + \frac{2}{13}x^2(ax + bx^3)^{3/2} + \dots
 \end{aligned}$$

Mathematica [C] time = 0.0415998, size = 84, normalized size = 0.28

$$\frac{2x\sqrt{x(a + bx^2)}\left((a + bx^2)^2\sqrt{\frac{bx^2}{a} + 1} - a^2 {}_2F_1\left(-\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)\right)}{13b\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3)^(3/2), x]

[Out] (2*x*sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*sqrt[1 + (b*x^2)/a] - a^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b*x^2)/a]))/(13*b*sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.012, size = 217, normalized size = 0.7

$$\frac{2bx^5}{13}\sqrt{bx^3+ax} + \frac{10ax^3}{39}\sqrt{bx^3+ax} + \frac{8a^2x}{195b}\sqrt{bx^3+ax} - \frac{4a^3}{65b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x)^(3/2), x)

[Out] $2/13*b*x^5*(b*x^3+a*x)^{(1/2)}+10/39*a*x^3*(b*x^3+a*x)^{(1/2)}+8/195*a^2*x*(b*x^3+a*x)^{(1/2)}/b-4/65/b^2*a^3*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + ax^2\right)\sqrt{bx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^(3/2), x, algorithm="fricas")

[Out] integral((b*x^4 + a*x^2)*sqrt(b*x^3 + a*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(x (a + bx^2) \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a*x)**(3/2),x)

[Out] Integral(x*(x*(a + b*x**2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)

3.48 $\int (ax + bx^3)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{4a^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}} + \frac{8a^2\sqrt{ax+bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax+bx^3} + \frac{2}{11}x(ax+bx^3)^{3/2}$$

[Out] $(8*a^2*\text{Sqrt}[a*x + b*x^3])/(77*b) + (12*a*x^2*\text{Sqrt}[a*x + b*x^3])/77 + (2*x*(a*x + b*x^3)^{(3/2)})/11 - (4*a^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(77*b^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.133583, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2004, 2021, 2024, 2011, 329, 220}

$$\frac{4a^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}} + \frac{8a^2\sqrt{ax+bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax+bx^3} + \frac{2}{11}x(ax+bx^3)^{3/2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^3)^{(3/2)}, x]$

[Out] $(8*a^2*\text{Sqrt}[a*x + b*x^3])/(77*b) + (12*a*x^2*\text{Sqrt}[a*x + b*x^3])/77 + (2*x*(a*x + b*x^3)^{(3/2)})/11 - (4*a^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(77*b^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2004

$\text{Int}[(a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)}]^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*(n - j)*p)/(n*p + 1), \text{Int}[x^j*(a*x^j + b*x^n)^{(p - 1)}, x], x] /;$ FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2021

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m + 1)}*(a*x^j + b*x^n)^p]/(c*(m + n*p + 1)), x] + \text{Dist}[(a$

$(n - j)p / (c^j(m + np + 1))$, $\text{Int}[(c*x)^{(m + j)}*(a*x^j + b*x^n)^{(p - 1)}$,
 $x]$, $x]$ /; $\text{FreeQ}\{a, b, c, m\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[m + np + 1, 0]$

Rule 2024

$\text{Int}[((c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, x_Symbol
 $] \text{:> } \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a*x^j + b*x^n)^{(p + 1)} / (b*(m + np + 1))$,
 $x] - \text{Dist}[(a*c^{(n - j)}*(m + j*p - n + j + 1)) / (b*(m + np + 1))$, $\text{Int}[(c*x)^{(m - (n - j))}$
 $*(a*x^j + b*x^n)^p, x]$, $x]$ /; $\text{FreeQ}\{a, b, c, m, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}$
 $[m + j*p + 1 - n + j, 0] \&\& \text{NeQ}[m + np + 1, 0]$

Rule 2011

$\text{Int}[((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, $x_Symbol]$ $\text{:> } \text{Dist}[(a*x^j +$
 $b*x^n)^{\text{FracPart}[p]} / (x^{(j*\text{FracPart}[p])}*(a + b*x^{(n - j)})^{\text{FracPart}[p])}$, $\text{Int}[x^{(j*p)}$
 $*(a + b*x^{(n - j)})^p, x]$, $x]$ /; $\text{FreeQ}\{a, b, j, n, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 329

$\text{Int}[((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}$, $x_Symbol]$ $\text{:> } \text{With}\{k =$
 $\text{Denominator}[m]\}$, $\text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})) / c^{(n)}$
 $^p, x]$, $x, (c*x)^{(1/k)}$, $x]]$ /; $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)*(x_*)^4]$, $x_Symbol]$ $\text{:> } \text{With}\{q = \text{Rt}[b/a, 4]\}$, $\text{Simp}[($
 $(1 + q^2*x^2)*\text{Sqrt}[a + b*x^4] / (a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x]$
 $, 1/2)] / (2*q*\text{Sqrt}[a + b*x^4])$, $x]]$ /; $\text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[b/a]$

Rubi steps

$$\begin{aligned}
\int (ax + bx^3)^{3/2} dx &= \frac{2}{11}x(ax + bx^3)^{3/2} + \frac{1}{11}(6a) \int x\sqrt{ax + bx^3} dx \\
&= \frac{12}{77}ax^2\sqrt{ax + bx^3} + \frac{2}{11}x(ax + bx^3)^{3/2} + \frac{1}{77}(12a^2) \int \frac{x^2}{\sqrt{ax + bx^3}} dx \\
&= \frac{8a^2\sqrt{ax + bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax + bx^3} + \frac{2}{11}x(ax + bx^3)^{3/2} - \frac{(4a^3) \int \frac{1}{\sqrt{ax+bx^3}} dx}{77b} \\
&= \frac{8a^2\sqrt{ax + bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax + bx^3} + \frac{2}{11}x(ax + bx^3)^{3/2} - \frac{(4a^3\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{77b\sqrt{ax + bx^3}} \\
&= \frac{8a^2\sqrt{ax + bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax + bx^3} + \frac{2}{11}x(ax + bx^3)^{3/2} - \frac{(8a^3\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\sqrt{a+bx^2}\right)}{77b\sqrt{ax + bx^3}} \\
&= \frac{8a^2\sqrt{ax + bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax + bx^3} + \frac{2}{11}x(ax + bx^3)^{3/2} - \frac{4a^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{77b^{5/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0354283, size = 83, normalized size = 0.53

$$\frac{2\sqrt{x(a + bx^2)}\left((a + bx^2)^2\sqrt{\frac{bx^2}{a} + 1} - a^2 {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{bx^2}{a}\right)\right)}{11b\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2), x]

[Out] (2*sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*sqrt[1 + (b*x^2)/a] - a^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^2)/a]))/(11*b*sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.011, size = 166, normalized size = 1.1

$$\frac{2bx^4}{11}\sqrt{bx^3 + ax} + \frac{26ax^2}{77}\sqrt{bx^3 + ax} + \frac{8a^2}{77b}\sqrt{bx^3 + ax} - \frac{4a^3}{77b^2}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a*x)^(3/2),x)`

[Out] $2/11*b*x^4*(b*x^3+a*x)^{(1/2)}+26/77*a*x^2*(b*x^3+a*x)^{(1/2)}+8/77*a^2*(b*x^3+a*x)^{(1/2)}/b-4/77/b^2*a^3*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3 + ax\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^3 + a*x)^(3/2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax + bx^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

$$3.49 \quad \int \frac{(ax+bx^3)^{3/2}}{x} dx$$

Optimal. Leaf size=275

$$\frac{4a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} - \frac{8a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}$$

[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.224599, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} - \frac{8a^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} + \frac{1}{15\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x, x]

[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3])

Rule 2021

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a

$(n - j)p / (c^j (m + n p + 1))$, $\text{Int}[(c x)^{(m + j)} (a x^j + b x^n)^{(p - 1)}, x]$, x /; $\text{FreeQ}\{a, b, c, m\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \mid \mid \text{GtQ}[c, 0])$ && $\text{GtQ}[p, 0]$ && $\text{NeQ}[m + n p + 1, 0]$

Rule 2004

$\text{Int}[(a x^j + b x^n)^p, x]$ $\text{Symbol} \rightarrow \text{Simp}[(x (a x^j + b x^n)^p) / (n p + 1), x] + \text{Dist}[(a (n - j) p) / (n p + 1), \text{Int}[x^j (a x^j + b x^n)^{(p - 1)}, x], x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $\text{GtQ}[p, 0]$ && $\text{NeQ}[n p + 1, 0]$

Rule 2032

$\text{Int}[(c x)^m (a x^j + b x^n)^p, x]$ $\text{Symbol} \rightarrow \text{Dist}[(c^{\text{IntPart}[m]} (c x)^{\text{FracPart}[m]} (a x^j + b x^n)^{\text{FracPart}[p]}) / (x^{\text{FracPart}[m] + j \text{FracPart}[p]} (a + b x^{(n - j)})^{\text{FracPart}[p]})$, $\text{Int}[x^{(m + j p)} (a + b x^{(n - j)})^p, x]$, x /; $\text{FreeQ}\{a, b, c, j, m, n, p\}, x$ && $\text{!IntegerQ}[p]$ && $\text{NeQ}[n, j]$ && $\text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c x)^m (a + b x^n)^p, x]$ $\text{Symbol} \rightarrow \text{With}\{k = \text{Denominator}[m]\}$, $\text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k(m + 1) - 1)} (a + (b x^{(k n)}) / c^n)^p, x], x, (c x)^{(1/k)}]$, $x]$ /; $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{FractionQ}[m]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

$\text{Int}[x^2 / \text{Sqrt}[a + b x^4], x]$ $\text{Symbol} \rightarrow \text{With}\{q = \text{Rt}[b/a, 2]\}$, $\text{Dist}[1/q, \text{Int}[1 / \text{Sqrt}[a + b x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q x^2) / \text{Sqrt}[a + b x^4], x], x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[b/a]$

Rule 220

$\text{Int}[1 / \text{Sqrt}[a + b x^4], x]$ $\text{Symbol} \rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}$, $\text{Simp}[(1 + q^2 x^2) \text{Sqrt}[a + b x^4] / (a (1 + q^2 x^2)^2) * \text{EllipticF}[2 \text{ArcTan}[q x], 1/2]] / (2 q \text{Sqrt}[a + b x^4])$, $x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d + e x^2) / \text{Sqrt}[a + c x^4], x]$ $\text{Symbol} \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}$, $-\text{Simp}[(d x \text{Sqrt}[a + c x^4]) / (a (1 + q^2 x^2)), x] + \text{Simp}[(d (1 + q^2 x^2) \text{Sqrt}[a + c x^4] / (a (1 + q^2 x^2)^2) * \text{EllipticE}[2 \text{ArcTan}[q x], 1/2]] / (q \text{Sqrt}[a + c x^4])$, $x]$ /; $\text{EqQ}[e + d q^2, 0]$ /; $\text{FreeQ}\{a, c, d, e\}$,

x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3)^{3/2}}{x} dx &= \frac{2}{9} (ax + bx^3)^{3/2} + \frac{1}{3} (2a) \int \sqrt{ax + bx^3} dx \\
 &= \frac{4}{15} ax \sqrt{ax + bx^3} + \frac{2}{9} (ax + bx^3)^{3/2} + \frac{1}{15} (4a^2) \int \frac{x}{\sqrt{ax + bx^3}} dx \\
 &= \frac{4}{15} ax \sqrt{ax + bx^3} + \frac{2}{9} (ax + bx^3)^{3/2} + \frac{(4a^2 \sqrt{x} \sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{15 \sqrt{ax + bx^3}} \\
 &= \frac{4}{15} ax \sqrt{ax + bx^3} + \frac{2}{9} (ax + bx^3)^{3/2} + \frac{(8a^2 \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{15 \sqrt{ax + bx^3}} \\
 &= \frac{4}{15} ax \sqrt{ax + bx^3} + \frac{2}{9} (ax + bx^3)^{3/2} + \frac{(8a^{5/2} \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x} \right)}{15 \sqrt{b} \sqrt{ax + bx^3}} - \frac{(8a^{5/2} \sqrt{x} \sqrt{a + bx^2})}{15 \sqrt{b} \sqrt{ax + bx^3}} \\
 &= \frac{8a^2 x (a + bx^2)}{15 \sqrt{b} (\sqrt{a} + \sqrt{bx}) \sqrt{ax + bx^3}} + \frac{4}{15} ax \sqrt{ax + bx^3} + \frac{2}{9} (ax + bx^3)^{3/2} - \frac{8a^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})}}}{15 b^{3/4} \sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0122047, size = 52, normalized size = 0.19

$$\frac{2ax \sqrt{x(a + bx^2)} {}_2F_1 \left(-\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a} \right)}{3 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x,x]

[Out] (2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b*x^2)/a]) / (3*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.011, size = 195, normalized size = 0.7

$$\frac{2bx^3}{9}\sqrt{bx^3+ax} + \frac{22ax}{45}\sqrt{bx^3+ax} + \frac{4a^2}{15b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-a}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x,x)

[Out] $2/9*b*x^3*(b*x^3+a*x)^{(1/2)}+22/45*a*x*(b*x^3+a*x)^{(1/2)}+4/15*a^2/b*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3+ax}(bx^2+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x)**(3/2)/x,x)

[Out] Integral((x*(a + b*x**2))**(3/2)/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")

[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)

$$3.50 \quad \int \frac{(ax+bx^3)^{3/2}}{x^2} dx$$

Optimal. Leaf size=134

$$\frac{4a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{4}{7}a\sqrt{ax+bx^3} + \frac{2(ax+bx^3)^{3/2}}{7x}$$

[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.131994, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2021, 2011, 329, 220}

$$\frac{4a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{4}{7}a\sqrt{ax+bx^3} + \frac{2(ax+bx^3)^{3/2}}{7x}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x^2, x]

[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x

$(j*p)*(a + b*x^(n - j))^p, x]$, x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3)^{3/2}}{x^2} dx &= \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{1}{7}(6a) \int \frac{\sqrt{ax + bx^3}}{x} dx \\
 &= \frac{4}{7}a\sqrt{ax + bx^3} + \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{1}{7}(4a^2) \int \frac{1}{\sqrt{ax + bx^3}} dx \\
 &= \frac{4}{7}a\sqrt{ax + bx^3} + \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{(4a^2\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{7\sqrt{ax + bx^3}} \\
 &= \frac{4}{7}a\sqrt{ax + bx^3} + \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{(8a^2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{7\sqrt{ax + bx^3}} \\
 &= \frac{4}{7}a\sqrt{ax + bx^3} + \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{4a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0144247, size = 49, normalized size = 0.37

$$\frac{2a\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^2,x]

[Out] (2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]/Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.012, size = 144, normalized size = 1.1

$$\frac{2bx^2}{7}\sqrt{bx^3+ax} + \frac{6a}{7}\sqrt{bx^3+ax} + \frac{4a^2}{7b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^2,x)

[Out] 2/7*b*x^2*(b*x^3+a*x)^(1/2)+6/7*a*(b*x^3+a*x)^(1/2)+4/7*a^2/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```


3.51

$$\int \frac{(ax+bx^3)^{3/2}}{x^3} dx$$

Optimal. Leaf size=274

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5\sqrt{ax+bx^3}}$$

[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.232554, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2020, 2004, 2032, 329, 305, 220, 1196}

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x^3,x]

[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3])

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*

$p*(n - j)/(c^n*(m + j*p + 1))$, Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2004

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2032

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},

x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3)^{3/2}}{x^3} dx &= -\frac{2(ax + bx^3)^{3/2}}{x^2} + (6b) \int \sqrt{ax + bx^3} dx \\
 &= \frac{12}{5} bx \sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{x^2} + \frac{1}{5} (12ab) \int \frac{x}{\sqrt{ax + bx^3}} dx \\
 &= \frac{12}{5} bx \sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{x^2} + \frac{(12ab \sqrt{x} \sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{5 \sqrt{ax + bx^3}} \\
 &= \frac{12}{5} bx \sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{x^2} + \frac{(24ab \sqrt{x} \sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{5 \sqrt{ax + bx^3}} \\
 &= \frac{12}{5} bx \sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{x^2} + \frac{(24a^{3/2} \sqrt{b} \sqrt{x} \sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{5 \sqrt{ax + bx^3}} - \frac{(24a^3)}{5 \sqrt{ax + bx^3}} \\
 &= \frac{24a \sqrt{bx} (a + bx^2)}{5 (\sqrt{a} + \sqrt{bx}) \sqrt{ax + bx^3}} + \frac{12}{5} bx \sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{x^2} - \frac{24a^{5/4} \sqrt[4]{b} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{a + bx^4}}}{5 \sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.014335, size = 52, normalized size = 0.19

$$\frac{2a \sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{x \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^3,x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/(x*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.016, size = 194, normalized size = 0.7

$$-2 \frac{(bx^2 + a)a}{\sqrt{x(bx^2 + a)}} + \frac{2bx}{5} \sqrt{bx^3 + ax} + \frac{12a}{5} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b} \right)} \sqrt{-bx \frac{1}{\sqrt{-ab}} \left(-2 \frac{\sqrt{-ab}}{b} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^3,x)

[Out] $-2*(b*x^2+a)*a/(x*(b*x^2+a))^{(1/2)}+2/5*b*x*(b*x^3+a*x)^{(1/2)}+12/5*a*(-a*b)^{(1/2)}*((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x)**(3/2)/x**3,x)

[Out] Integral((x*(a + b*x**2))**(3/2)/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)

$$3.52 \quad \int \frac{(ax+bx^3)^{3/2}}{x^4} dx$$

Optimal. Leaf size=134

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt{ax+bx^3}} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4}{3}b\sqrt{ax+bx^3}$$

[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.131137, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4}{3}b\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x^4, x]

[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3])

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2021

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3)^{3/2}}{x^4} dx &= -\frac{2(ax + bx^3)^{3/2}}{3x^3} + (2b) \int \frac{\sqrt{ax + bx^3}}{x} dx \\
&= \frac{4}{3}b\sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{3x^3} + \frac{1}{3}(4ab) \int \frac{1}{\sqrt{ax + bx^3}} dx \\
&= \frac{4}{3}b\sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{3x^3} + \frac{(4ab\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{3\sqrt{ax + bx^3}} \\
&= \frac{4}{3}b\sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{3x^3} + \frac{(8ab\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{3\sqrt{ax + bx^3}} \\
&= \frac{4}{3}b\sqrt{ax + bx^3} - \frac{2(ax + bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0153651, size = 54, normalized size = 0.4

$$-\frac{2a\sqrt{x(a+bx^2)} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^4,x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -(b*x^2)/a])/(3*x^2*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.017, size = 139, normalized size = 1.

$$-\frac{2a}{3x^2}\sqrt{bx^3+ax} + \frac{2b}{3}\sqrt{bx^3+ax} + \frac{4a}{3}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{\frac{bx^3+ax}{bx^3+ax}}, \sqrt{-ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^4,x)


```
[Out] -2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(
1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((x+1/b*
(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

3.53

$$\int \frac{(ax+bx^3)^{3/2}}{x^5} dx$$

Optimal. Leaf size=277

$$\frac{12\sqrt[4]{ab^{5/4}}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5\sqrt{ax+bx^3}} + \frac{24b^{3/2}x(a+bx^2)}{5(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{24\sqrt[4]{ab^{5/4}}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{5\sqrt{ax+bx^3}}$$

[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (12*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.260832, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{24b^{3/2}x(a+bx^2)}{5(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} + \frac{12\sqrt[4]{ab^{5/4}}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5\sqrt{ax+bx^3}} - \frac{24\sqrt[4]{ab^{5/4}}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{5\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x^5, x]

[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (12*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3])

Rule 2020

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*

$p*(n - j))/(c^n*(m + j*p + 1)), \text{Int}[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2032

$\text{Int}[((c_.)*(x_.))^(m_.)*((a_.)*(x_.)^(j_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]})/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])*(a + b*x^{(n - j)})^{\text{FracPart}[p]})}, \text{Int}[x^{(m + j*p)}*(a + b*x^{(n - j)})^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

$\text{Int}[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

$\text{Int}[(x_)^2/\text{Sqrt}[(a_) + (b_.)*(x_)^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b/a, 2]\}, \text{Dist}[1/q, \text{Int}[1/\text{Sqrt}[a + b*x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^4], x], x]] /;$ FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x]] /;$ FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

$\text{Int}[((d_) + (e_.)*(x_)^2)/\text{Sqrt}[(a_) + (c_.)*(x_)^4], x_Symbol] := \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2]]/(q*\text{Sqrt}[a + c*x^4]), x] /;$ EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3)^{3/2}}{x^5} dx &= -\frac{2(ax + bx^3)^{3/2}}{5x^4} + \frac{1}{5}(6b) \int \frac{\sqrt{ax + bx^3}}{x^2} dx \\
&= -\frac{12b\sqrt{ax + bx^3}}{5x} - \frac{2(ax + bx^3)^{3/2}}{5x^4} + \frac{1}{5}(12b^2) \int \frac{x}{\sqrt{ax + bx^3}} dx \\
&= -\frac{12b\sqrt{ax + bx^3}}{5x} - \frac{2(ax + bx^3)^{3/2}}{5x^4} + \frac{(12b^2\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{5\sqrt{ax + bx^3}} \\
&= -\frac{12b\sqrt{ax + bx^3}}{5x} - \frac{2(ax + bx^3)^{3/2}}{5x^4} + \frac{(24b^2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5\sqrt{ax + bx^3}} \\
&= -\frac{12b\sqrt{ax + bx^3}}{5x} - \frac{2(ax + bx^3)^{3/2}}{5x^4} + \frac{(24\sqrt{a}b^{3/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5\sqrt{ax + bx^3}} - \frac{(24\sqrt{a}b^{3/2}\sqrt{x}\sqrt{a + bx^2})}{5\sqrt{ax + bx^3}} \\
&= \frac{24b^{3/2}x(a + bx^2)}{5(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{12b\sqrt{ax + bx^3}}{5x} - \frac{2(ax + bx^3)^{3/2}}{5x^4} - \frac{24\sqrt{a}b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx}{(\sqrt{a} + \sqrt{bx})}}}{5\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.013007, size = 54, normalized size = 0.19

$$-\frac{2a\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{3}{2}, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5x^3\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^5,x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -(b*x^2)/a])/ (5*x^3*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.016, size = 196, normalized size = 0.7

$$-\frac{2a}{5x^3}\sqrt{bx^3 + ax} - \frac{(14bx^2 + 14a)b}{5} \frac{1}{\sqrt{x(bx^2 + a)}} + \frac{12b}{5}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx - \frac{\sqrt{-ab}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a*x)^(3/2)/x^5,x)`

[Out]
$$-2/5*a*(b*x^3+a*x)^{(1/2)}/x^3-14/5*(b*x^2+a)*b/(x*(b*x^2+a))^{(1/2)}+12/5*b*(-a*b)^{(1/2)}*((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b))^{(1/2)})*b/(-a*b)^{(1/2)}^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b))^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b))^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x)^(3/2)/x^5, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^4, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x)**(3/2)/x**5,x)

[Out] Integral((x*(a + b*x**2))**(3/2)/x**5, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)

$$3.54 \quad \int \frac{(ax+bx^3)^{3/2}}{x^6} dx$$

Optimal. Leaf size=137

$$\frac{4b^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5}$$

[Out] $(-4*b*\text{Sqrt}[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^{(3/2)})/(7*x^5) + (4*b^{7/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(7*a^{(1/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.134207, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2020, 2011, 329, 220}

$$\frac{4b^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^3)^{(3/2)}/x^6, x]$

[Out] $(-4*b*\text{Sqrt}[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^{(3/2)})/(7*x^5) + (4*b^{7/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(7*a^{(1/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2020

$\text{Int}[(c_*)*(x_)^{(m_)}*((a_*)*(x_)^{(j_)} + (b_*)*(x_)^{(n_)})^{(p_)}, x_Symbol]$
 $:\> \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j*p+1, 0]

Rule 2011


```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3)^{3/2}}{x^6} dx &= -\frac{2(ax + bx^3)^{3/2}}{7x^5} + \frac{1}{7}(6b) \int \frac{\sqrt{ax + bx^3}}{x^3} dx \\
&= -\frac{4b\sqrt{ax + bx^3}}{7x^2} - \frac{2(ax + bx^3)^{3/2}}{7x^5} + \frac{1}{7}(4b^2) \int \frac{1}{\sqrt{ax + bx^3}} dx \\
&= -\frac{4b\sqrt{ax + bx^3}}{7x^2} - \frac{2(ax + bx^3)^{3/2}}{7x^5} + \frac{(4b^2\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{7\sqrt{ax + bx^3}} \\
&= -\frac{4b\sqrt{ax + bx^3}}{7x^2} - \frac{2(ax + bx^3)^{3/2}}{7x^5} + \frac{(8b^2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{7\sqrt{ax + bx^3}} \\
&= -\frac{4b\sqrt{ax + bx^3}}{7x^2} - \frac{2(ax + bx^3)^{3/2}}{7x^5} + \frac{4b^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.016064, size = 54, normalized size = 0.39

$$\frac{2a\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{7}{4}, -\frac{3}{2}; -\frac{3}{4}; -\frac{bx^2}{a}\right)}{7x^4\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^6,x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)]/(7*x^4*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.015, size = 142, normalized size = 1.

$$-\frac{2a}{7x^4}\sqrt{bx^3+ax}-\frac{6b}{7x^2}\sqrt{bx^3+ax}+\frac{4b}{7}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^6,x)

[Out] -2/7*a*(b*x^3+a*x)^(1/2)/x^4-6/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}(bx^2+a)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^5, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**(3/2)/x**6,x)`

[Out] `Integral((x*(a + b*x**2))**(3/2)/x**6, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a*x)^(3/2)/x^6, x)`

$$3.55 \quad \int \frac{(ax+bx^3)^{3/2}}{x^7} dx$$

Optimal. Leaf size=306

$$\frac{4b^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} - \frac{8b^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$

[Out] $(8*b^{(5/2)}*x*(a + b*x^2))/(15*a*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (4*b*\text{Sqrt}[a*x + b*x^3])/(15*x^3) - (8*b^2*\text{Sqrt}[a*x + b*x^3])/(15*a*x) - (2*(a*x + b*x^3)^{(3/2)})/(9*x^6) - (8*b^{(9/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (4*b^{(9/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.311964, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} - \frac{8b^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} + \frac{1}{15a}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(3/2)/x^7, x]

[Out] $(8*b^{(5/2)}*x*(a + b*x^2))/(15*a*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (4*b*\text{Sqrt}[a*x + b*x^3])/(15*x^3) - (8*b^2*\text{Sqrt}[a*x + b*x^3])/(15*a*x) - (2*(a*x + b*x^3)^{(3/2)})/(9*x^6) - (8*b^{(9/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3]) + (4*b^{(9/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*

$p*(n - j)/(c^n*(m + j*p + 1))$, Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2025

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2032

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],

1/2]]/(q*sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3)^{3/2}}{x^7} dx &= -\frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{1}{3}(2b) \int \frac{\sqrt{ax + bx^3}}{x^4} dx \\
 &= -\frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{1}{15}(4b^2) \int \frac{1}{x\sqrt{ax + bx^3}} dx \\
 &= -\frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{8b^2\sqrt{ax + bx^3}}{15ax} - \frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{(4b^3) \int \frac{x}{\sqrt{ax + bx^3}} dx}{15a} \\
 &= -\frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{8b^2\sqrt{ax + bx^3}}{15ax} - \frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{(4b^3\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{15a\sqrt{ax + bx^3}} \\
 &= -\frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{8b^2\sqrt{ax + bx^3}}{15ax} - \frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{(8b^3\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{15a\sqrt{ax + bx^3}} \\
 &= -\frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{8b^2\sqrt{ax + bx^3}}{15ax} - \frac{2(ax + bx^3)^{3/2}}{9x^6} + \frac{(8b^{5/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{15\sqrt{a}\sqrt{ax + bx^3}} \\
 &= \frac{8b^{5/2}x(a + bx^2)}{15a(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{4b\sqrt{ax + bx^3}}{15x^3} - \frac{8b^2\sqrt{ax + bx^3}}{15ax} - \frac{2(ax + bx^3)^{3/2}}{9x^6} - \frac{8b^{9/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{15a\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0158659, size = 54, normalized size = 0.18

$$\frac{2a\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{bx^2}{a}\right)}{9x^5\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^7, x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -(b*x^2)/a])/(9*x^5*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.018, size = 223, normalized size = 0.7

$$-\frac{2a}{9x^5}\sqrt{bx^3+ax}-\frac{22b}{45x^3}\sqrt{bx^3+ax}-\frac{(8bx^2+8a)b^2}{15a}\frac{1}{\sqrt{x(bx^2+a)}}+\frac{4b^2}{15a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x+\frac{1}{b}\sqrt{-ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^7,x)

[Out]
$$-2/9*a*(b*x^3+a*x)^{(1/2)}/x^5-22/45*b*(b*x^3+a*x)^{(1/2)}/x^3-8/15*(b*x^2+a)/a*b^2/(x*(b*x^2+a))^{(1/2)}+4/15/a*b^2*(-a*b)^{(1/2)}*((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}(bx^2+a)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")

[Out] `integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^6, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x)**(3/2)/x**7,x)`

[Out] `Integral((x*(a + b*x**2))**(3/2)/x**7, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a*x)^(3/2)/x^7, x)`

$$3.56 \quad \int \frac{(ax+bx^3)^{3/2}}{x^8} dx$$

Optimal. Leaf size=163

$$\frac{4b^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{2(ax+bx^3)^{3/2}}{11x^7}$$

[Out] $(-12*b*\text{Sqrt}[a*x + b*x^3])/(77*x^4) - (8*b^2*\text{Sqrt}[a*x + b*x^3])/(77*a*x^2) - (2*(a*x + b*x^3)^{(3/2)})/(11*x^7) - (4*b^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(77*a^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.178191, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2020, 2025, 2011, 329, 220}

$$\frac{4b^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{2(ax+bx^3)^{3/2}}{11x^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^3)^{(3/2)}/x^8, x]$

[Out] $(-12*b*\text{Sqrt}[a*x + b*x^3])/(77*x^4) - (8*b^2*\text{Sqrt}[a*x + b*x^3])/(77*a*x^2) - (2*(a*x + b*x^3)^{(3/2)})/(11*x^7) - (4*b^{(11/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(77*a^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2020

$\text{Int}[(c_*)*(x_*)^{(m_*)}((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:\> \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \ || \ \text{GtQ}[c, 0]) \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m+j*p+1, 0]$

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3)^{3/2}}{x^8} dx &= -\frac{2(ax + bx^3)^{3/2}}{11x^7} + \frac{1}{11}(6b) \int \frac{\sqrt{ax + bx^3}}{x^5} dx \\
&= -\frac{12b\sqrt{ax + bx^3}}{77x^4} - \frac{2(ax + bx^3)^{3/2}}{11x^7} + \frac{1}{77}(12b^2) \int \frac{1}{x^2\sqrt{ax + bx^3}} dx \\
&= -\frac{12b\sqrt{ax + bx^3}}{77x^4} - \frac{8b^2\sqrt{ax + bx^3}}{77ax^2} - \frac{2(ax + bx^3)^{3/2}}{11x^7} - \frac{(4b^3) \int \frac{1}{\sqrt{ax+bx^3}} dx}{77a} \\
&= -\frac{12b\sqrt{ax + bx^3}}{77x^4} - \frac{8b^2\sqrt{ax + bx^3}}{77ax^2} - \frac{2(ax + bx^3)^{3/2}}{11x^7} - \frac{(4b^3\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{77a\sqrt{ax + bx^3}} \\
&= -\frac{12b\sqrt{ax + bx^3}}{77x^4} - \frac{8b^2\sqrt{ax + bx^3}}{77ax^2} - \frac{2(ax + bx^3)^{3/2}}{11x^7} - \frac{(8b^3\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{77a\sqrt{ax + bx^3}} \\
&= -\frac{12b\sqrt{ax + bx^3}}{77x^4} - \frac{8b^2\sqrt{ax + bx^3}}{77ax^2} - \frac{2(ax + bx^3)^{3/2}}{11x^7} - \frac{4b^{11/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}+\sqrt{bx}}\right)\right)}{77a^{5/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0159306, size = 54, normalized size = 0.33

$$\frac{2a\sqrt{x(a + bx^2)} {}_2F_1\left(-\frac{11}{4}, -\frac{3}{2}; -\frac{7}{4}; -\frac{bx^2}{a}\right)}{11x^6\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(3/2)/x^8,x]

[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)])/(11*x^6*Sqrt[1 + (b*x^2)/a])

Maple [A] time = 0.019, size = 169, normalized size = 1.

$$-\frac{2a}{11x^6}\sqrt{bx^3 + ax} - \frac{26b}{77x^4}\sqrt{bx^3 + ax} - \frac{8b^2}{77ax^2}\sqrt{bx^3 + ax} - \frac{4b^2}{77a}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a*x)^(3/2)/x^8,x)`

[Out]
$$-2/11*a*(b*x^3+a*x)^{(1/2)}/x^6-26/77*b*(b*x^3+a*x)^{(1/2)}/x^4-8/77*b^2*(b*x^3+a*x)^{(1/2)}/a/x^2-4/77/a*b^2*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x)^(3/2)/x^8, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}(bx^2 + a)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^7, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

$$3.57 \quad \int \frac{x^4}{\sqrt{ax+bx^3}} dx$$

Optimal. Leaf size=140

$$\frac{5a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}} - \frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b}$$

[Out] $(-10*a*\text{Sqrt}[a*x + b*x^3])/(21*b^2) + (2*x^2*\text{Sqrt}[a*x + b*x^3])/(7*b) + (5*a^{7/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[x])/a^{1/4}], 1/2])/(21*b^{9/4}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.13725, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2024, 2011, 329, 220}

$$\frac{5a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}} - \frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/\text{Sqrt}[a*x + b*x^3], x]$

[Out] $(-10*a*\text{Sqrt}[a*x + b*x^3])/(21*b^2) + (2*x^2*\text{Sqrt}[a*x + b*x^3])/(7*b) + (5*a^{7/4}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[x])/a^{1/4}], 1/2])/(21*b^{9/4}*\text{Sqrt}[a*x + b*x^3])$

Rule 2024

$\text{Int}[\left((c_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(j_.)} + (b_.)*(x_.)^{(n_.)})^{(p_.)}\right), x_Symbol] \rightarrow \text{Simp}[\left(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)}\right)/(b*(m+n*p+1)), x] - \text{Dist}[\left(a*c^{(n-j)}*(m+j*p-n+j+1)\right)/(b*(m+n*p+1)), \text{Int}[\left((c*x)^{(m-(n-j))}*(a*x^j + b*x^n)^p, x\right), x] /;$ $\text{FreeQ}\{a, b, c, m, p, x\}$ && $\text{IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$ && $\text{GtQ}[m + j*p + 1 - n + j, 0]$ && $\text{NeQ}[m + n*p + 1, 0]$

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^(1/k), x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\sqrt{ax + bx^3}} dx &= \frac{2x^2\sqrt{ax + bx^3}}{7b} - \frac{(5a) \int \frac{x^2}{\sqrt{ax + bx^3}} dx}{7b} \\
 &= -\frac{10a\sqrt{ax + bx^3}}{21b^2} + \frac{2x^2\sqrt{ax + bx^3}}{7b} + \frac{(5a^2) \int \frac{1}{\sqrt{ax + bx^3}} dx}{21b^2} \\
 &= -\frac{10a\sqrt{ax + bx^3}}{21b^2} + \frac{2x^2\sqrt{ax + bx^3}}{7b} + \frac{(5a^2\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{21b^2\sqrt{ax + bx^3}} \\
 &= -\frac{10a\sqrt{ax + bx^3}}{21b^2} + \frac{2x^2\sqrt{ax + bx^3}}{7b} + \frac{(10a^2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{21b^2\sqrt{ax + bx^3}} \\
 &= -\frac{10a\sqrt{ax + bx^3}}{21b^2} + \frac{2x^2\sqrt{ax + bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{21b^{9/4}\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0316225, size = 80, normalized size = 0.57

$$\frac{2x \left(5a^2 \sqrt{\frac{bx^2}{a}} + {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a} \right) - 5a^2 - 2abx^2 + 3b^2x^4 \right)}{21b^2 \sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a*x + b*x^3],x]

[Out] (2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^2)/a]))/(21*b^2*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.013, size = 149, normalized size = 1.1

$$\frac{2x^2}{7b} \sqrt{bx^3 + ax} - \frac{10a}{21b^2} \sqrt{bx^3 + ax} + \frac{5a^2}{21b^3} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b} \right)} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF} \left(\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x)^(1/2),x)

[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)/b-10/21*a*(b*x^3+a*x)^(1/2)/b^2+5/21*a^2/b^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax^3}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^3/(b*x^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**3+a*x)**(1/2),x)`

[Out] `Integral(x**4/sqrt(x*(a + b*x**2)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^4/sqrt(b*x^3 + a*x), x)`

3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}} dx$

Optimal. Leaf size=258

$$\frac{3a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}} + \frac{6a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5b^{7/4}\sqrt{ax+bx^3}}$$

[Out] $(-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])$

Rubi [A] time = 0.205405, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$\frac{3a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)\left|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}} + \frac{6a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)\left|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}} - \frac{5b^{7/4}\sqrt{ax+bx^3}}{5b^{7/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x + b*x^3], x]

[Out] $(-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])$

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In

$t[(c*x)^{(m - (n - j))}*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p], Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax+bx^3}} dx &= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(3a) \int \frac{x}{\sqrt{ax+bx^3}} dx}{5b} \\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(3a\sqrt{x}\sqrt{a+bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{5b\sqrt{ax+bx^3}} \\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(6a\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5b\sqrt{ax+bx^3}} \\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}} + \frac{(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1-\sqrt{b}}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}} \\
&= -\frac{6ax(a+bx^2)}{5b^{3/2}(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} + \frac{2x\sqrt{ax+bx^3}}{5b} + \frac{6a^{5/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{5b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0261342, size = 66, normalized size = 0.26

$$\frac{2x^2 \left(-a\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) + a + bx^2 \right)}{5b\sqrt{x}(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x + b*x^3], x]

[Out] (2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(b*x^2)/a]))/(5*b*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.011, size = 178, normalized size = 0.7

$$\frac{2x}{5b}\sqrt{bx^3+ax} - \frac{3a}{5b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\left(-2\frac{\sqrt{-ab}}{b}\text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^3+a*x)^(1/2),x)`

[Out] $2/5*x*(b*x^3+a*x)^{(1/2)}/b-3/5/b^2*a*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}))+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/sqrt(b*x^3 + a*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + axx^2}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^2/(b*x^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

$$3.59 \quad \int \frac{x^2}{\sqrt{ax+bx^3}} dx$$

Optimal. Leaf size=116

$$\frac{2\sqrt{ax+bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.091224, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2024, 2011, 329, 220}

$$\frac{2\sqrt{ax+bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x + b*x^3],x]

[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :-> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a*x^j + b*x^n)^(p+1))/(b*(m+n*p+1)), x]
  - Dist[(a*c^(n-j)*(m+j*p-n+j+1))/(b*(m+n*p+1)), Int[(c*x)^(m-(n-j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
  && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m+j*p+1-n+j, 0] && NeQ[m+n*p+1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax + bx^3}} dx &= \frac{2\sqrt{ax + bx^3}}{3b} - \frac{a \int \frac{1}{\sqrt{ax + bx^3}} dx}{3b} \\
&= \frac{2\sqrt{ax + bx^3}}{3b} - \frac{(a\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{3b\sqrt{ax + bx^3}} \\
&= \frac{2\sqrt{ax + bx^3}}{3b} - \frac{(2a\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{3b\sqrt{ax + bx^3}} \\
&= \frac{2\sqrt{ax + bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3b^{5/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0254226, size = 64, normalized size = 0.55

$$\frac{2x \left(-a\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a + bx^2 \right)}{3b\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x + b*x^3],x]

[Out] (2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^2)/a]))/(3*b*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.013, size = 127, normalized size = 1.1

$$\frac{2}{3b}\sqrt{bx^3+ax}-\frac{a}{3b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x)^(1/2),x)

[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3/b^2*a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3+ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

3.60 $\int \frac{x}{\sqrt{ax+bx^3}} dx$

Optimal. Leaf size=229

$$\frac{\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}$$

[Out] $(2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^{1/4}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^{1/4}*Sqrt[x])/a^{1/4}], 1/2])/(b^{3/4}*Sqrt[a*x + b*x^3]) + (a^{1/4}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^{1/4}*Sqrt[x])/a^{1/4}], 1/2])/(b^{3/4}*Sqrt[a*x + b*x^3])$

Rubi [A] time = 0.149676, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \frac{1}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x + b*x^3], x]

[Out] $(2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^{1/4}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^{1/4}*Sqrt[x])/a^{1/4}], 1/2])/(b^{3/4}*Sqrt[a*x + b*x^3]) + (a^{1/4}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^{1/4}*Sqrt[x])/a^{1/4}], 1/2])/(b^{3/4}*Sqrt[a*x + b*x^3])$

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p], Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ

erQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{ax+bx^3}} dx &= \frac{(\sqrt{x}\sqrt{a+bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{\sqrt{ax+bx^3}} \\
&= \frac{(2\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= \frac{(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} - \frac{(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1-\frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} \\
&= \frac{2x(a+bx^2)}{\sqrt{b}(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{a}\sqrt{x}(\sqrt{a}+\sqrt{bx})}{b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0129953, size = 53, normalized size = 0.23

$$\frac{2x^2 \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x + b*x^3], x]

[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/(3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.011, size = 158, normalized size = 0.7

$$\frac{1}{b} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \left(-2 \frac{\sqrt{-ab}}{b} \text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}} \left(x + \frac{\sqrt{-ab}}{b}\right)}, \frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x)^(1/2), x)

```
[Out] 1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2))*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

3.61 $\int \frac{1}{\sqrt{ax+bx^3}} dx$

Optimal. Leaf size=92

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.0488648, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2011, 329, 220}

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x + b*x^3], x]

[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*x + b*x^3])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^(p), x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax + bx^3}} dx &= \frac{(\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{\sqrt{ax + bx^3}} \\ &= \frac{(2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{ax + bx^3}} \\ &= \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0095926, size = 49, normalized size = 0.53

$$\frac{2x\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right)}{\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x + b*x^3], x]

[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]/Sqrt[x*(a + b*x^2)]

Maple [A] time = 0.013, size = 108, normalized size = 1.2

$$\frac{1}{b}\sqrt{-ab}\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}, \frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{bx^3 +}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^3+a*x)^(1/2),x)`

[Out] `1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(b*x^3 + a*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^3 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(1/sqrt(b*x^3 + a*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

$$3.62 \quad \int \frac{1}{x\sqrt{ax+bx^3}} dx$$

Optimal. Leaf size=253

$$\frac{\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}$$

[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.200493, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} + \frac{2\sqrt{a}}{a(\sqrt{a} + \sqrt{bx})}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a*x + b*x^3]),x]

[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])

Rule 2025

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)
*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2])*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax+bx^3}} dx &= -\frac{2\sqrt{ax+bx^3}}{ax} + \frac{b \int \frac{x}{\sqrt{ax+bx^3}} dx}{a} \\
&= -\frac{2\sqrt{ax+bx^3}}{ax} + \frac{(b\sqrt{x}\sqrt{a+bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{a\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{ax} + \frac{(2b\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{ax} + \frac{(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}} - \frac{(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1-\sqrt{b}}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}} \\
&= \frac{2\sqrt{bx}(a+bx^2)}{a(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{ax} - \frac{2^4\sqrt{b}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.0122931, size = 48, normalized size = 0.19

$$\frac{2\sqrt{\frac{bx^2}{a}+1} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x + b*x^3]),x]

[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)]/Sqrt[x*(a + b*x^2)]

Maple [A] time = 0.015, size = 182, normalized size = 0.7

$$-2 \frac{bx^2+a}{a\sqrt{x(bx^2+a)}} + \frac{1}{a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-ab}}{b}\text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^3+a*x)^(1/2),x)`

[Out]
$$\frac{-2*(b*x^2+a)/a/(x*(b*x^2+a))^{1/2}+1/a*(-a*b)^{1/2}*((x+1/b*(-a*b)^{1/2})*b/(-a*b)^{1/2})^{1/2}*(-2*(x-1/b*(-a*b)^{1/2})*b/(-a*b)^{1/2})^{1/2}*(-x*b/(-a*b)^{1/2})^{1/2}/(b*x^3+a*x)^{1/2}*(-2/b*(-a*b)^{1/2}*EllipticE((x+1/b*(-a*b)^{1/2})*b/(-a*b)^{1/2})^{1/2},1/2*2^{1/2}))+1/b*(-a*b)^{1/2}*EllipticF((x+1/b*(-a*b)^{1/2})*b/(-a*b)^{1/2})^{1/2},1/2*2^{1/2}))}{1}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x)*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^4 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)/(b*x^4 + a*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x(a+bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + axx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```


3.63 $\int \frac{1}{x^2 \sqrt{ax+bx^3}} dx$

Optimal. Leaf size=119

$$\frac{b^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3a^{5/4} \sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3ax^2}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^3])/(3*a*x^2) - (b^{(3/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(3*a^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.0893417, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2025, 2011, 329, 220}

$$\frac{b^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3a^{5/4} \sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3ax^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[a*x + b*x^3]), x]$

[Out] $(-2*\text{Sqrt}[a*x + b*x^3])/(3*a*x^2) - (b^{(3/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(3*a^{(5/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2025

$\text{Int}[(c_*)*(x_*)^{(m_*)}((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[m + j*p + 1, 0]$

Rule 2011

$\text{Int}[(a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(a*x^j + b*x^n)^{\text{FracPart}[p]}/(x^{(j*\text{FracPart}[p])}*(a + b*x^{(n-j)})^{\text{FracPart}[p]}), \text{Int}[x$

$(j*p)*(a + b*x^(n - j))^p, x]$ /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{ax + bx^3}} dx &= -\frac{2\sqrt{ax + bx^3}}{3ax^2} - \frac{b \int \frac{1}{\sqrt{ax + bx^3}} dx}{3a} \\ &= -\frac{2\sqrt{ax + bx^3}}{3ax^2} - \frac{(b\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{3a\sqrt{ax + bx^3}} \\ &= -\frac{2\sqrt{ax + bx^3}}{3ax^2} - \frac{(2b\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{3a\sqrt{ax + bx^3}} \\ &= -\frac{2\sqrt{ax + bx^3}}{3ax^2} - \frac{b^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3a^{5/4} \sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0150918, size = 53, normalized size = 0.45

$$\frac{2\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]

[Out] $(-2\sqrt{1 + (bx^2)/a} \text{Hypergeometric2F1}[-3/4, 1/2, 1/4, -((bx^2)/a)]) / (3 \sqrt{x} \sqrt{x(a + bx^2)})$

Maple [A] time = 0.014, size = 129, normalized size = 1.1

$$-\frac{2}{3ax^2} \sqrt{bx^3 + ax} - \frac{1}{3a} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b} \right)} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF} \left(\sqrt{b \left(x + \frac{1}{b} \sqrt{-ab} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^3+a*x)^(1/2),x)`

[Out] $-2/3*(bx^3+ax)^{(1/2)}/a/x^2-1/3/a*(-ab)^{(1/2)}*((x+1/b*(-ab)^{(1/2)})b/(-ab)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-ab)^{(1/2)})b/(-ab)^{(1/2)})^{(1/2)}*(-x*b/(-ab)^{(1/2)})^{(1/2)}/(bx^3+ax)^{(1/2)}*\text{EllipticF}(((x+1/b*(-ab)^{(1/2)})b/(-ab)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + axx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^3 + ax}}{bx^5 + ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)/(b*x^5 + a*x^3), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**3+a*x)**(1/2),x)`

[Out] `Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)`

$$3.64 \quad \int \frac{1}{x^3 \sqrt{ax+bx^3}} dx$$

Optimal. Leaf size=286

$$\frac{3b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5a^{7/4} \sqrt{ax+bx^3}} - \frac{6b^{3/2} x (a+bx^2)}{5a^2 (\sqrt{a} + \sqrt{bx}) \sqrt{ax+bx^3}} + \frac{6b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx})}{5a^{7/4} \sqrt{ax}}$$

[Out] $(-6*b^{(3/2)}*x*(a + b*x^2))/(5*a^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (2*\text{Sqrt}[a*x + b*x^3])/(5*a*x^3) + (6*b*\text{Sqrt}[a*x + b*x^3])/(5*a^2*x) + (6*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[a*x + b*x^3]) - (3*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.254296, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{6b^{3/2} x (a+bx^2)}{5a^2 (\sqrt{a} + \sqrt{bx}) \sqrt{ax+bx^3}} - \frac{3b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5a^{7/4} \sqrt{ax+bx^3}} + \frac{6b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}}{5a^{7/4} \sqrt{ax}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a*x + b*x^3]),x]

[Out] $(-6*b^{(3/2)}*x*(a + b*x^2))/(5*a^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) - (2*\text{Sqrt}[a*x + b*x^3])/(5*a*x^3) + (6*b*\text{Sqrt}[a*x + b*x^3])/(5*a^2*x) + (6*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[a*x + b*x^3]) - (3*b^{(5/4)}*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[a*x + b*x^3])$

Rule 2025

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p

```

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p], x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 305

```

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

```

Rule 220

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

```

Rule 1196

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{ax + bx^3}} dx &= -\frac{2\sqrt{ax + bx^3}}{5ax^3} - \frac{(3b) \int \frac{1}{x\sqrt{ax+bx^3}} dx}{5a} \\
&= -\frac{2\sqrt{ax + bx^3}}{5ax^3} + \frac{6b\sqrt{ax + bx^3}}{5a^2x} - \frac{(3b^2) \int \frac{x}{\sqrt{ax+bx^3}} dx}{5a^2} \\
&= -\frac{2\sqrt{ax + bx^3}}{5ax^3} + \frac{6b\sqrt{ax + bx^3}}{5a^2x} - \frac{(3b^2 \sqrt{x} \sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{5a^2 \sqrt{ax + bx^3}} \\
&= -\frac{2\sqrt{ax + bx^3}}{5ax^3} + \frac{6b\sqrt{ax + bx^3}}{5a^2x} - \frac{(6b^2 \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x} \right)}{5a^2 \sqrt{ax + bx^3}} \\
&= -\frac{2\sqrt{ax + bx^3}}{5ax^3} + \frac{6b\sqrt{ax + bx^3}}{5a^2x} - \frac{(6b^{3/2} \sqrt{x} \sqrt{a + bx^2}) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x} \right)}{5a^{3/2} \sqrt{ax + bx^3}} + \frac{(6b^{3/2} \sqrt{x} \sqrt{a + bx^2})}{5a^{3/2} \sqrt{ax + bx^3}} \\
&= -\frac{6b^{3/2}x(a + bx^2)}{5a^2(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{2\sqrt{ax + bx^3}}{5ax^3} + \frac{6b\sqrt{ax + bx^3}}{5a^2x} + \frac{6b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}}}{5a^{7/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0138129, size = 53, normalized size = 0.19

$$-\frac{2\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5x^2 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*sqrt[a*x + b*x^3]),x]

[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/(5*x^2*sqrt[x*(a + b*x^2)])

Maple [A] time = 0.016, size = 204, normalized size = 0.7

$$-\frac{2}{5ax^3} \sqrt{bx^3 + ax} + \frac{(6bx^2 + 6a)b}{5a^2} \frac{1}{\sqrt{x(bx^2 + a)}} - \frac{3b}{5a^2} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx} \sqrt{-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^3+a*x)^(1/2),x)`

[Out]
$$-2/5*(b*x^3+a*x)^{(1/2)}/a/x^3+6/5*(b*x^2+a)*b/a^2/(x*(b*x^2+a))^{(1/2)}-3/5/a^2*b*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^6 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)/(b*x^6 + a*x^4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{x(a + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**3+a*x)**(1/2),x)`

[Out] `Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)`

$$3.65 \quad \int \frac{x^7}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=161

$$\frac{15a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} - \frac{15a\sqrt{ax+bx^3}}{7b^3} - \frac{x^5}{b\sqrt{ax+bx^3}}$$

[Out] $-(x^5/(b\sqrt{a*x + b*x^3})) - (15*a*\sqrt{a*x + b*x^3})/(7*b^3) + (9*x^2*\sqrt{a*x + b*x^3})/(7*b^2) + (15*a^{(7/4)}*\sqrt{x}*(\sqrt{a} + \sqrt{b}*x)*\sqrt{(a + b*x^2)/(\sqrt{a} + \sqrt{b}*x)^2}*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\sqrt{x})/a^{(1/4)}], 1/2])/(14*b^{(13/4)}*\sqrt{a*x + b*x^3})$

Rubi [A] time = 0.193408, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2022, 2024, 2011, 329, 220}

$$\frac{15a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} - \frac{15a\sqrt{ax+bx^3}}{7b^3} - \frac{x^5}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x + b*x^3)^(3/2), x]

[Out] $-(x^5/(b\sqrt{a*x + b*x^3})) - (15*a*\sqrt{a*x + b*x^3})/(7*b^3) + (9*x^2*\sqrt{a*x + b*x^3})/(7*b^2) + (15*a^{(7/4)}*\sqrt{x}*(\sqrt{a} + \sqrt{b}*x)*\sqrt{(a + b*x^2)/(\sqrt{a} + \sqrt{b}*x)^2}*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\sqrt{x})/a^{(1/4)}], 1/2])/(14*b^{(13/4)}*\sqrt{a*x + b*x^3})$

Rule 2022

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a*x^j + b*x^n)^(p+1))/(b*(n-j)*(p+1)), x] - Dist[(c^n*(m+j*p-n+j+1))/(b*(n-j)*(p+1)), Int[(c*x)^(m-n)*(a*x^j + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax + bx^3)^{3/2}} dx &= -\frac{x^5}{b\sqrt{ax + bx^3}} + \frac{9 \int \frac{x^4}{\sqrt{ax+bx^3}} dx}{2b} \\
&= -\frac{x^5}{b\sqrt{ax + bx^3}} + \frac{9x^2\sqrt{ax + bx^3}}{7b^2} - \frac{(45a) \int \frac{x^2}{\sqrt{ax+bx^3}} dx}{14b^2} \\
&= -\frac{x^5}{b\sqrt{ax + bx^3}} - \frac{15a\sqrt{ax + bx^3}}{7b^3} + \frac{9x^2\sqrt{ax + bx^3}}{7b^2} + \frac{(15a^2) \int \frac{1}{\sqrt{ax+bx^3}} dx}{14b^3} \\
&= -\frac{x^5}{b\sqrt{ax + bx^3}} - \frac{15a\sqrt{ax + bx^3}}{7b^3} + \frac{9x^2\sqrt{ax + bx^3}}{7b^2} + \frac{(15a^2\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{14b^3\sqrt{ax + bx^3}} \\
&= -\frac{x^5}{b\sqrt{ax + bx^3}} - \frac{15a\sqrt{ax + bx^3}}{7b^3} + \frac{9x^2\sqrt{ax + bx^3}}{7b^2} + \frac{(15a^2\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{7b^3\sqrt{ax + bx^3}} \\
&= -\frac{x^5}{b\sqrt{ax + bx^3}} - \frac{15a\sqrt{ax + bx^3}}{7b^3} + \frac{9x^2\sqrt{ax + bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}+\sqrt{bx}}\right)\right)}{14b^{13/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0332699, size = 80, normalized size = 0.5

$$\frac{x \left(15a^2 \sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right) - 15a^2 - 6abx^2 + 2b^2x^4 \right)}{7b^3 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x + b*x^3)^(3/2), x]

[Out] (x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^2)/a]))/(7*b^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.018, size = 172, normalized size = 1.1

$$-\frac{a^2x}{b^3} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{2x^2}{7b^2} \sqrt{bx^3 + ax} - \frac{8a}{7b^3} \sqrt{bx^3 + ax} + \frac{15a^2}{14b^4} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^3+a*x)^(3/2),x)`

[Out]
$$-1/b^3*x*a^2/((1/b*a+x^2)*b*x)^{(1/2)}+2/7*x^2*(b*x^3+a*x)^{(1/2)}/b^2-8/7*a*(b*x^3+a*x)^{(1/2)}/b^3+15/14*a^2/b^4*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*EllipticF(((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^7/(b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}x^5}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^5/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**3+a*x)**(3/2),x)

[Out] Integral(x**7/(x*(a + b*x**2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)

3.66

$$\int \frac{x^6}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=279

$$\frac{21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}} + \frac{21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5b^{11/4}\sqrt{ax+bx^3}}$$

[Out] $-(x^4/(b\sqrt{ax+bx^3})) - (21a^{5/4}(a+bx^2)/(5b^{5/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3})) + (7x\sqrt{ax+bx^3})/(5b^2) + (21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})E\text{llipticE}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(5b^{11/4}\sqrt{ax+bx^3}) - (21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})E\text{llipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(10b^{11/4}\sqrt{ax+bx^3})$

Rubi [A] time = 0.268844, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2022, 2024, 2032, 329, 305, 220, 1196}

$$\frac{21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}} + \frac{21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{5b^{11/4}\sqrt{ax+bx^3}} +$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x + b*x^3)^(3/2), x]

[Out] $-(x^4/(b\sqrt{ax+bx^3})) - (21a^{5/4}(a+bx^2)/(5b^{5/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3})) + (7x\sqrt{ax+bx^3})/(5b^2) + (21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})E\text{llipticE}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(5b^{11/4}\sqrt{ax+bx^3}) - (21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})E\text{llipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(10b^{11/4}\sqrt{ax+bx^3})$

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*(p + 1)), x]
- Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x]
- Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x]
/; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]
/; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196


```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax + bx^3)^{3/2}} dx &= -\frac{x^4}{b\sqrt{ax + bx^3}} + \frac{7 \int \frac{x^3}{\sqrt{ax + bx^3}} dx}{2b} \\
&= -\frac{x^4}{b\sqrt{ax + bx^3}} + \frac{7x\sqrt{ax + bx^3}}{5b^2} - \frac{(21a) \int \frac{x}{\sqrt{ax + bx^3}} dx}{10b^2} \\
&= -\frac{x^4}{b\sqrt{ax + bx^3}} + \frac{7x\sqrt{ax + bx^3}}{5b^2} - \frac{(21a\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a + bx^2}} dx}{10b^2\sqrt{ax + bx^3}} \\
&= -\frac{x^4}{b\sqrt{ax + bx^3}} + \frac{7x\sqrt{ax + bx^3}}{5b^2} - \frac{(21a\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{5b^2\sqrt{ax + bx^3}} \\
&= -\frac{x^4}{b\sqrt{ax + bx^3}} + \frac{7x\sqrt{ax + bx^3}}{5b^2} - \frac{(21a^{3/2}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{ax + bx^3}} + \frac{(21a^{3/2}\sqrt{x}\sqrt{a + bx^2})}{5b^{5/2}\sqrt{ax + bx^3}} \\
&= -\frac{x^4}{b\sqrt{ax + bx^3}} - \frac{21ax(a + bx^2)}{5b^{5/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{7x\sqrt{ax + bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})}}}{5b^{11/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0262226, size = 68, normalized size = 0.24

$$\frac{2x^2 \left(7a\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 7a + bx^2 \right)}{5b^2\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x + b*x^3)^(3/2), x]

[Out] $(2*x^2*(-7*a + b*x^2 + 7*a*\text{Sqrt}[1 + (b*x^2)/a]*\text{Hypergeometric2F1}[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(5*b^2*\text{Sqrt}[x*(a + b*x^2)])$

Maple [A] time = 0.015, size = 200, normalized size = 0.7

$$\frac{ax^2}{b^2} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{2x}{5b^2} \sqrt{bx^3 + ax} - \frac{21a}{10b^3} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx\frac{1}{\sqrt{-ab}}\left(-2\frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^3+a*x)^(3/2),x)`

[Out] $1/b^2*x^2*a/((1/b*a+x^2)*b*x)^{(1/2)}+2/5*x*(b*x^3+a*x)^{(1/2)}/b^2-21/10/b^3*a*(-a*b)^{(1/2)}*((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*\text{EllipticE}(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*\text{EllipticF}(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^6/(b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}x^4}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

$$3.67 \quad \int \frac{x^5}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=137

$$\frac{5a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}} + \frac{5\sqrt{ax+bx^3}}{3b^2} - \frac{x^3}{b\sqrt{ax+bx^3}}$$

[Out] $-(x^3/(b\sqrt{a*x + b*x^3})) + (5*\sqrt{a*x + b*x^3})/(3*b^2) - (5*a^{(3/4)}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^{(1/4)}*Sqrt[x])/a^{(1/4)}], 1/2])/(6*b^{(9/4)}*Sqrt[a*x + b*x^3])$

Rubi [A] time = 0.147964, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2022, 2024, 2011, 329, 220}

$$\frac{5a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}} + \frac{5\sqrt{ax+bx^3}}{3b^2} - \frac{x^3}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x + b*x^3)^(3/2), x]

[Out] $-(x^3/(b\sqrt{a*x + b*x^3})) + (5*\sqrt{a*x + b*x^3})/(3*b^2) - (5*a^{(3/4)}*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^{(1/4)}*Sqrt[x])/a^{(1/4)}], 1/2])/(6*b^{(9/4)}*Sqrt[a*x + b*x^3])$

Rule 2022

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(ax + bx^3)^{3/2}} dx &= -\frac{x^3}{b\sqrt{ax + bx^3}} + \frac{5 \int \frac{x^2}{\sqrt{ax + bx^3}} dx}{2b} \\
&= -\frac{x^3}{b\sqrt{ax + bx^3}} + \frac{5\sqrt{ax + bx^3}}{3b^2} - \frac{(5a) \int \frac{1}{\sqrt{ax + bx^3}} dx}{6b^2} \\
&= -\frac{x^3}{b\sqrt{ax + bx^3}} + \frac{5\sqrt{ax + bx^3}}{3b^2} - \frac{(5a\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{6b^2\sqrt{ax + bx^3}} \\
&= -\frac{x^3}{b\sqrt{ax + bx^3}} + \frac{5\sqrt{ax + bx^3}}{3b^2} - \frac{(5a\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{3b^2\sqrt{ax + bx^3}} \\
&= -\frac{x^3}{b\sqrt{ax + bx^3}} + \frac{5\sqrt{ax + bx^3}}{3b^2} - \frac{5a^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6b^{9/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0287506, size = 67, normalized size = 0.49

$$\frac{x \left(-5a \sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right) + 5a + 2bx^2 \right)}{3b^2 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x + b*x^3)^(3/2), x]

[Out] (x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^2)/a]))/(3*b^2*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.015, size = 147, normalized size = 1.1

$$\frac{ax}{b^2} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{2}{3b^2} \sqrt{bx^3 + ax} - \frac{5a}{6b^3} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^3+a*x)^(3/2),x)`

[Out] $\frac{1}{b^2} \frac{x^5}{(bx^3+ax)^{3/2}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(bx^3 + ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^5/(b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}x^3}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^3/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(x(a + bx^2))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```


$$3.68 \quad \int \frac{x^4}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=253

$$\frac{3\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}} + \frac{3x(a+bx^2)}{b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}}{b^{7/4}\sqrt{ax+bx^3}}$$

[Out] $-(x^2/(b\sqrt{ax+bx^3})) + (3*x*(a+bx^2)/(b^{3/2}*(\sqrt{a} + \sqrt{bx})*\sqrt{ax+bx^3})) - (3*a^{1/4}*\sqrt{x}*(\sqrt{a} + \sqrt{bx})*\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})*x})^2*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\sqrt{x})/a^{1/4}], 1/2])/(b^{7/4}*\sqrt{ax+bx^3}) + (3*a^{1/4}*\sqrt{x}*(\sqrt{a} + \sqrt{bx})*\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})*x})^2*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\sqrt{x})/a^{1/4}], 1/2])/(2*b^{7/4}*\sqrt{ax+bx^3})$

Rubi [A] time = 0.213289, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2022, 2032, 329, 305, 220, 1196}

$$\frac{3x(a+bx^2)}{b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} + \frac{3\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}}{b^{7/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x + b*x^3)^(3/2), x]

[Out] $-(x^2/(b\sqrt{ax+bx^3})) + (3*x*(a+bx^2)/(b^{3/2}*(\sqrt{a} + \sqrt{bx})*\sqrt{ax+bx^3})) - (3*a^{1/4}*\sqrt{x}*(\sqrt{a} + \sqrt{bx})*\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})*x})^2*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\sqrt{x})/a^{1/4}], 1/2])/(b^{7/4}*\sqrt{ax+bx^3}) + (3*a^{1/4}*\sqrt{x}*(\sqrt{a} + \sqrt{bx})*\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})*x})^2*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\sqrt{x})/a^{1/4}], 1/2])/(2*b^{7/4}*\sqrt{ax+bx^3})$

Rule 2022

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a*x^j + b*x^n)^(p+1))/(b*(n-j)*

$(p + 1)), x] - \text{Dist}[(c^n * (m + j * p - n + j + 1)) / (b * (n - j) * (p + 1)), \text{Int}[(c * x)^{m - n} * (a * x^j + b * x^n)^{p + 1}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j * p + 1, n - j]

Rule 2032

$\text{Int}[(c * x)^m * (a * x^j + b * x^n)^p, x_Symbol] := \text{Dist}[(c^{\text{IntPart}[m]} * (c * x)^{\text{FracPart}[m]} * (a * x^j + b * x^n)^{\text{FracPart}[p]}) / (x^{\text{FracPart}[m] + j * \text{FracPart}[p]} * (a + b * x^{n - j})^{\text{FracPart}[p]}), \text{Int}[x^{m + j * p} * (a + b * x^{n - j})^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

$\text{Int}[(c * x)^m * (a + b * x^n)^p, x_Symbol] := \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k * (m + 1) - 1} * (a + (b * x^{k * n})) / c^n]^p, x], x, (c * x)^{1/k}], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

$\text{Int}[x^2 / \text{Sqrt}[a + b * x^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b/a, 2]\}, \text{Dist}[1/q, \text{Int}[1 / \text{Sqrt}[a + b * x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q * x^2) / \text{Sqrt}[a + b * x^4], x], x] /;$ FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

$\text{Int}[1 / \text{Sqrt}[a + b * x^4], x_Symbol] := \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2 * x^2) * \text{Sqrt}[(a + b * x^4) / (a * (1 + q^2 * x^2)^2)] * \text{EllipticF}[2 * \text{ArcTan}[q * x], 1/2]) / (2 * q * \text{Sqrt}[a + b * x^4]), x] /;$ FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

$\text{Int}[(d + e * x^2) / \text{Sqrt}[a + c * x^4], x_Symbol] := \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d * x * \text{Sqrt}[a + c * x^4]) / (a * (1 + q^2 * x^2)), x] + \text{Simp}[(d * (1 + q^2 * x^2) * \text{Sqrt}[(a + c * x^4) / (a * (1 + q^2 * x^2)^2)] * \text{EllipticE}[2 * \text{ArcTan}[q * x], 1/2]) / (q * \text{Sqrt}[a + c * x^4]), x] /;$ EqQ[e + d * q^2, 0] /;

FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(ax + bx^3)^{3/2}} dx &= -\frac{x^2}{b\sqrt{ax + bx^3}} + \frac{3 \int \frac{x}{\sqrt{ax+bx^3}} dx}{2b} \\
&= -\frac{x^2}{b\sqrt{ax + bx^3}} + \frac{(3\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{2b\sqrt{ax + bx^3}} \\
&= -\frac{x^2}{b\sqrt{ax + bx^3}} + \frac{(3\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{b\sqrt{ax + bx^3}} \\
&= -\frac{x^2}{b\sqrt{ax + bx^3}} + \frac{(3\sqrt{a}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{ax + bx^3}} - \frac{(3\sqrt{a}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{ax + bx^3}} \\
&= -\frac{x^2}{b\sqrt{ax + bx^3}} + \frac{3x(a + bx^2)}{b^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{b^{7/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0216261, size = 57, normalized size = 0.23

$$-\frac{2x^2 \left(\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 1 \right)}{b\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x + b*x^3)^(3/2), x]

[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a])*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])/(b*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.016, size = 182, normalized size = 0.7

$$-\frac{x^2}{b} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{3}{2b^2} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx\frac{1}{\sqrt{-ab}}} \left(-2\frac{\sqrt{-ab}}{b} \text{EllipticE}\left(\sqrt{\frac{a+bx^2}{a+bx^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^3+a*x)^(3/2),x)`

[Out]
$$-1/b*x^2/((1/b*a+x^2)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*\text{EllipticE}(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*\text{EllipticF}(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}x^2}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

$$3.69 \quad \int \frac{x^3}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{ab^{5/4}}\sqrt{ax+bx^3}} - \frac{x}{b\sqrt{ax+bx^3}}$$

[Out] $-(x/(b\sqrt{ax+bx^3})) + (\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(2a^{1/4}b^{5/4}\sqrt{ax+bx^3})$

Rubi [A] time = 0.0998051, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2022, 2011, 329, 220}

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{ab^{5/4}}\sqrt{ax+bx^3}} - \frac{x}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/(a*x + b*x^3)^{(3/2)}, x]$

[Out] $-(x/(b\sqrt{ax+bx^3})) + (\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{(a+bx^2)/(\sqrt{a} + \sqrt{bx})^2})\text{EllipticF}[2\text{ArcTan}[(b^{1/4}\sqrt{x})/a^{1/4}], 1/2])/(2a^{1/4}b^{5/4}\sqrt{ax+bx^3})$

Rule 2022

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] := \text{Simp}[(c^{(n-1)})(c*x)^{(m-n+1)}(a*x^j + b*x^n)^{(p+1)}/(b*(n-j)*(p+1)), x] - \text{Dist}[(c^n*(m+j*p-n+j+1))/(b*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-n)}(a*x^j + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+j*p+1, n-j]$

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(ax + bx^3)^{3/2}} dx &= -\frac{x}{b\sqrt{ax + bx^3}} + \frac{\int \frac{1}{\sqrt{ax + bx^3}} dx}{2b} \\ &= -\frac{x}{b\sqrt{ax + bx^3}} + \frac{(\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{2b\sqrt{ax + bx^3}} \\ &= -\frac{x}{b\sqrt{ax + bx^3}} + \frac{(\sqrt{x}\sqrt{a + bx^2}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{b\sqrt{ax + bx^3}} \\ &= -\frac{x}{b\sqrt{ax + bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{ab^{5/4}}\sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0196673, size = 54, normalized size = 0.47

$$\frac{x \left(\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right) - 1 \right)}{b\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3)^(3/2),x]

[Out] (x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])))/(b*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.015, size = 130, normalized size = 1.1

$$-\frac{x}{b} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{1}{2b^2} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx} \frac{1}{\sqrt{-ab}} \text{EllipticF}\left(\sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x)^(3/2),x)

[Out] -1/b*x/((1/b*a+x^2)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**3+a*x)**(3/2),x)`

[Out] `Integral(x**3/(x*(a + b*x**2))**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^3/(b*x^3 + a*x)^(3/2), x)`

3.70 $\int \frac{x^2}{(ax+bx^3)^{3/2}} dx$

Optimal. Leaf size=254

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}} + \frac{x^2}{a\sqrt{ax+bx^3}}$$

[Out] $x^2/(a*\text{Sqrt}[a*x + b*x^3]) - (x*(a + b*x^2))/(a*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) + (\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[x])/a^(1/4)], 1/2])/ (a^(3/4)*b^(3/4)*\text{Sqrt}[a*x + b*x^3]) - (\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[x])/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.209029, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2023, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}} + \frac{x^2}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(a*x + b*x^3)^(3/2), x]$

[Out] $x^2/(a*\text{Sqrt}[a*x + b*x^3]) - (x*(a + b*x^2))/(a*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[a*x + b*x^3]) + (\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[x])/a^(1/4)], 1/2])/ (a^(3/4)*b^(3/4)*\text{Sqrt}[a*x + b*x^3]) - (\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[x])/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*\text{Sqrt}[a*x + b*x^3])$

Rule 2023

$\text{Int}[\frac{(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}}{(c_*)^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)}}/(a*(n-j)*(p+1)), x] + \text{Dist}[\frac{(c^j*(m+n*p+n-j+1))}{(a*(n-j)*(p+1))}, \text{Int}[\frac{(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}}{(c_*)^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)}}/(a*(n-j)*(p+1)), x]$

```
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2])*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(ax + bx^3)^{3/2}} dx &= \frac{x^2}{a\sqrt{ax + bx^3}} - \frac{\int \frac{x}{\sqrt{ax+bx^3}} dx}{2a} \\
&= \frac{x^2}{a\sqrt{ax + bx^3}} - \frac{(\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{2a\sqrt{ax + bx^3}} \\
&= \frac{x^2}{a\sqrt{ax + bx^3}} - \frac{(\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{a\sqrt{ax + bx^3}} \\
&= \frac{x^2}{a\sqrt{ax + bx^3}} - \frac{(\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax + bx^3}} + \frac{(\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1-\frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax + bx^3}} \\
&= \frac{x^2}{a\sqrt{ax + bx^3}} - \frac{x(a + bx^2)}{a\sqrt{b}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0152682, size = 56, normalized size = 0.22

$$\frac{2x^2\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3a\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x + b*x^3)^(3/2),x]

[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -(b*x^2)/a])/(3*a*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.015, size = 184, normalized size = 0.7

$$\frac{x^2}{a} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} - \frac{1}{2ab} \sqrt{-ab} \sqrt{b\left(x + \frac{1}{b}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{b}{\sqrt{-ab}}\left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx\frac{1}{\sqrt{-ab}}\left(-2\frac{\sqrt{-ab}}{b}\text{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-a}}}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^3+a*x)^(3/2),x)`

[Out] $x^2/a/((1/b*a+x^2)*b*x)^{(1/2)}-1/2/a/b*(-a*b)^{(1/2)}*((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF((x+1/b*(-a*b)^{(1/2)})*b/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^3 + a*x)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^3 + a*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

$$3.71 \quad \int \frac{x}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=114

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{x}{a\sqrt{ax+bx^3}}$$

[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.083586, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2023, 2011, 329, 220}

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{x}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3)^(3/2), x]

[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(ax + bx^3)^{3/2}} dx &= \frac{x}{a\sqrt{ax + bx^3}} + \frac{\int \frac{1}{\sqrt{ax + bx^3}} dx}{2a} \\ &= \frac{x}{a\sqrt{ax + bx^3}} + \frac{(\sqrt{x}\sqrt{a + bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a + bx^2}} dx}{2a\sqrt{ax + bx^3}} \\ &= \frac{x}{a\sqrt{ax + bx^3}} + \frac{(\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^4}} dx, x, \sqrt{x}\right)}{a\sqrt{ax + bx^3}} \\ &= \frac{x}{a\sqrt{ax + bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a + bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax + bx^3}} \end{aligned}$$

Mathematica [C] time = 0.0192975, size = 54, normalized size = 0.47

$$\frac{x\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^2}{a}\right) + x}{a\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3)^(3/2),x]

[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(a*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.016, size = 132, normalized size = 1.2

$$\frac{x}{a} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right)bx}} + \frac{1}{2ab} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx} \frac{1}{\sqrt{-ab}} \text{EllipticF} \left(\sqrt{b \left(x + \frac{1}{b} \sqrt{-ab}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x)^(3/2),x)

[Out] x/a/((1/b*a+x^2)*b*x)^(1/2)+1/2/a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^3 + ax}}{b^2x^5 + 2abx^3 + a^2x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^5 + 2*a*b*x^3 + a^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x/(x*(a + b*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

$$3.72 \quad \int \frac{1}{(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=273

$$\frac{3\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}} + \frac{3\sqrt{bx}(a+bx^2)}{a^2(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{2a^{7/4}\sqrt{ax+bx^3}}$$

[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.223823, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2006, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{bx}(a+bx^2)}{a^2(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{3\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{b}\sqrt{x}(\sqrt{a} + \sqrt{bx})}{2a^{7/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)^(-3/2), x]

[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a*x + b*x^3])

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(a*x^j +
  b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
  (a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
  b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
  (1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

$1 + q^2 x^2) \sqrt{(a + c x^4)/(a(1 + q^2 x^2)^2)} \text{EllipticE}[2 \text{ArcTan}[q x], 1/2]) / (q \sqrt{a + c x^4}), x] /; \text{EqQ}[e + d q^2, 0] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(ax + bx^3)^{3/2}} dx &= \frac{1}{a\sqrt{ax + bx^3}} + \frac{3 \int \frac{1}{x\sqrt{ax+bx^3}} dx}{2a} \\
 &= \frac{1}{a\sqrt{ax + bx^3}} - \frac{3\sqrt{ax + bx^3}}{a^2 x} + \frac{(3b) \int \frac{x}{\sqrt{ax+bx^3}} dx}{2a^2} \\
 &= \frac{1}{a\sqrt{ax + bx^3}} - \frac{3\sqrt{ax + bx^3}}{a^2 x} + \frac{(3b\sqrt{x}\sqrt{a + bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{2a^2\sqrt{ax + bx^3}} \\
 &= \frac{1}{a\sqrt{ax + bx^3}} - \frac{3\sqrt{ax + bx^3}}{a^2 x} + \frac{(3b\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{a^2\sqrt{ax + bx^3}} \\
 &= \frac{1}{a\sqrt{ax + bx^3}} - \frac{3\sqrt{ax + bx^3}}{a^2 x} + \frac{(3\sqrt{b}\sqrt{x}\sqrt{a + bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{a^{3/2}\sqrt{ax + bx^3}} - \frac{(3\sqrt{b}\sqrt{x}\sqrt{a + bx^2})}{a} \\
 &= \frac{1}{a\sqrt{ax + bx^3}} + \frac{3\sqrt{bx}(a + bx^2)}{a^2(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{3\sqrt{ax + bx^3}}{a^2 x} - \frac{3^4\sqrt{b}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \arctan\left(\frac{\sqrt{bx}}{\sqrt{a} + \sqrt{bx}}\right)\right)}{a^{7/4}\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [C] time = 0.0127408, size = 51, normalized size = 0.19

$$\frac{2\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(-\frac{1}{4}, \frac{3}{2}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{a\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)^(-3/2), x]

[Out] $(-2\sqrt{1 + (b*x^2)/a} \text{Hypergeometric2F1}[-1/4, 3/2, 3/4, -((b*x^2)/a)]) / (a \sqrt{x(a + b*x^2)})$

Maple [A] time = 0.018, size = 206, normalized size = 0.8

$$-2 \frac{bx^2 + a}{a^2 \sqrt{x(bx^2 + a)}} - \frac{bx^2}{a^2} \frac{1}{\sqrt{\left(\frac{a}{b} + x^2\right) bx}} + \frac{3}{2a^2} \sqrt{-ab} \sqrt{b \left(x + \frac{1}{b} \sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2 \frac{b}{\sqrt{-ab}} \left(x - \frac{\sqrt{-ab}}{b}\right)} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \left(-2 \sqrt{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a*x)^(3/2),x)

[Out] $-2*(b*x^2+a)/a^2/(x*(b*x^2+a))^{(1/2)} - b*x^2/a^2/((1/b*a+x^2)*b*x)^{(1/2)} + 3/2/a^2*(-a*b)^{(1/2)}*((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x-1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}/(b*x^3+a*x)^{(1/2)}*(-2/b*(-a*b)^{(1/2)}*EllipticE(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})+1/b*(-a*b)^{(1/2)}*EllipticF(((x+1/b*(-a*b))^{(1/2)}*b/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^6 + 2abx^4 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + bx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

$$3.73 \quad \int \frac{1}{x(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=139

$$\frac{5b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}\text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} + \frac{1}{ax\sqrt{ax+bx^3}}$$

[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.14217, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2023, 2025, 2011, 329, 220}

$$\frac{5b^{3/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} + \frac{1}{ax\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3)^(3/2)), x]

[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x^3])

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x]
+ Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
```


Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax+bx^3)^{3/2}} dx &= \frac{1}{ax\sqrt{ax+bx^3}} + \frac{5 \int \frac{1}{x^2\sqrt{ax+bx^3}} dx}{2a} \\
&= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{(5b) \int \frac{1}{\sqrt{ax+bx^3}} dx}{6a^2} \\
&= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{(5b\sqrt{x}\sqrt{a+bx^2}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^2}} dx}{6a^2\sqrt{ax+bx^3}} \\
&= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{(5b\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{x}\right)}{3a^2\sqrt{ax+bx^3}} \\
&= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0168496, size = 56, normalized size = 0.4

$$\frac{2\sqrt{\frac{bx^2}{a}+1} {}_2F_1\left(-\frac{3}{4}, \frac{3}{2}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3ax\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]

[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(3*a*x*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.018, size = 150, normalized size = 1.1

$$-\frac{2}{3a^2x^2}\sqrt{bx^3+ax} - \frac{bx}{a^2}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}} - \frac{5}{6a^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x)^(3/2),x)

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-b*x/a^2/((1/b*a+x^2)*b*x)^(1/2)-5/6/a^2*(-a*
b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2)
))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Ellipt
icF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^7 + 2abx^5 + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

[Out] Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)

$$3.74 \quad \int \frac{1}{x^2(ax+bx^3)^{3/2}} dx$$

Optimal. Leaf size=306

$$\frac{21b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} + \frac{21b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticE}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}$$

[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*Sqrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])

Rubi [A] time = 0.322572, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{21b^{3/2}x(a+bx^2)}{5a^3(\sqrt{a} + \sqrt{bx})\sqrt{ax+bx^3}} - \frac{21b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}} + \frac{21b^{5/4}\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \text{EllipticE}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]

[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*Sqrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])

Rule 2023

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax+bx^3)^{3/2}} dx &= \frac{1}{ax^2\sqrt{ax+bx^3}} + \frac{7 \int \frac{1}{x^3\sqrt{ax+bx^3}} dx}{2a} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} - \frac{(21b) \int \frac{1}{x\sqrt{ax+bx^3}} dx}{10a^2} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^2) \int \frac{x}{\sqrt{ax+bx^3}} dx}{10a^3} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^2\sqrt{x}\sqrt{a+bx^2}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2}} dx}{10a^3\sqrt{ax+bx^3}} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^2\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}} dx, x, \sqrt{ax+bx^3}\right)}{5a^3\sqrt{ax+bx^3}} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} dx, x, \sqrt{ax+bx^3}\right)}{5a^{5/2}\sqrt{ax+bx^3}} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} + \frac{21b^{5/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})}{5a^3\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.0162168, size = 56, normalized size = 0.18

$$\frac{2\sqrt{\frac{bx^2}{a}} + 1 {}_2F_1\left(-\frac{5}{4}, \frac{3}{2}; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5ax^2\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3)^(3/2)), x]

[Out] $(-2\sqrt{1 + (b*x^2)/a} * \text{Hypergeometric2F1}[-5/4, 3/2, -1/4, -((b*x^2)/a)]) / (5*a*x^2*\sqrt{x*(a + b*x^2)})$

Maple [A] time = 0.02, size = 228, normalized size = 0.8

$$-\frac{2}{5x^3a^2}\sqrt{bx^3+ax} + \frac{(16bx^2+16a)b}{5a^3}\frac{1}{\sqrt{x(bx^2+a)}} + \frac{b^2x^2}{a^3}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}} - \frac{21b}{10a^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{b}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^3+a*x)^(3/2),x)`

[Out] $-2/5*(b*x^3+a*x)^(1/2)/x^3/a^2+16/5*(b*x^2+a)*b/a^3/(x*(b*x^2+a))^(1/2)+b^2*x^2/a^3/((1/b*a+x^2)*b*x)^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+1/b*(-a*b))^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b))^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*\text{EllipticE}((x+1/b*(-a*b))^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*\text{EllipticF}((x+1/b*(-a*b))^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^8+2abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (x(a + bx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

$$3.75 \quad \int \frac{x^{29/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=159

$$\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}} - \frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{(25/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (9*x^{(19/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (3*x^{(13/2)})/(5*b^3*(a*x + b*x^3)^{(3/2)}) - (3*x^{(7/2)})/(b^4*\text{Sqrt}[a*x + b*x^3]) + (9*\text{Sqrt}[x]*\text{Sqrt}[a*x + b*x^3])/(2*b^5) - (9*a*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x + b*x^3]])/(2*b^{(11/2)})$

Rubi [A] time = 0.248376, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2022, 2024, 2029, 206}

$$\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}} - \frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(29/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $-x^{(25/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (9*x^{(19/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (3*x^{(13/2)})/(5*b^3*(a*x + b*x^3)^{(3/2)}) - (3*x^{(7/2)})/(b^4*\text{Sqrt}[a*x + b*x^3]) + (9*\text{Sqrt}[x]*\text{Sqrt}[a*x + b*x^3])/(2*b^5) - (9*a*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x + b*x^3]])/(2*b^{(11/2)})$

Rule 2022

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)})/(b*(n-j)*(p+1)), x] - \text{Dist}[(c^n*(m+j*p-n+j+1))/(b*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+j*p+1, n-j]$

Rule 2024

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

```

Rule 2029

```

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{29/2}}{(ax + bx^3)^{9/2}} dx &= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} + \frac{9 \int \frac{x^{23/2}}{(ax+bx^3)^{7/2}} dx}{7b} \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} + \frac{9 \int \frac{x^{17/2}}{(ax+bx^3)^{5/2}} dx}{5b^2} \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax + bx^3)^{3/2}} + \frac{3 \int \frac{x^{11/2}}{(ax+bx^3)^{3/2}} dx}{b^3} \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax + bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax + bx^3}} + \frac{9 \int \frac{x^{5/2}}{\sqrt{ax+bx^3}} dx}{b^4} \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax + bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax + bx^3}} + \frac{9\sqrt{x}\sqrt{ax + bx^3}}{2b^5} - \dots \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax + bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax + bx^3}} + \frac{9\sqrt{x}\sqrt{ax + bx^3}}{2b^5} - \dots \\
&= -\frac{x^{25/2}}{7b(ax + bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax + bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax + bx^3}} + \frac{9\sqrt{x}\sqrt{ax + bx^3}}{2b^5} - \dots
\end{aligned}$$

Mathematica [A] time = 0.18079, size = 130, normalized size = 0.82

$$\frac{\sqrt{x} \left(\sqrt{bx} (1218a^2b^2x^4 + 1050a^3bx^2 + 315a^4 + 528ab^3x^6 + 35b^4x^8) - \frac{315\sqrt{a}(a+bx^2)^4 \sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}} \right)}{70b^{11/2} (a + bx^2)^3 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2), x]

[Out] (Sqrt[x]*(Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^3*x^6 + 35*b^4*x^8) - (315*Sqrt[a]*(a + b*x^2)^4*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a])/(70*b^(11/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.04, size = 212, normalized size = 1.3

$$-\frac{1}{70(bx^2+a)^4} \sqrt{x(bx^2+a)} \left(-35x^9b^{9/2} + 315 \ln(x\sqrt{b} + \sqrt{bx^2+a}) x^6ab^3\sqrt{bx^2+a} - 528b^{7/2}x^7a + 945 \ln(x\sqrt{b} + \sqrt{bx^2+a}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(29/2)/(b*x^3+a*x)^(9/2), x)

[Out]
$$-1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*\ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^6*a*b^3*(b*x^2+a)^(1/2)-528*b^(7/2)*x^7*a+945*\ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*b^(5/2)*x^5*a^2+945*\ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^2*a^3*b*(b*x^2+a)^(1/2)-1050*b^(3/2)*x^3*a^3+315*\ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*x*a^4)/x^(1/2)/(b*x^2+a)^4$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{29}{2}}}{(bx^3+ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2), x, algorithm="maxima")

[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.58916, size = 845, normalized size = 5.31

$$\left[\frac{315(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5)\sqrt{b} \log(2bx^2 - 2\sqrt{bx^3+ax}\sqrt{b}\sqrt{x} + a) + 2(35b^5x^8 + 528ab^4x^6 + 1218a^2b^3x^4 + 1050a^3b^2x^2 + 315a^4b)x^7}{140(b^{10}x^8 + 4ab^9x^6 + 6a^2b^8x^4 + 4a^3b^7x^2 + a^4b^6)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2)) + (35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.32596, size = 123, normalized size = 0.77

$$\frac{\left(\left(x^2\left(\frac{35x^2}{b} + \frac{528a}{b^2}\right) + \frac{1218a^2}{b^3}\right)x^2 + \frac{1050a^3}{b^4}\right)x^2 + \frac{315a^4}{b^5}}{70(bx^2 + a)^{\frac{7}{2}}} + \frac{9a \log\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{2b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2 + 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)

$$3.76 \quad \int \frac{x^{27/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=126

$$-\frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}} - \frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{23/2}/(7*b*(a*x + b*x^3)^{7/2}) - (8*x^{17/2})/(35*b^2*(a*x + b*x^3)^{5/2}) - (16*x^{11/2})/(35*b^3*(a*x + b*x^3)^{3/2}) - (64*x^{5/2})/(35*b^4*\text{Sqrt}[a*x + b*x^3]) + (128*\text{Sqrt}[a*x + b*x^3])/(35*b^5*\text{Sqrt}[x])$

Rubi [A] time = 0.200279, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$-\frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}} - \frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{27/2}/(a*x + b*x^3)^{9/2}, x]$

[Out] $-x^{23/2}/(7*b*(a*x + b*x^3)^{7/2}) - (8*x^{17/2})/(35*b^2*(a*x + b*x^3)^{5/2}) - (16*x^{11/2})/(35*b^3*(a*x + b*x^3)^{3/2}) - (64*x^{5/2})/(35*b^4*\text{Sqrt}[a*x + b*x^3]) + (128*\text{Sqrt}[a*x + b*x^3])/(35*b^5*\text{Sqrt}[x])$

Rule 2015

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c, j, m, n\}, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[j] \parallel \text{GtQ}[c, 0])$

Rule 2014

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)$

*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^{27/2}}{(ax + bx^3)^{9/2}} dx &= -\frac{x^{23/2}}{7b(ax + bx^3)^{7/2}} + \frac{8 \int \frac{x^{21/2}}{(ax+bx^3)^{7/2}} dx}{7b} \\
 &= -\frac{x^{23/2}}{7b(ax + bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax + bx^3)^{5/2}} + \frac{48 \int \frac{x^{15/2}}{(ax+bx^3)^{5/2}} dx}{35b^2} \\
 &= -\frac{x^{23/2}}{7b(ax + bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax + bx^3)^{3/2}} + \frac{64 \int \frac{x^{9/2}}{(ax+bx^3)^{3/2}} dx}{35b^3} \\
 &= -\frac{x^{23/2}}{7b(ax + bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax + bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax + bx^3}} + \frac{128 \int \frac{x^{3/2}}{\sqrt{ax+bx^3}} dx}{35b^4} \\
 &= -\frac{x^{23/2}}{7b(ax + bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax + bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax + bx^3}} + \frac{128\sqrt{ax + bx^3}}{35b^5\sqrt{x}}
 \end{aligned}$$

Mathematica [A] time = 0.0366427, size = 77, normalized size = 0.61

$$\frac{\sqrt{x} (560a^2b^2x^4 + 448a^3bx^2 + 128a^4 + 280ab^3x^6 + 35b^4x^8)}{35b^5(a + bx^2)^3 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2), x]

[Out] (Sqrt[x]*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8))/(35*b^5*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.007, size = 70, normalized size = 0.6

$$\frac{(bx^2 + a)(35x^8b^4 + 280ax^6b^3 + 560a^2x^4b^2 + 448a^3x^2b + 128a^4)}{35b^5} x^{\frac{9}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(27/2)/(b*x^3+a*x)^(9/2),x)`

[Out] $\frac{1}{35}(b^4x^8 + 280ab^3x^6 + 560a^2b^2x^4 + 448a^3bx^2 + 128a^4)x^{9/2}/b^5/(b^3x^3 + ax)^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{27}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")`

[Out] `integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)`

Fricas [A] time = 1.46616, size = 234, normalized size = 1.86

$$\frac{(35b^4x^8 + 280ab^3x^6 + 560a^2b^2x^4 + 448a^3bx^2 + 128a^4)\sqrt{bx^3 + ax}\sqrt{x}}{35(b^9x^9 + 4ab^8x^7 + 6a^2b^7x^5 + 4a^3b^6x^3 + a^4b^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")`

[Out] $\frac{1}{35}(35b^4x^8 + 280ab^3x^6 + 560a^2b^2x^4 + 448a^3bx^2 + 128a^4)\sqrt{bx^3 + ax}\sqrt{x}/(b^9x^9 + 4a^3b^6x^3 + a^4b^5x)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28052, size = 108, normalized size = 0.86

$$\frac{35\sqrt{bx^2+a} + \frac{140(bx^2+a)^3 a - 70(bx^2+a)^2 a^2 + 28(bx^2+a)a^3 - 5a^4}{(bx^2+a)^{\frac{7}{2}}}}{35b^5} - \frac{128\sqrt{a}}{35b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(35*sqrt(b*x^2 + a) + (140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/(b*x^2 + a)^(7/2))/b^5 - 128/35*sqrt(a)/b^5
```

$$3.77 \quad \int \frac{x^{25/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=130

$$-\frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}} - \frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{21/2}/(7*b*(a*x + b*x^3)^{(7/2)}) - x^{15/2}/(5*b^2*(a*x + b*x^3)^{(5/2)}) - x^{9/2}/(3*b^3*(a*x + b*x^3)^{(3/2)}) - x^{3/2}/(b^4*\text{Sqrt}[a*x + b*x^3]) + \text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x + b*x^3]]/b^{(9/2)}$

Rubi [A] time = 0.205549, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2022, 2029, 206}

$$-\frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}} - \frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(25/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $-x^{21/2}/(7*b*(a*x + b*x^3)^{(7/2)}) - x^{15/2}/(5*b^2*(a*x + b*x^3)^{(5/2)}) - x^{9/2}/(3*b^3*(a*x + b*x^3)^{(3/2)}) - x^{3/2}/(b^4*\text{Sqrt}[a*x + b*x^3]) + \text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x + b*x^3]]/b^{(9/2)}$

Rule 2022

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)})/(b*(n-j)*(p+1)), x] - \text{Dist}[(c^n*(m+j*p-n+j+1))/(b*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]

Rule 2029

$\text{Int}[(x_*)^{(m_*)}/\text{Sqrt}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[-2/(n-j), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]],$

$x] /; \text{FreeQ}[\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
 \int \frac{x^{25/2}}{(ax + bx^3)^{9/2}} dx &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} + \frac{\int \frac{x^{19/2}}{(ax + bx^3)^{7/2}} dx}{b} \\
 &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax + bx^3)^{5/2}} + \frac{\int \frac{x^{13/2}}{(ax + bx^3)^{5/2}} dx}{b^2} \\
 &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax + bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax + bx^3)^{3/2}} + \frac{\int \frac{x^{7/2}}{(ax + bx^3)^{3/2}} dx}{b^3} \\
 &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax + bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax + bx^3)^{3/2}} - \frac{x^{3/2}}{b^4 \sqrt{ax + bx^3}} + \frac{\int \frac{\sqrt{x}}{\sqrt{ax + bx^3}} dx}{b^4} \\
 &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax + bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax + bx^3)^{3/2}} - \frac{x^{3/2}}{b^4 \sqrt{ax + bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \right)}{b^4} \\
 &= -\frac{x^{21/2}}{7b(ax + bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax + bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax + bx^3)^{3/2}} - \frac{x^{3/2}}{b^4 \sqrt{ax + bx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx^3/2}}{\sqrt{ax + bx^3}}\right)}{b^{9/2}}
 \end{aligned}$$

Mathematica [A] time = 0.252861, size = 120, normalized size = 0.92

$$\frac{\sqrt{x} \left(105 \sqrt{a} (a + bx^2)^3 \sqrt{\frac{bx^2}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{a}} \right) - \sqrt{bx} (350a^2 bx^2 + 105a^3 + 406ab^2 x^4 + 176b^3 x^6) \right)}{105b^{9/2} (a + bx^2)^3 \sqrt{x} (a + bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2), x]

[Out] $(\text{Sqrt}[x] * (-\text{Sqrt}[b] * x * (105 * a^3 + 350 * a^2 * b * x^2 + 406 * a * b^2 * x^4 + 176 * b^3 * x^6)) + 105 * \text{Sqrt}[a] * (a + b * x^2)^3 * \text{Sqrt}[1 + (b * x^2) / a] * \text{ArcSinh}[(\text{Sqrt}[b] * x) / \text{Sqrt}[a]]) / (105 * b^{(9/2)} * (a + b * x^2)^3 * \text{Sqrt}[x * (a + b * x^2)])$

Maple [A] time = 0.022, size = 198, normalized size = 1.5

$$\frac{1}{105 (bx^2 + a)^4} \sqrt{x(bx^2 + a)} \left(105 \ln(x\sqrt{b} + \sqrt{bx^2 + a}) x^6 b^3 \sqrt{bx^2 + a} - 176 x^7 b^{7/2} + 315 \ln(x\sqrt{b} + \sqrt{bx^2 + a}) x^4 a b^2 \sqrt{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(25/2)} / (b * x^3 + a * x)^{(9/2)}, x)$

[Out] $1/105 * (x * (b * x^2 + a))^{(1/2)} / b^{(9/2)} * (105 * \ln(x * b^{(1/2)} + (b * x^2 + a)^{(1/2)}) * x^6 * b^3 * (b * x^2 + a)^{(1/2)} - 176 * x^7 * b^{(7/2)} + 315 * \ln(x * b^{(1/2)} + (b * x^2 + a)^{(1/2)}) * x^4 * a * b^2 * (b * x^2 + a)^{(1/2)} - 406 * b^{(5/2)} * x^5 * a + 315 * \ln(x * b^{(1/2)} + (b * x^2 + a)^{(1/2)}) * x^2 * a^2 * b * (b * x^2 + a)^{(1/2)} - 350 * b^{(3/2)} * x^3 * a^2 + 105 * \ln(x * b^{(1/2)} + (b * x^2 + a)^{(1/2)}) * a^3 * (b * x^2 + a)^{(1/2)} - 105 * b^{(1/2)} * x * a^3) / x^{(1/2)} / (b * x^2 + a)^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{25}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(25/2)} / (b * x^3 + a * x)^{(9/2)}, x, \text{algorithm} = \text{"maxima"})$

[Out] $\text{integrate}(x^{(25/2)} / (b * x^3 + a * x)^{(9/2)}, x)$

Fricas [A] time = 1.47245, size = 783, normalized size = 6.02

$$\left[\frac{105 (b^4 x^8 + 4 a b^3 x^6 + 6 a^2 b^2 x^4 + 4 a^3 b x^2 + a^4) \sqrt{b} \log(2 b x^2 + 2 \sqrt{b x^3 + a x} \sqrt{b} \sqrt{x} + a) - 2 (176 b^4 x^6 + 406 a b^3 x^4 + 350 a^2 b^2 x^2 + 105 a^3) \sqrt{b} \sqrt{x}}{210 (b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2)))) + (176*b^4*x^6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40829, size = 105, normalized size = 0.81

$$\frac{\left(2\left(x^2\left(\frac{88x^2}{b} + \frac{203a}{b^2}\right) + \frac{175a^2}{b^3}\right)x^2 + \frac{105a^3}{b^4}\right)x}{105(bx^2 + a)^{\frac{7}{2}}} - \frac{\log\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

$$3.78 \quad \int \frac{x^{23/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=101

$$-\frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}} - \frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{(19/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (6*x^{(13/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (8*x^{(7/2)})/(35*b^3*(a*x + b*x^3)^{(3/2)}) - (16*sqrt[x])/(35*b^4*sqrt[a*x + b*x^3])$

Rubi [A] time = 0.161451, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$-\frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}} - \frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]

[Out] $-x^{(19/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (6*x^{(13/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (8*x^{(7/2)})/(35*b^3*(a*x + b*x^3)^{(3/2)}) - (16*sqrt[x])/(35*b^4*sqrt[a*x + b*x^3])$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c
*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^{23/2}}{(ax + bx^3)^{9/2}} dx &= -\frac{x^{19/2}}{7b(ax + bx^3)^{7/2}} + \frac{6 \int \frac{x^{17/2}}{(ax + bx^3)^{7/2}} dx}{7b} \\
 &= -\frac{x^{19/2}}{7b(ax + bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax + bx^3)^{5/2}} + \frac{24 \int \frac{x^{11/2}}{(ax + bx^3)^{5/2}} dx}{35b^2} \\
 &= -\frac{x^{19/2}}{7b(ax + bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax + bx^3)^{3/2}} + \frac{16 \int \frac{x^{5/2}}{(ax + bx^3)^{3/2}} dx}{35b^3} \\
 &= -\frac{x^{19/2}}{7b(ax + bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax + bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [A] time = 0.0302392, size = 66, normalized size = 0.65

$$-\frac{\sqrt{x}(56a^2bx^2 + 16a^3 + 70ab^2x^4 + 35b^3x^6)}{35b^4(a + bx^2)^3\sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2), x]

[Out] -(Sqrt[x]*(16*a^3 + 56*a^2*b*x^2 + 70*a*b^2*x^4 + 35*b^3*x^6))/(35*b^4*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.005, size = 59, normalized size = 0.6

$$-\frac{(bx^2 + a)(35x^6b^3 + 70ax^4b^2 + 56a^2x^2b + 16a^3)}{35b^4}x^{\frac{9}{2}}(bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(23/2)/(b*x^3+a*x)^(9/2),x)`

[Out] $-1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^{9/2}/b^4/(b*x^3+a*x)^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{23}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")`

[Out] `integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)`

Fricas [A] time = 1.4131, size = 207, normalized size = 2.05

$$\frac{(35b^3x^6 + 70ab^2x^4 + 56a^2bx^2 + 16a^3)\sqrt{bx^3 + ax}\sqrt{x}}{35(b^8x^9 + 4ab^7x^7 + 6a^2b^6x^5 + 4a^3b^5x^3 + a^4b^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")`

[Out] $-1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*\text{sqrt}(b*x^3 + a*x)*\text{sqrt}(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27062, size = 86, normalized size = 0.85

$$\frac{16}{35\sqrt{a}b^4} - \frac{35(bx^2 + a)^3 - 35(bx^2 + a)^2a + 21(bx^2 + a)a^2 - 5a^3}{35(bx^2 + a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

$$3.79 \quad \int \frac{x^{21/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=25

$$\frac{x^{21/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(21/2)}/(7*a*(a*x + b*x^3)^{(7/2)})$

Rubi [A] time = 0.0369776, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$\frac{x^{21/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(21/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $x^{(21/2)}/(7*a*(a*x + b*x^3)^{(7/2)})$

Rule 2014

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}} dx = \frac{x^{21/2}}{7a(ax+bx^3)^{7/2}}$$

Mathematica [A] time = 0.0168311, size = 25, normalized size = 1.

$$\frac{x^{21/2}}{7a(x(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))

Maple [A] time = 0.004, size = 27, normalized size = 1.1

$$\frac{bx^2 + a}{7a} x^{\frac{23}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x)

[Out] 1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{21}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [B] time = 1.42907, size = 130, normalized size = 5.2

$$\frac{\sqrt{bx^3 + ax} x^{\frac{13}{2}}}{7(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
4*a^4*b*x^2 + a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2361, size = 23, normalized size = 0.92

$$\frac{x^7}{7(bx^2 + a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

$$3.80 \quad \int \frac{x^{19/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=76

$$-\frac{4x^{9/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{3/2}}{105b^3(ax+bx^3)^{3/2}} - \frac{x^{15/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{(15/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (4*x^{(9/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (8*x^{(3/2)})/(105*b^3*(a*x + b*x^3)^{(3/2)})$

Rubi [A] time = 0.118776, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$-\frac{4x^{9/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{3/2}}{105b^3(ax+bx^3)^{3/2}} - \frac{x^{15/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]

[Out] $-x^{(15/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (4*x^{(9/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)}) - (8*x^{(3/2)})/(105*b^3*(a*x + b*x^3)^{(3/2)})$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{19/2}}{(ax + bx^3)^{9/2}} dx &= -\frac{x^{15/2}}{7b(ax + bx^3)^{7/2}} + \frac{4 \int \frac{x^{13/2}}{(ax+bx^3)^{7/2}} dx}{7b} \\
&= -\frac{x^{15/2}}{7b(ax + bx^3)^{7/2}} - \frac{4x^{9/2}}{35b^2(ax + bx^3)^{5/2}} + \frac{8 \int \frac{x^{7/2}}{(ax+bx^3)^{5/2}} dx}{35b^2} \\
&= -\frac{x^{15/2}}{7b(ax + bx^3)^{7/2}} - \frac{4x^{9/2}}{35b^2(ax + bx^3)^{5/2}} - \frac{8x^{3/2}}{105b^3(ax + bx^3)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0256023, size = 55, normalized size = 0.72

$$-\frac{\sqrt{x}(8a^2 + 28abx^2 + 35b^2x^4)}{105b^3(a + bx^2)^3 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2), x]

[Out] -(Sqrt[x]*(8*a^2 + 28*a*b*x^2 + 35*b^2*x^4))/(105*b^3*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.005, size = 48, normalized size = 0.6

$$-\frac{(bx^2 + a)(35x^4b^2 + 28ax^2b + 8a^2)}{105b^3} x^{\frac{9}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(19/2)/(b*x^3+a*x)^(9/2), x)

[Out] -1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{19}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.42644, size = 184, normalized size = 2.42

$$-\frac{(35b^2x^4 + 28abx^2 + 8a^2)\sqrt{bx^3 + ax}\sqrt{x}}{105(b^7x^9 + 4ab^6x^7 + 6a^2b^5x^5 + 4a^3b^4x^3 + a^4b^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.24878, size = 68, normalized size = 0.89

$$\frac{8}{105 a^{\frac{3}{2}} b^3} - \frac{35 (bx^2 + a)^2 - 42 (bx^2 + a)a + 15 a^2}{105 (bx^2 + a)^{\frac{7}{2}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^3)

$$3.81 \quad \int \frac{x^{17/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=51

$$\frac{2x^{15/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{x^{17/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(17/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (2*x^{(15/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)})$

Rubi [A] time = 0.0747788, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$\frac{2x^{15/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{x^{17/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]

[Out] $x^{(17/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (2*x^{(15/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)})$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{17/2}}{(ax + bx^3)^{9/2}} dx = \frac{x^{17/2}}{7a(ax + bx^3)^{7/2}} + \frac{2 \int \frac{x^{15/2}}{(ax+bx^3)^{7/2}} dx}{7a}$$

$$= \frac{x^{17/2}}{7a(ax + bx^3)^{7/2}} + \frac{2x^{15/2}}{35a^2(ax + bx^3)^{5/2}}$$

Mathematica [A] time = 0.0210368, size = 44, normalized size = 0.86

$$\frac{x^{9/2} \sqrt{x(a + bx^2)} (7a + 2bx^2)}{35a^2 (a + bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2), x]

[Out] (x^(9/2)*Sqrt[x*(a + b*x^2)]*(7*a + 2*b*x^2))/(35*a^2*(a + b*x^2)^4)

Maple [A] time = 0.004, size = 37, normalized size = 0.7

$$\frac{(bx^2 + a)(2bx^2 + 7a)}{35a^2} x^{19/2} (bx^3 + ax)^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(17/2)/(b*x^3+a*x)^(9/2), x)

[Out] 1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{17/2}}{(bx^3 + ax)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.43253, size = 159, normalized size = 3.12

$$\frac{(2bx^6 + 7ax^4)\sqrt{bx^3 + ax}\sqrt{x}}{35(a^2b^4x^8 + 4a^3b^3x^6 + 6a^4b^2x^4 + 4a^5bx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.33704, size = 39, normalized size = 0.76

$$\frac{x^5\left(\frac{2bx^2}{a^2} + \frac{7}{a}\right)}{35(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

$$3.82 \quad \int \frac{x^{15/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=51

$$-\frac{2x^{5/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{x^{11/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{(11/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (2*x^{(5/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)})$

Rubi [A] time = 0.0736555, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$-\frac{2x^{5/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{x^{11/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]

[Out] $-x^{(11/2)}/(7*b*(a*x + b*x^3)^{(7/2)}) - (2*x^{(5/2)})/(35*b^2*(a*x + b*x^3)^{(5/2)})$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{15/2}}{(ax + bx^3)^{9/2}} dx = -\frac{x^{11/2}}{7b(ax + bx^3)^{7/2}} + \frac{2 \int \frac{x^{9/2}}{(ax+bx^3)^{7/2}} dx}{7b}$$

$$= -\frac{x^{11/2}}{7b(ax + bx^3)^{7/2}} - \frac{2x^{5/2}}{35b^2(ax + bx^3)^{5/2}}$$

Mathematica [A] time = 0.0195845, size = 44, normalized size = 0.86

$$-\frac{\sqrt{x}(2a + 7bx^2)}{35b^2(a + bx^2)^3 \sqrt{x(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2), x]

[Out] -(Sqrt[x]*(2*a + 7*b*x^2))/(35*b^2*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])

Maple [A] time = 0.006, size = 37, normalized size = 0.7

$$-\frac{(bx^2 + a)(7bx^2 + 2a)}{35b^2} x^{\frac{9}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(15/2)/(b*x^3+a*x)^(9/2), x)

[Out] -1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.59191, size = 158, normalized size = 3.1

$$\frac{\sqrt{bx^3 + ax}(7bx^2 + 2a)\sqrt{x}}{35(b^6x^9 + 4ab^5x^7 + 6a^2b^4x^5 + 4a^3b^3x^3 + a^4b^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.26349, size = 45, normalized size = 0.88

$$-\frac{7bx^2 + 2a}{35(bx^2 + a)^{\frac{7}{2}}b^2} + \frac{2}{35a^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] $-1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^{(7/2)}*b^2) + 2/35/(a^{(5/2)}*b^2)$

$$3.83 \quad \int \frac{x^{13/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=76

$$\frac{4x^{11/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3(ax+bx^3)^{3/2}} + \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(13/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (4*x^{(11/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (8*x^{(9/2)})/(105*a^3*(a*x + b*x^3)^{(3/2)})$

Rubi [A] time = 0.112928, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$\frac{4x^{11/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3(ax+bx^3)^{3/2}} + \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]

[Out] $x^{(13/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (4*x^{(11/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (8*x^{(9/2)})/(105*a^3*(a*x + b*x^3)^{(3/2)})$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}} dx &= \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}} + \frac{4 \int \frac{x^{11/2}}{(ax+bx^3)^{7/2}} dx}{7a} \\
&= \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8 \int \frac{x^{9/2}}{(ax+bx^3)^{5/2}} dx}{35a^2} \\
&= \frac{x^{13/2}}{7a(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3(ax+bx^3)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.020886, size = 55, normalized size = 0.72

$$\frac{x^{5/2} \sqrt{x(a+bx^2)} (35a^2 + 28abx^2 + 8b^2x^4)}{105a^3(a+bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2), x]

[Out] (x^(5/2)*Sqrt[x*(a + b*x^2)]*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a + b*x^2)^4)

Maple [A] time = 0.007, size = 48, normalized size = 0.6

$$\frac{(bx^2 + a)(8b^2x^4 + 28abx^2 + 35a^2)}{105a^3} x^{15/2} (bx^3 + ax)^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)/(b*x^3+a*x)^(9/2), x)

[Out] 1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.65983, size = 185, normalized size = 2.43

$$\frac{(8b^2x^6 + 28abx^4 + 35a^2x^2)\sqrt{bx^3 + ax}\sqrt{x}}{105(a^3b^4x^8 + 4a^4b^3x^6 + 6a^5b^2x^4 + 4a^6bx^2 + a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.27394, size = 58, normalized size = 0.76

$$\frac{\left(4x^2\left(\frac{2b^2x^2}{a^3} + \frac{7b}{a^2}\right) + \frac{35}{a}\right)x^3}{105(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)

$$3.84 \quad \int \frac{x^{11/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=25

$$-\frac{x^{7/2}}{7b(ax+bx^3)^{7/2}}$$

[Out] $-x^{(7/2)}/(7*b*(a*x + b*x^3)^{(7/2)})$

Rubi [A] time = 0.0370535, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$-\frac{x^{7/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(11/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $-x^{(7/2)}/(7*b*(a*x + b*x^3)^{(7/2)})$

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}} dx = -\frac{x^{7/2}}{7b(ax+bx^3)^{7/2}}$$

Mathematica [A] time = 0.0135167, size = 25, normalized size = 1.

$$-\frac{x^{7/2}}{7b(x(ax+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]

[Out] $-x^{(7/2)}/(7*b*(x*(a + b*x^2))^{(7/2)})$

Maple [A] time = 0.003, size = 27, normalized size = 1.1

$$-\frac{bx^2 + a}{7b} x^{\frac{9}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x)

[Out] $-1/7*(b*x^2+a)/b*x^{(9/2)}/(b*x^3+a*x)^{(9/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [B] time = 1.70886, size = 132, normalized size = 5.28

$$-\frac{\sqrt{bx^3 + ax}\sqrt{x}}{7(b^5x^9 + 4ab^4x^7 + 6a^2b^3x^5 + 4a^3b^2x^3 + a^4bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a^3*b^2*x^3 + a^4*b*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27413, size = 31, normalized size = 1.24

$$-\frac{1}{7(bx^2 + a)^{\frac{7}{2}}b} + \frac{1}{7a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```


$$3.85 \quad \int \frac{x^{9/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=101

$$\frac{6x^{7/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3(ax+bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}} + \frac{x^{9/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(9/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (6*x^{(7/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (8*x^{(5/2)})/(35*a^3*(a*x + b*x^3)^{(3/2)}) + (16*x^{(3/2)})/(35*a^4*\text{Sqrt}[a*x + b*x^3])$

Rubi [A] time = 0.157272, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$\frac{6x^{7/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3(ax+bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}} + \frac{x^{9/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]

[Out] $x^{(9/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (6*x^{(7/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (8*x^{(5/2)})/(35*a^3*(a*x + b*x^3)^{(3/2)}) + (16*x^{(3/2)})/(35*a^4*\text{Sqrt}[a*x + b*x^3])$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
  *(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c
  *x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
  && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
  j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^{9/2}}{(ax + bx^3)^{9/2}} dx &= \frac{x^{9/2}}{7a(ax + bx^3)^{7/2}} + \frac{6 \int \frac{x^{7/2}}{(ax + bx^3)^{7/2}} dx}{7a} \\
 &= \frac{x^{9/2}}{7a(ax + bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{24 \int \frac{x^{5/2}}{(ax + bx^3)^{5/2}} dx}{35a^2} \\
 &= \frac{x^{9/2}}{7a(ax + bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3(ax + bx^3)^{3/2}} + \frac{16 \int \frac{x^{3/2}}{(ax + bx^3)^{3/2}} dx}{35a^3} \\
 &= \frac{x^{9/2}}{7a(ax + bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3(ax + bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax + bx^3}}
 \end{aligned}$$

Mathematica [A] time = 0.0231574, size = 66, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{x(a + bx^2)}(70a^2bx^2 + 35a^3 + 56ab^2x^4 + 16b^3x^6)}{35a^4(a + bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2), x]

[Out] (Sqrt[x]*Sqrt[x*(a + b*x^2)]*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)^4)

Maple [A] time = 0.005, size = 59, normalized size = 0.6

$$\frac{(bx^2 + a)(16b^3x^6 + 56b^2x^4a + 70bx^2a^2 + 35a^3)}{35a^4} x^{\frac{11}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(9/2)/(b*x^3+a*x)^(9/2),x)`

[Out] $\frac{1}{35}(b*x^2+a)*x^{11/2}*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b*x^3+a*x)^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")`

[Out] `integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)`

Fricas [A] time = 1.50133, size = 203, normalized size = 2.01

$$\frac{(16b^3x^6 + 56ab^2x^4 + 70a^2bx^2 + 35a^3)\sqrt{bx^3 + ax}\sqrt{x}}{35(a^4b^4x^8 + 4a^5b^3x^6 + 6a^6b^2x^4 + 4a^7bx^2 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")`

[Out] $\frac{1}{35}(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*\text{sqrt}(b*x^3 + a*x)*\text{sqrt}(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)`

[Out] Timed out

Giac [A] time = 1.31053, size = 74, normalized size = 0.73

$$\frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4} + \frac{7b^2}{a^3}\right) + \frac{35b}{a^2}\right)x^2 + \frac{35}{a}\right)x}{35\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2 + a)^(7/2)

$$3.86 \quad \int \frac{x^{7/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=130

$$\frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}} + \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(7/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + x^{(5/2)}/(5*a^2*(a*x + b*x^3)^{(5/2)}) + x^{(3/2)}/(3*a^3*(a*x + b*x^3)^{(3/2)}) + \text{Sqrt}[x]/(a^4*\text{Sqrt}[a*x + b*x^3]) - \text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[a*x + b*x^3])]/a^{(9/2)}$

Rubi [A] time = 0.202778, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2023, 2029, 206}

$$\frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}} + \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(7/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $x^{(7/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + x^{(5/2)}/(5*a^2*(a*x + b*x^3)^{(5/2)}) + x^{(3/2)}/(3*a^3*(a*x + b*x^3)^{(3/2)}) + \text{Sqrt}[x]/(a^4*\text{Sqrt}[a*x + b*x^3]) - \text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[a*x + b*x^3])]/a^{(9/2)}$

Rule 2023

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \|\| \text{GtQ}[c, 0]) \&\& \text{LtQ}[p, -1]$

Rule 2029

$\text{Int}[(x_*)^{(m_*)}/\text{Sqrt}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[-2/(n-j), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]],$

$x] /; \text{FreeQ}[\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x^{7/2}}{(ax+bx^3)^{9/2}} dx &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{\int \frac{x^{5/2}}{(ax+bx^3)^{7/2}} dx}{a} \\ &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{\int \frac{x^{3/2}}{(ax+bx^3)^{5/2}} dx}{a^2} \\ &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(ax+bx^3)^{3/2}} dx}{a^3} \\ &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3}} dx}{a^4} \\ &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{ax+bx^3}}{\sqrt{x}}\right)}{a^4} \\ &= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}} \end{aligned}$$

Mathematica [C] time = 0.0117814, size = 43, normalized size = 0.33

$$\frac{x^{7/2} {}_2F_1\left(-\frac{7}{2}, 1; -\frac{5}{2}; \frac{bx^2}{a} + 1\right)}{7a(x(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2), x]

[Out] $(x^{7/2} \text{Hypergeometric2F1}[-7/2, 1, -5/2, 1 + (b*x^2)/a]) / (7*a*(x*(a + b*x^2))^{7/2})$

Maple [B] time = 0.012, size = 217, normalized size = 1.7

$$-\frac{1}{105 (bx^2 + a)^4} \sqrt{x (bx^2 + a)} \left(105 \ln \left(2 \frac{\sqrt{a} \sqrt{bx^2 + a} + a}{x} \right) x^6 b^3 \sqrt{bx^2 + a} - 105 \sqrt{ax} b^3 + 315 \ln \left(2 \frac{\sqrt{a} \sqrt{bx^2 + a} + a}{x} \right) x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{7/2}/(b*x^3+a*x)^{9/2}, x)$

[Out] $-1/105*(x*(b*x^2+a))^{1/2}/a^{9/2}*(105*\ln(2*(a^{1/2}*(b*x^2+a)^{1/2}+a)/x)*x^6*b^3*(b*x^2+a)^{1/2}-105*a^{1/2}*x^6*b^3+315*\ln(2*(a^{1/2}*(b*x^2+a)^{1/2}+a)/x)*x^4*a*b^2*(b*x^2+a)^{1/2}-350*a^{3/2}*x^4*b^2+315*\ln(2*(a^{1/2}*(b*x^2+a)^{1/2}+a)/x)*x^2*a^2*b*(b*x^2+a)^{1/2}-406*a^{5/2}*x^2*b+105*\ln(2*(a^{1/2}*(b*x^2+a)^{1/2}+a)/x)*a^3*(b*x^2+a)^{1/2}-176*a^{7/2})/x^{1/2}/(b*x^2+a)^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{7/2}/(b*x^3+a*x)^{9/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(x^{7/2}/(b*x^3 + a*x)^{9/2}, x)$

Fricas [A] time = 1.29511, size = 803, normalized size = 6.18

$$\left[\frac{105 (b^4 x^9 + 4 a b^3 x^7 + 6 a^2 b^2 x^5 + 4 a^3 b x^3 + a^4 x) \sqrt{a} \log \left(\frac{bx^3 + 2ax - 2\sqrt{bx^3 + ax} \sqrt{a} \sqrt{x}}{x^3} \right) + 2 (105 a b^3 x^6 + 350 a^2 b^2 x^4 + 406 a^3 b x^2 + 176 a^4 x) \sqrt{a} \sqrt{x}}{210 (a^5 b^4 x^9 + 4 a^6 b^3 x^7 + 6 a^7 b^2 x^5 + 4 a^8 b x^3 + a^9 x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x))*sqrt(a)*sqrt(x))/x^3) + 2*(105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.28796, size = 154, normalized size = 1.18

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^4}} - \frac{105\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 176\sqrt{-a}}{105\sqrt{-aa^2}} + \frac{105(bx^2+a)^3 + 35(bx^2+a)^2a + 21(bx^2+a)a^2 + 15a^3}{105(bx^2+a)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 + a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2))*a^4)

$$3.87 \quad \int \frac{x^{5/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=126

$$\frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}} + \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(5/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (8*x^{(3/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (16*\text{Sqrt}[x])/(35*a^3*(a*x + b*x^3)^{(3/2)}) + 64/(35*a^4*\text{Sqrt}[x]*\text{Sqrt}[a*x + b*x^3]) - (128*\text{Sqrt}[a*x + b*x^3])/(35*a^5*x^{(3/2)})$

Rubi [A] time = 0.192445, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$\frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}} + \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(5/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $x^{(5/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (8*x^{(3/2)})/(35*a^2*(a*x + b*x^3)^{(5/2)}) + (16*\text{Sqrt}[x])/(35*a^3*(a*x + b*x^3)^{(3/2)}) + 64/(35*a^4*\text{Sqrt}[x]*\text{Sqrt}[a*x + b*x^3]) - (128*\text{Sqrt}[a*x + b*x^3])/(35*a^5*x^{(3/2)})$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^{5/2}}{(ax + bx^3)^{9/2}} dx &= \frac{x^{5/2}}{7a(ax + bx^3)^{7/2}} + \frac{8 \int \frac{x^{3/2}}{(ax+bx^3)^{7/2}} dx}{7a} \\
 &= \frac{x^{5/2}}{7a(ax + bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{48 \int \frac{\sqrt{x}}{(ax+bx^3)^{5/2}} dx}{35a^2} \\
 &= \frac{x^{5/2}}{7a(ax + bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax + bx^3)^{3/2}} + \frac{64 \int \frac{1}{\sqrt{x}(ax+bx^3)^{3/2}} dx}{35a^3} \\
 &= \frac{x^{5/2}}{7a(ax + bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax + bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax + bx^3}} + \frac{128 \int \frac{1}{x^{3/2}\sqrt{ax+b}}}{35a^4} \\
 &= \frac{x^{5/2}}{7a(ax + bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax + bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax + bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax + bx^3}} - \frac{128\sqrt{ax + bx^3}}{35a^5x^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0297992, size = 77, normalized size = 0.61

$$\frac{\sqrt{x(a + bx^2)}(560a^2b^2x^4 + 280a^3bx^2 + 35a^4 + 448ab^3x^6 + 128b^4x^8)}{35a^5x^{3/2}(a + bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2), x]

[Out] -(Sqrt[x*(a + b*x^2)]*(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*a*b^3*x^6 + 128*b^4*x^8))/(35*a^5*x^(3/2)*(a + b*x^2)^4)

Maple [A] time = 0.004, size = 70, normalized size = 0.6

$$\frac{(bx^2 + a)(128b^4x^8 + 448b^3x^6a + 560b^2x^4a^2 + 280bx^2a^3 + 35a^4)}{35a^5} x^{\frac{7}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)/(b*x^3+a*x)^(9/2),x)`

[Out] $-1/35*x^{7/2}*(b*x^2+a)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*b*x^2+35*a^4)/a^5/(b*x^3+a*x)^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")`

[Out] `integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)`

Fricas [A] time = 1.67136, size = 239, normalized size = 1.9

$$\frac{(128b^4x^8 + 448ab^3x^6 + 560a^2b^2x^4 + 280a^3bx^2 + 35a^4)\sqrt{bx^3 + ax}\sqrt{x}}{35(a^5b^4x^{10} + 4a^6b^3x^8 + 6a^7b^2x^6 + 4a^8bx^4 + a^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")`

[Out] $-1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a^4)*\text{sqrt}(b*x^3 + a*x)*\text{sqrt}(x)/(a^5*b^4*x^{10} + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.44085, size = 96, normalized size = 0.76

$$-\frac{\left(x^2\left(\frac{93b^4x^2}{a^5} + \frac{308b^3}{a^4}\right) + \frac{350b^2}{a^3}\right)x^2 + \frac{140b}{a^2}x}{35(bx^2 + a)^{\frac{7}{2}}} - \frac{\sqrt{b + \frac{a}{x^2}}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) - sqrt(b + a/x^2)/a^5

$$3.88 \quad \int \frac{x^{3/2}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=159

$$\frac{9\sqrt{x}}{35a^2(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}} + \frac{9b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}} + \frac{x^{3/2}}{7a(ax+bx^3)^{7/2}}$$

[Out] $x^{(3/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (9*\text{Sqrt}[x])/(35*a^2*(a*x + b*x^3)^{(5/2)}) + 3/(5*a^3*\text{Sqrt}[x]*(a*x + b*x^3)^{(3/2)}) + 3/(a^4*x^{(3/2)}*\text{Sqrt}[a*x + b*x^3]) - (9*\text{Sqrt}[a*x + b*x^3])/(2*a^5*x^{(5/2)}) + (9*b*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^3]])/(2*a^{(11/2)})$

Rubi [A] time = 0.240548, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$\frac{9\sqrt{x}}{35a^2(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}} + \frac{9b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}} + \frac{x^{3/2}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(3/2)}/(a*x + b*x^3)^{(9/2)}, x]$

[Out] $x^{(3/2)}/(7*a*(a*x + b*x^3)^{(7/2)}) + (9*\text{Sqrt}[x])/(35*a^2*(a*x + b*x^3)^{(5/2)}) + 3/(5*a^3*\text{Sqrt}[x]*(a*x + b*x^3)^{(3/2)}) + 3/(a^4*x^{(3/2)}*\text{Sqrt}[a*x + b*x^3]) - (9*\text{Sqrt}[a*x + b*x^3])/(2*a^5*x^{(5/2)}) + (9*b*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^3]])/(2*a^{(11/2)})$

Rule 2023

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x \&\amp; \text{IntegerQ}[p] \&\amp; \text{LtQ}[0, j, n] \&\amp; (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\amp; \text{LtQ}[p, -1]$

Rule 2025

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2029

```

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(ax + bx^3)^{9/2}} dx &= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9 \int \frac{\sqrt{x}}{(ax+bx^3)^{7/2}} dx}{7a} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{9 \int \frac{1}{\sqrt{x}(ax+bx^3)^{5/2}} dx}{5a^2} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax + bx^3)^{3/2}} + \frac{3 \int \frac{1}{x^{3/2}(ax+bx^3)^{3/2}} dx}{a^3} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax + bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax + bx^3}} + \frac{9 \int \frac{1}{x^{5/2}\sqrt{ax+bx^3}} dx}{a^4} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax + bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax + bx^3}} - \frac{9\sqrt{ax + bx^3}}{2a^5x^{5/2}} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax + bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax + bx^3}} - \frac{9\sqrt{ax + bx^3}}{2a^5x^{5/2}} \\
&= \frac{x^{3/2}}{7a(ax + bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2(ax + bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}(ax + bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax + bx^3}} - \frac{9\sqrt{ax + bx^3}}{2a^5x^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0196505, size = 44, normalized size = 0.28

$$\frac{bx^{7/2} {}_2F_1\left(-\frac{7}{2}, 2; -\frac{5}{2}; \frac{bx^2}{a} + 1\right)}{7a^2(x(a + bx^2))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2), x]

[Out] -(b*x^(7/2)*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(7*a^2*(x*(a + b*x^2))^(7/2))

Maple [A] time = 0.013, size = 234, normalized size = 1.5

$$\frac{1}{70 (bx^2 + a)^4} \sqrt{x(bx^2 + a)} \left(315 \ln \left(2 \frac{\sqrt{a}\sqrt{bx^2 + a} + a}{x} \right) x^8 b^4 \sqrt{bx^2 + a} - 315 \sqrt{ax^8} b^4 + 945 \ln \left(2 \frac{\sqrt{a}\sqrt{bx^2 + a} + a}{x} \right) x^6 ab^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^3+a*x)^(9/2), x)

[Out] 1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^8*b^4*(b*x^2+a)^(1/2)-315*a^(1/2)*x^8*b^4+945*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^6*a*b^3*(b*x^2+a)^(1/2)-1050*a^(3/2)*x^6*b^3+945*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*a^(5/2)*x^4*b^2+315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^2*a^3*b*(b*x^2+a)^(1/2)-528*a^(7/2)*x^2*b-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2), x, algorithm="maxima")

[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.3772, size = 884, normalized size = 5.56

$$\left[\frac{315 (b^5 x^{11} + 4 ab^4 x^9 + 6 a^2 b^3 x^7 + 4 a^3 b^2 x^5 + a^4 b x^3) \sqrt{a} \log \left(\frac{bx^3 + 2ax + 2\sqrt{bx^3 + ax}\sqrt{a}\sqrt{x}}{x^3} \right) - 2 (315 ab^4 x^8 + 1050 a^2 b^3 x^6 + 1218 a^3 b^2 x^4 + 528 a^4 b x^2 + 35 a^5) \sqrt{a}}{140 (a^6 b^4 x^{11} + 4 a^7 b^3 x^9 + 6 a^8 b^2 x^7 + 4 a^9 b x^5 + a^{10} x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2), x, algorithm="fricas")


```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x))*sqrt(a)*sqrt(x))/x^3) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32919, size = 142, normalized size = 0.89

$$-\frac{1}{70}b \left(\frac{315 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5} + \frac{2\left(140(bx^2+a)^3 + 35(bx^2+a)^2a + 14(bx^2+a)a^2 + 5a^3\right)}{(bx^2+a)^{\frac{7}{2}}a^5} + \frac{35\sqrt{bx^2+a}}{a^5bx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*b*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) + 2*(140*(b*x^2 + a)^3 + 35*(b*x^2 + a)^2*a + 14*(b*x^2 + a)*a^2 + 5*a^3)/((b*x^2 + a)^(7/2)*a^5) + 35*sqrt(b*x^2 + a)/(a^5*b*x^2))
```

$$3.89 \quad \int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=152

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}} - \frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} + \frac{16}{21a^3x^{3/2}(ax+bx^3)^{3/2}} + \frac{2}{7a^2\sqrt{x}(ax+bx^3)^{5/2}} + \frac{\sqrt{x}}{7a(ax+bx^3)^{7/2}}$$

[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) + 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])/(21*a^6*x^(3/2))

Rubi [A] time = 0.233374, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 2014}

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}} - \frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} + \frac{16}{21a^3x^{3/2}(ax+bx^3)^{3/2}} + \frac{2}{7a^2\sqrt{x}(ax+bx^3)^{5/2}} + \frac{\sqrt{x}}{7a(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2), x]

[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) + 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])/(21*a^6*x^(3/2))

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
]:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
]:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{(ax + bx^3)^{9/2}} dx &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{10 \int \frac{1}{\sqrt{x}(ax + bx^3)^{7/2}} dx}{7a} \\ &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax + bx^3)^{5/2}} + \frac{16 \int \frac{1}{x^{3/2}(ax + bx^3)^{5/2}} dx}{7a^2} \\ &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax + bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax + bx^3)^{3/2}} + \frac{32 \int \frac{1}{x^{5/2}(ax + bx^3)^{3/2}} dx}{7a^3} \\ &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax + bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax + bx^3)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax + bx^3}} + \frac{128 \int \frac{1}{x^7}}{21a^5} \\ &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax + bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax + bx^3)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax + bx^3}} - \frac{128\sqrt{ax}}{21a^5} \\ &= \frac{\sqrt{x}}{7a(ax + bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax + bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax + bx^3)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax + bx^3}} - \frac{128\sqrt{ax}}{21a^5} \end{aligned}$$

Mathematica [A] time = 0.024839, size = 88, normalized size = 0.58

$$\frac{\sqrt{x(a + bx^2)}(1120a^2b^3x^6 + 560a^3b^2x^4 + 70a^4bx^2 - 7a^5 + 896ab^4x^8 + 256b^5x^{10})}{21a^6x^{7/2}(a + bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]

[Out] (Sqrt[x*(a + b*x^2)]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*x^(7/2)*(a + b*x^2)^4)

Maple [A] time = 0.006, size = 81, normalized size = 0.5

$$\frac{(bx^2 + a)(-256b^5x^{10} - 896b^4x^8a - 1120b^3x^6a^2 - 560b^2x^4a^3 - 70bx^2a^4 + 7a^5)}{21a^6} x^{\frac{3}{2}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x)

[Out] -1/21*x^(3/2)*(b*x^2+a)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(bx^3 + ax)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)

Fricas [A] time = 1.85057, size = 265, normalized size = 1.74

$$\frac{(256b^5x^{10} + 896ab^4x^8 + 1120a^2b^3x^6 + 560a^3b^2x^4 + 70a^4bx^2 - 7a^5)\sqrt{bx^3 + ax}\sqrt{x}}{21(a^6b^4x^{12} + 4a^7b^3x^{10} + 6a^8b^2x^8 + 4a^9bx^6 + a^{10}x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] $\frac{1}{21} \cdot (256b^5x^{10} + 896a^2b^4x^8 + 1120a^2b^3x^6 + 560a^3b^2x^4 + 70a^4b^2x^2 - 7a^5) \sqrt{bx^3 + ax} \sqrt{x} / (a^6b^4x^{12} + 4a^7b^3x^{10} + 6a^8b^2x^8 + 4a^9bx^6 + a^{10}x^4)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.38026, size = 116, normalized size = 0.76

$$\frac{\left(\left(x^2 \left(\frac{158b^5x^2}{a^6} + \frac{511b^4}{a^5} \right) + \frac{560b^3}{a^4} \right) x^2 + \frac{210b^2}{a^3} \right) x}{21 (bx^2 + a)^{\frac{7}{2}}} - \frac{\left(b + \frac{a}{x^2} \right)^{\frac{3}{2}} - 15 \sqrt{b + \frac{a}{x^2}} b}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] $\frac{1}{21} \cdot ((x^2 \cdot (158b^5x^2/a^6 + 511b^4/a^5) + 560b^3/a^4) \cdot x^2 + 210b^2/a^3) \cdot x / (bx^2 + a)^{(7/2)} - 1/3 \cdot ((b + a/x^2)^{(3/2)} - 15 \cdot \sqrt{b + a/x^2} \cdot b) / a^6$

$$3.90 \quad \int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=189

$$-\frac{99b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}} + \frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}}$$

[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x + b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3])/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(8*a^(13/2))

Rubi [A] time = 0.292728, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$-\frac{99b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}} + \frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)), x]

[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x + b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3])/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(8*a^(13/2))

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}} dx &= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11 \int \frac{1}{x^{3/2}(ax+bx^3)^{7/2}} dx}{7a} \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{99 \int \frac{1}{x^{5/2}(ax+bx^3)^{5/2}} dx}{35a^2} \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33 \int \frac{1}{x^{7/2}(ax+bx^3)^{3/2}} dx}{5a^3} \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} + \dots \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \dots \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \dots \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \dots \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \dots \\
&= \frac{1}{7a\sqrt{x}(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}(ax+bx^3)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.0193527, size = 46, normalized size = 0.24

$$\frac{b^2x^{7/2} {}_2F_1\left(-\frac{7}{2}, 3; -\frac{5}{2}; \frac{bx^2}{a} + 1\right)}{7a^3(x(ax+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)), x]

[Out] (b^2*x^(7/2)*Hypergeometric2F1[-7/2, 3, -5/2, 1 + (b*x^2)/a])/(7*a^3*(x*(a + b*x^2))^(7/2))

Maple [A] time = 0.013, size = 247, normalized size = 1.3

$$-\frac{1}{280 (bx^2 + a)^4} \sqrt{x (bx^2 + a)} \left(3465 \ln \left(2 \frac{\sqrt{a} \sqrt{bx^2 + a} + a}{x} \right) x^{10} b^5 \sqrt{bx^2 + a} - 3465 \sqrt{ax}^{10} b^5 + 10395 \ln \left(2 \frac{\sqrt{a} \sqrt{bx^2 + a}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x)`

[Out] $-1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*\ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^{10}*b^5*(b*x^2+a)^(1/2)-3465*a^(1/2)*x^{10}*b^5+10395*\ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^8*a*b^4*(b*x^2+a)^(1/2)-11550*a^(3/2)*x^8*b^4+10395*\ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^6*a^2*b^3*(b*x^2+a)^(1/2)-13398*a^(5/2)*x^6*b^3+3465*\ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^4*a^3*b^2*(b*x^2+a)^(1/2)-5808*a^(7/2)*x^4*b^2-385*a^(9/2)*x^2*b+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{9}{2}} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)`

Fricas [A] time = 1.42645, size = 963, normalized size = 5.1

$$\frac{3465 (b^6 x^{13} + 4 a b^5 x^{11} + 6 a^2 b^4 x^9 + 4 a^3 b^3 x^7 + a^4 b^2 x^5) \sqrt{a} \log \left(\frac{bx^3 + 2ax - 2\sqrt{bx^3 + ax} \sqrt{a} \sqrt{x}}{x^3} \right) + 2 (3465 ab^5 x^{10} + 11550 a^2 b^4 x^8 + 11550 a^3 b^3 x^6 + 5808 a^4 b^2 x^4 + 385 a^5 b x^2 + 70 a^6)}{560 (a^7 b^4 x^{13} + 4 a^8 b^3 x^{11} + 6 a^9 b^2 x^9 + 4 a^{10} b x^7 + a^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")`

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35876, size = 165, normalized size = 0.87

$$\frac{1}{280} b^2 \left(\frac{3465 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^6} + \frac{8\left(350(bx^2+a)^3 + 70(bx^2+a)^2a + 21(bx^2+a)a^2 + 5a^3\right)}{(bx^2+a)^{\frac{7}{2}}a^6} + \frac{35\left(19(bx^2+a)^{\frac{3}{2}} - 21\sqrt{bx^2+a}\right)}{a^6b^2x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/280*b^2*(3465*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 8*(350*(b
*x^2 + a)^3 + 70*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 5*a^3)/((b*x^2 + a)
^(7/2)*a^6) + 35*(19*(b*x^2 + a)^(3/2) - 21*sqrt(b*x^2 + a)*a)/(a^6*b^2*x^4
))
```

$$3.91 \quad \int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}} dx$$

Optimal. Leaf size=180

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}} + \frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}} - \frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}}$$

[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))

Rubi [A] time = 0.284987, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 2014}

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}} + \frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}} - \frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]

[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))

Rule 2015

```
Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c
*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_.))^(m_.)*((a_.)*(x_.)^(j_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_.))^(m_.)*((a_.)*(x_.)^(j_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j
)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}} dx &= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12 \int \frac{1}{x^{5/2}(ax+bx^3)^{7/2}} dx}{7a} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{24 \int \frac{1}{x^{7/2}(ax+bx^3)^{5/2}} dx}{7a^2} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64 \int \frac{1}{x^{9/2}(ax+bx^3)^{3/2}} dx}{7a^3} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} + \frac{3}{7a^4x^{9/2}\sqrt{ax+bx^3}} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{7a^4x^{9/2}\sqrt{ax+bx^3}} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{7a^4x^{9/2}\sqrt{ax+bx^3}} \\
&= \frac{1}{7ax^{3/2}(ax+bx^3)^{7/2}} + \frac{12}{35a^2x^{5/2}(ax+bx^3)^{5/2}} + \frac{8}{7a^3x^{7/2}(ax+bx^3)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{7a^4x^{9/2}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.0310446, size = 99, normalized size = 0.55

$$\frac{\sqrt{x(a+bx^2)}(4480a^2b^4x^8 + 2240a^3b^3x^6 + 280a^4b^2x^4 - 28a^5bx^2 + 7a^6 + 3584ab^5x^{10} + 1024b^6x^{12})}{35a^7x^{11/2}(a+bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x]

[Out] -(Sqrt[x*(a + b*x^2)]*(7*a^6 - 28*a^5*b*x^2 + 280*a^4*b^2*x^4 + 2240*a^3*b^3*x^6 + 4480*a^2*b^4*x^8 + 3584*a*b^5*x^10 + 1024*b^6*x^12))/(35*a^7*x^(11/2)*(a + b*x^2)^4)

Maple [A] time = 0.007, size = 92, normalized size = 0.5

$$\frac{(bx^2 + a)(1024b^6x^{12} + 3584b^5x^{10}a + 4480b^4x^8a^2 + 2240b^3x^6a^3 + 280b^2x^4a^4 - 28bx^2a^5 + 7a^6)}{35a^7} \frac{1}{\sqrt{x}} (bx^3 + ax)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2), x)

[Out] -1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b^3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax)^{\frac{9}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2), x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)

Fricas [A] time = 2.28488, size = 298, normalized size = 1.66

$$\frac{(1024 b^6 x^{12} + 3584 a b^5 x^{10} + 4480 a^2 b^4 x^8 + 2240 a^3 b^3 x^6 + 280 a^4 b^2 x^4 - 28 a^5 b x^2 + 7 a^6) \sqrt{b x^3 + a x} \sqrt{x}}{35 (a^7 b^4 x^{14} + 4 a^8 b^3 x^{12} + 6 a^9 b^2 x^{10} + 4 a^{10} b x^8 + a^{11} x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")

[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)

[Out] Timed out

Giac [A] time = 1.40183, size = 136, normalized size = 0.76

$$\frac{\left(2 x^2 \left(\frac{281 b^6 x^2}{a^7} + \frac{896 b^5}{a^6}\right) + \frac{1925 b^4}{a^5}\right) x^2 + \frac{700 b^3}{a^4}}{35 (b x^2 + a)^{\frac{7}{2}}} - \frac{\left(b + \frac{a}{x^2}\right)^{\frac{5}{2}} - 10 \left(b + \frac{a}{x^2}\right)^{\frac{3}{2}} b + 75 \sqrt{b + \frac{a}{x^2}} b^2}{5 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")

[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3/a^4)*x/(b*x^2 + a)^(7/2) - 1/5*((b + a/x^2)^(5/2) - 10*(b + a/x^2)^(3/2)*b + 75*sqrt(b + a/x^2)*b^2)/a^7

$$3.92 \quad \int \frac{x^4}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=55

$$\frac{x\sqrt{ax+bx^4}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*b^(3/2))

Rubi [A] time = 0.0698222, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2024, 2029, 206}

$$\frac{x\sqrt{ax+bx^4}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[a*x + b*x^4], x]

[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*b^(3/2))

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sqrt{ax+bx^4}} dx &= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a \int \frac{x}{\sqrt{ax+bx^4}} dx}{2b} \\ &= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b} \\ &= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0327577, size = 81, normalized size = 1.47

$$\frac{\sqrt{bx^2}(a+bx^3) - a\sqrt{x}\sqrt{a+bx^3} \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4], x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

Maple [C] time = 0.032, size = 997, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x)^(1/2), x)
```

```
[Out] 1/3*x*(b*x^4+a*x)^(1/2)/b-a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-
```


$$b^2 a^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3} / (x - 1/b (-b^2 a)^{1/3})^{1/2} * (x - 1/b (-b^2 a)^{1/3})^2 (1/b (-b^2 a)^{1/3} * (x + 1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (-1/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (x - 1/b (-b^2 a)^{1/3})^{1/2} * (1/b (-b^2 a)^{1/3} * (x + 1/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (-1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (x - 1/b (-b^2 a)^{1/3})^{1/2} / (-3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (-b^2 a)^{1/3} / (b x * (x - 1/b (-b^2 a)^{1/3}) * (x + 1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) * (x + 1/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}))^{1/2} * (1/b (-b^2 a)^{1/3} * \text{EllipticF}(((-3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) * x / (-1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (x - 1/b (-b^2 a)^{1/3}))^{1/2}, ((3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) * (1/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (3/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}))^{1/2}) - 1/b (-b^2 a)^{1/3} * \text{EllipticPi}(((-3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) * x / (-1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (x - 1/b (-b^2 a)^{1/3}))^{1/2}, (-1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (-3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}), ((3/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) * (1/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (1/2/b (-b^2 a)^{1/3} + 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}) / (3/2/b (-b^2 a)^{1/3} - 1/2 I 3^{1/2} / b (-b^2 a)^{1/3}))^{1/2}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)

Fricas [A] time = 1.97972, size = 315, normalized size = 5.73

$$\left[\frac{4 \sqrt{bx^4 + ax} bx + a \sqrt{b} \log\left(-8 b^2 x^6 - 8 abx^3 - a^2 + 4(2bx^4 + ax)\sqrt{bx^4 + ax}\sqrt{b}\right)}{12 b^2}, \frac{2 \sqrt{bx^4 + ax} bx + a \sqrt{-b} \arctan\left(\frac{2 \sqrt{bx^4 + ax}}{2}\right)}{6 b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
+ 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

Giac [A] time = 1.32322, size = 61, normalized size = 1.11

$$\frac{\sqrt{bx^4 + axx}}{3b} + \frac{a \arctan\left(\frac{\sqrt{b + \frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-bb}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

$$3.93 \quad \int \frac{x}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])

Rubi [A] time = 0.0322918, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x + b*x^4],x]

[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{x}{\sqrt{ax + bx^4}} dx = \frac{2}{3} \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^2}{\sqrt{ax + bx^4}} \right)$$

$$= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{ax + bx^4}} \right)}{3\sqrt{b}}$$

Mathematica [A] time = 0.0131221, size = 61, normalized size = 1.91

$$\frac{2\sqrt{x}\sqrt{a + bx^3} \tanh^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a + bx^3}} \right)}{3\sqrt{b}\sqrt{x(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x + b*x^4],x]

[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt[x*(a + b*x^3)])

Maple [C] time = 0.015, size = 979, normalized size = 30.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^4+a*x)^(1/2),x)

[Out] 2*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/((-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))/((-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))/((-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*b/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*EllipticF(((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/((-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3))))^(1/2))

$$3^{1/2}/b*(-b^2*a)^{1/3}*x/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2}, ((3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})*(1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(3/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3}))^{1/2}-1/b*(-b^2*a)^{1/3}*EllipticPi(((3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})*x/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3}))^{1/2}, (-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3}), ((3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})*(1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})/(3/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3}))^{1/2}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(b*x^4 + a*x), x)

Fricas [A] time = 1.89581, size = 225, normalized size = 7.03

$$\left[\frac{\log\left(-8b^2x^6 - 8abx^3 - a^2 - 4(2bx^4 + ax)\sqrt{bx^4 + ax}\sqrt{b}\right)}{6\sqrt{b}}, -\frac{\sqrt{-b} \arctan\left(\frac{2\sqrt{bx^4 + ax}\sqrt{-bx}}{2bx^3 + a}\right)}{3b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a))/b]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**4+a*x)**(1/2),x)

[Out] Integral(x/sqrt(x*(a + b*x**3)), x)

Giac [A] time = 1.30927, size = 31, normalized size = 0.97

$$-\frac{2 \arctan\left(\frac{\sqrt{b + \frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)

$$3.94 \quad \int \frac{1}{x^2 \sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(3*a*x^2)$

Rubi [A] time = 0.0337861, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2014}

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[a*x + b*x^4]), x]$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(3*a*x^2)$

Rule 2014

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\int \frac{1}{x^2 \sqrt{ax+bx^4}} dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Mathematica [A] time = 0.0106894, size = 23, normalized size = 1.

$$-\frac{2\sqrt{x(a+bx^3)}}{3ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)

Maple [A] time = 0.003, size = 27, normalized size = 1.2

$$-\frac{2bx^3 + 2a}{3ax} \frac{1}{\sqrt{bx^4 + ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^4+a*x)^(1/2),x)

[Out] -2/3*(b*x^3+a)/x/a/(b*x^4+a*x)^(1/2)

Maxima [A] time = 1.06738, size = 35, normalized size = 1.52

$$-\frac{2(bx^4 + ax)}{3\sqrt{bx^3 + aax^2}^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))

Fricas [A] time = 1.26757, size = 43, normalized size = 1.87

$$-\frac{2\sqrt{bx^4 + ax}}{3ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] $-2/3\sqrt{b*x^4 + a*x}/(a*x^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**4+a*x)**(1/2),x)`

[Out] `Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)`

Giac [A] time = 1.19666, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{b + \frac{a}{x^3}}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")`

[Out] $-2/3\sqrt{b + a/x^3}/a$

$$3.95 \quad \int \frac{1}{x^5 \sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=48

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2} - \frac{2\sqrt{ax+bx^4}}{9ax^5}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(9*a*x^5) + (4*b*\text{Sqrt}[a*x + b*x^4])/(9*a^2*x^2)$

Rubi [A] time = 0.0664935, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2} - \frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^5*\text{Sqrt}[a*x + b*x^4]),x]$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(9*a*x^5) + (4*b*\text{Sqrt}[a*x + b*x^4])/(9*a^2*x^2)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x]
  - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x]
  /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x]
  /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5 \sqrt{ax + bx^4}} dx = -\frac{2\sqrt{ax + bx^4}}{9ax^5} - \frac{(2b) \int \frac{1}{x^2 \sqrt{ax + bx^4}} dx}{3a}$$

$$= -\frac{2\sqrt{ax + bx^4}}{9ax^5} + \frac{4b\sqrt{ax + bx^4}}{9a^2x^2}$$

Mathematica [A] time = 0.0137281, size = 31, normalized size = 0.65

$$-\frac{2(a - 2bx^3)\sqrt{x(a + bx^3)}}{9a^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]

[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)

Maple [A] time = 0.004, size = 35, normalized size = 0.7

$$-\frac{(2bx^3 + 2a)(-2bx^3 + a)}{9a^2x^4} \frac{1}{\sqrt{bx^4 + ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^4+a*x)^(1/2),x)

[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^4/a^2/(b*x^4+a*x)^(1/2)

Maxima [A] time = 1.01426, size = 51, normalized size = 1.06

$$\frac{2(2b^2x^7 + abx^4 - a^2x)}{9\sqrt{bx^3 + aa^2x^{\frac{11}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))

Fricas [A] time = 1.19889, size = 63, normalized size = 1.31

$$\frac{2\sqrt{bx^4 + ax}(2bx^3 - a)}{9a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)

[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)

Giac [A] time = 1.19973, size = 36, normalized size = 0.75

$$-\frac{2\left(\left(b + \frac{a}{x^3}\right)^{\frac{3}{2}} - 3\sqrt{b + \frac{a}{x^3}b}\right)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] -2/9*((b + a/x^3)^(3/2) - 3*sqrt(b + a/x^3)*b)/a^2

$$3.96 \quad \int \frac{1}{x^8 \sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=74

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2} + \frac{8b\sqrt{ax+bx^4}}{45a^2x^5} - \frac{2\sqrt{ax+bx^4}}{15ax^8}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(15*a*x^8) + (8*b*\text{Sqrt}[a*x + b*x^4])/(45*a^2*x^5) - (16*b^2*\text{Sqrt}[a*x + b*x^4])/(45*a^3*x^2)$

Rubi [A] time = 0.101669, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2} + \frac{8b\sqrt{ax+bx^4}}{45a^2x^5} - \frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(15*a*x^8) + (8*b*\text{Sqrt}[a*x + b*x^4])/(45*a^2*x^5) - (16*b^2*\text{Sqrt}[a*x + b*x^4])/(45*a^3*x^2)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8 \sqrt{ax + bx^4}} dx &= -\frac{2\sqrt{ax + bx^4}}{15ax^8} - \frac{(4b) \int \frac{1}{x^5 \sqrt{ax + bx^4}} dx}{5a} \\
&= -\frac{2\sqrt{ax + bx^4}}{15ax^8} + \frac{8b\sqrt{ax + bx^4}}{45a^2x^5} + \frac{(8b^2) \int \frac{1}{x^2 \sqrt{ax + bx^4}} dx}{15a^2} \\
&= -\frac{2\sqrt{ax + bx^4}}{15ax^8} + \frac{8b\sqrt{ax + bx^4}}{45a^2x^5} - \frac{16b^2\sqrt{ax + bx^4}}{45a^3x^2}
\end{aligned}$$

Mathematica [A] time = 0.0146873, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{x(a + bx^3)}(3a^2 - 4abx^3 + 8b^2x^6)}{45a^3x^8}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*sqrt[a*x + b*x^4]),x]

[Out] (-2*sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)

Maple [A] time = 0.005, size = 48, normalized size = 0.7

$$-\frac{(2bx^3 + 2a)(8b^2x^6 - 4abx^3 + 3a^2)}{45x^7a^3} \frac{1}{\sqrt{bx^4 + ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(b*x^4+a*x)^(1/2),x)

[Out] -2/45*(b*x^3+a)*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^7/a^3/(b*x^4+a*x)^(1/2)

Maxima [A] time = 1.08143, size = 68, normalized size = 0.92

$$-\frac{2(8b^3x^{10} + 4ab^2x^7 - a^2bx^4 + 3a^3x)}{45\sqrt{bx^3 + aa^3x^{\frac{17}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")`

[Out] $-2/45*(8*b^3*x^{10} + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3*x^{(17/2)})$

Fricas [A] time = 1.18574, size = 90, normalized size = 1.22

$$\frac{2(8b^2x^6 - 4abx^3 + 3a^2)\sqrt{bx^4 + ax}}{45a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")`

[Out] $-2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8 \sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**8/(b*x**4+a*x)**(1/2),x)`

[Out] `Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)`

Giac [A] time = 1.23487, size = 58, normalized size = 0.78

$$\frac{2\left(3\left(b + \frac{a}{x^3}\right)^{\frac{5}{2}} - 10\left(b + \frac{a}{x^3}\right)^{\frac{3}{2}}b + 15\sqrt{b + \frac{a}{x^3}}b^2\right)}{45a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(3/2)*b + 15*sqrt(b + a/x^3)*b^2)/a^3
```


$$3.97 \quad \int \frac{x^3}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=224

$$\frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax+bx^4}}$$

[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rubi [A] time = 0.212868, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2024, 2011, 329, 225}

$$\frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x + b*x^4], x]

[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p

```

+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

```

Rule 2011

```

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 225

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax+bx^4}} dx &= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a \int \frac{1}{\sqrt{ax+bx^4}} dx}{4b} \\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{(a\sqrt{x}\sqrt{a+bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{4b\sqrt{ax+bx^4}} \\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{(a\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{2b\sqrt{ax+bx^4}} \\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right) \middle| \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0230682, size = 64, normalized size = 0.29

$$\frac{x \left(-a\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{2b\sqrt{x(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x + b*x^4], x]

[Out] (x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -(b*x^3)/a]))/(2*b*Sqrt[x*(a + b*x^3)])

Maple [C] time = 0.014, size = 688, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a*x)^(1/2), x)

[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*

$$\begin{aligned} & (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3} / (x - 1/b * (-b^2a)^{1/3})^{1/2} \\ & * (x - 1/b * (-b^2a)^{1/3})^2 * (1/b * (-b^2a)^{1/3} * (x + 1/2/b * (-b^2a)^{1/3} + 1/2 * \\ & I * 3^{1/2} / b * (-b^2a)^{1/3}) / (-1/2/b * (-b^2a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}) \\ & / (x - 1/b * (-b^2a)^{1/3})^{1/2} * (1/b * (-b^2a)^{1/3} * (x + 1/2/b * (-b^2a)^{1/3} \\ & - 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}) / (-1/2/b * (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} / \\ & b * (-b^2a)^{1/3}) / (x - 1/b * (-b^2a)^{1/3})^{1/2} / (-3/2/b * (-b^2a)^{1/3} + 1/2 \\ & * I * 3^{1/2} / b * (-b^2a)^{1/3}) / (-b^2a)^{1/3} / (b * x * (x - 1/b * (-b^2a)^{1/3}) * (x \\ & + 1/2/b * (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}) * (x + 1/2/b * (-b^2a)^{1/3} \\ & - 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}))^{1/2} * \text{EllipticF}(((-3/2/b * (-b^2a)^{1/3} + \\ & 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}) * x / (-1/2/b * (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} / b * (- \\ & b^2a)^{1/3}) / (x - 1/b * (-b^2a)^{1/3}))^{1/2}, ((3/2/b * (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} \\ & / b * (-b^2a)^{1/3}) * (1/2/b * (-b^2a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3} \\ &)) / (1/2/b * (-b^2a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}) / (3/2/b * (-b^2a)^{1/3} \\ & - 1/2 * I * 3^{1/2} / b * (-b^2a)^{1/3}))^{1/2} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax^2}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**4+a*x)**(1/2), x)

[Out] Integral(x**3/sqrt(x*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a*x)^(1/2), x, algorithm="giac")

[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)

$$3.98 \quad \int \frac{1}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=197

$$\frac{x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{\sqrt[4]{3} \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}}$$

[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rubi [A] time = 0.130435, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2011, 329, 225}

$$\frac{x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}\right) \middle| \frac{1}{4}(2 + \sqrt{3})\right)}{\sqrt[4]{3} \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x + b*x^4], x]

[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x

$(j*p)*(a + b*x^{(n - j)})^p, x], x] /; \text{FreeQ}[\{a, b, j, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{PosQ}[n - j]$

Rule 329

$\text{Int}[\{(c_.)*(x_)^{(m_)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \ :> \ \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^{(n)})^p, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 225

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^6], x_Symbol] \ :> \ \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(x*(s + r*x^2)*\text{Sqrt}[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]*\text{EllipticF}[\text{ArcCos}[(s + (1 - \text{Sqrt}[3])*r*x^2)/(s + (1 + \text{Sqrt}[3])*r*x^2)], (2 + \text{Sqrt}[3])/4])/(2*3^{(1/4)}*s*\text{Sqrt}[a + b*x^6]*\text{Sqrt}[(r*x^2*(s + r*x^2))/(s + (1 + \text{Sqrt}[3])*r*x^2)^2]), x]] /; \text{FreeQ}[\{a, b\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax + bx^4}} dx &= \frac{\left(\sqrt{x}\sqrt{a + bx^3}\right) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{\sqrt{ax + bx^4}} \\ &= \frac{\left(2\sqrt{x}\sqrt{a + bx^3}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{\sqrt{ax + bx^4}} \\ &= \frac{x\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}\right)^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right) \middle| \frac{1}{4}(2 + \sqrt{3})\right)}{\sqrt[4]{3}\sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}\right)^2}} \sqrt{ax + bx^4}} \end{aligned}$$

Mathematica [C] time = 0.0096315, size = 49, normalized size = 0.25

$$\frac{2x\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{bx^3}{a}\right)}{\sqrt{x(a + bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]/Sqrt[x*(a + b*x^3)]
```

Maple [C] time = 0.016, size = 671, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*b/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*EllipticF((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```


[Out] integrate(1/sqrt(b*x^4 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^4 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*x^4 + a*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**4+a*x)**(1/2),x)

[Out] Integral(1/sqrt(a*x + b*x**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*x^4 + a*x), x)

$$3.99 \quad \int \frac{1}{x^3 \sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=225

$$\frac{2bx(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{5^4 \sqrt[3]{3} a^{4/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax + bx^4}} - \frac{2\sqrt{ax + bx^4}}{5ax^3}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(5*3^{(1/4)}*a^{(4/3)}*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x + b*x^4])$

Rubi [A] time = 0.186875, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2025, 2011, 329, 225}

$$\frac{2bx(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{5^4 \sqrt[3]{3} a^{4/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax + bx^4}} - \frac{2\sqrt{ax + bx^4}}{5ax^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^3*\text{Sqrt}[a*x + b*x^4]), x]$

[Out] $(-2*\text{Sqrt}[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(5*3^{(1/4)}*a^{(4/3)}*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x + b*x^4])$

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{ax + bx^4}} dx &= -\frac{2\sqrt{ax + bx^4}}{5ax^3} - \frac{(2b) \int \frac{1}{\sqrt{ax+bx^4}} dx}{5a} \\
&= -\frac{2\sqrt{ax + bx^4}}{5ax^3} - \frac{(2b\sqrt{x}\sqrt{a + bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{5a\sqrt{ax + bx^4}} \\
&= -\frac{2\sqrt{ax + bx^4}}{5ax^3} - \frac{(4b\sqrt{x}\sqrt{a + bx^3}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{5a\sqrt{ax + bx^4}} \\
&= -\frac{2\sqrt{ax + bx^4}}{5ax^3} - \frac{2bx(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right)\right) \frac{1}{4}(2 + \sqrt{3})}{5\sqrt[4]{3}a^{4/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.013207, size = 53, normalized size = 0.24

$$\frac{2\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; -\frac{bx^3}{a}\right)}{5x^2 \sqrt{x(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5*x^2*Sqrt[x*(a + b*x^3)])

Maple [C] time = 0.018, size = 696, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^4+a*x)^(1/2), x)

[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x

$$\begin{aligned} & /(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}) \\ & ^{(1/2)}*(x-1/b*(-b^2*a)^{(1/3)})^{(1/2)}*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)} \\ & +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b \\ & *(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)})^{(1/2)}*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b \\ & *(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I \\ & *3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)})^{(1/2)}/(-3/2/b*(-b^2*a)^{(1/3)} \\ & +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-b^2*a)^{(1/3)}/(b*x*(x-1/b*(-b^2*a)^{(1/3)}) \\ & *(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(x+1/2/b*(-b^2*a)^{(1/3)} \\ & -1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*EllipticF(((3/2/b*(-b^2*a)^{(1/3)} \\ & +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) \\ & /((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}}{bx^7 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a*x)/(b*x^7 + a*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**4+a*x)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)

$$3.100 \quad \int \frac{x^5}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=503

$$\frac{5(1-\sqrt{3})a^{4/3}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax+bx^4}} + \frac{5\sqrt[4]{3}a^{4/3}x(\sqrt[3]{a}+\sqrt[3]{bx})}{8b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax+bx^4}}$$

[Out] $(-5*(1 + \text{Sqrt}[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)*\text{Sqrt}[a*x + b*x^4]) + (x^2*\text{Sqrt}[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{EllipticE}[\text{ArcCos}[(a^(1/3) + (1 - \text{Sqrt}[3])*b^(1/3)*x)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)], (2 + \text{Sqrt}[3])/4])/ (8*b^(5/3)*\text{Sqrt}[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{Sqrt}[a*x + b*x^4]) + (5*(1 - \text{Sqrt}[3])*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^(1/3) + (1 - \text{Sqrt}[3])*b^(1/3)*x)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)], (2 + \text{Sqrt}[3])/4])/ (16*3^(1/4)*b^(5/3)*\text{Sqrt}[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{Sqrt}[a*x + b*x^4])$

Rubi [A] time = 0.514097, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5(1-\sqrt{3})a^{4/3}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax+bx^4}} + \frac{5\sqrt[4]{3}a^{4/3}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}}{8b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax+bx^4}}}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[a*x + b*x^4], x]

[Out] $(-5*(1 + \text{Sqrt}[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)*\text{Sqrt}[a*x + b*x^4]) + (x^2*\text{Sqrt}[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{EllipticE}[\text{ArcCos}[(a^(1/3) + (1 - \text{Sqrt}[3])*b^(1/3)*x)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)], (2 + \text{Sqrt}[3])/4])/ (8*b^(5/3)*\text{Sqrt}[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{Sqrt}[a*x + b*x^4]) + (5*(1 - \text{Sqrt}[3])*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^(1/3) + (1 - \text{Sqrt}[3])*b^(1/3)*x)/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)], (2 + \text{Sqrt}[3])/4])/ (16*3^(1/4)*b^(5/3)*\text{Sqrt}[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + \text{Sqrt}[3])*b^(1/3)*x)^2]*\text{Sqrt}[a*x + b*x^4])$

$$\begin{aligned} &)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)*x})^2 * \text{EllipticE}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)*x})/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)*x})], (2 + \text{Sqrt}[3])/4]]/ \\ &(8*b^{(5/3)}*\text{Sqrt}[(b^{(1/3)*x}*(a^{(1/3)} + b^{(1/3)*x})/(a^{(1/3)} + (1 + \text{Sqrt}[3])* \\ &b^{(1/3)*x})^2]*\text{Sqrt}[a*x + b*x^4]) + (5*(1 - \text{Sqrt}[3])*a^{(4/3)*x}*(a^{(1/3)} + b^{(1/3)*x})* \\ &\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2})/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)*x})^2] * \\ &\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)*x})/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)*x})], \\ &(2 + \text{Sqrt}[3])/4]]/(16*3^{(1/4)}*b^{(5/3)}*\text{Sqrt}[(b^{(1/3)*x}*(a^{(1/3)} + b^{(1/3)*x})/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)*x})^2] * \\ &\text{Sqrt}[a*x + b*x^4]) \end{aligned}$$

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqrt[a + b*x^6], x], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```



```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4)]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_)*(x_)^4)/Sqrt[(a_) + (b_)*(x_)^6], x_Symbol] := With[{r =
  Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4)]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{\sqrt{ax+bx^4}} dx &= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a) \int \frac{x^2}{\sqrt{ax+bx^4}} dx}{8b} \\
 &= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a\sqrt{x}\sqrt{a+bx^3}) \int \frac{x^{3/2}}{\sqrt{a+bx^3}} dx}{8b\sqrt{ax+bx^4}} \\
 &= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{4b\sqrt{ax+bx^4}} \\
 &= \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{(5a\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{(-1+\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}} + \frac{(5(1-\sqrt{3})a^{5/3}\sqrt{x}\sqrt{a+bx^3})}{8b^{5/3}\sqrt{ax+bx^4}} \\
 &= -\frac{5(1+\sqrt{3})ax(a+bx^3)}{8b^{5/3}(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{5^4\sqrt{3}a^{4/3}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}}{8b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{a+(1+\sqrt{3})\sqrt[3]{bx}})}}}
 \end{aligned}$$

Mathematica [C] time = 0.0239624, size = 66, normalized size = 0.13

$$\frac{x^3 \left(-a\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{4b\sqrt{x(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[a*x + b*x^4],x]

[Out] (x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])

Maple [C] time = 0.016, size = 1079, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^4+a*x)^(1/2),x)

[Out]
$$\frac{1}{4}x^2(bx^4+ax)^{1/2}/b-5/8/b*a*(x*(x+1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*(x+1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})+(1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*((-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*x/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2}*(x-1/b*(-b^2*a)^{1/3})^{2*(1/b*(-b^2*a)^{1/3}*(x+1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(-1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2}*(1/b*(-b^2*a)^{1/3}*(x+1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2})*(((1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/b*(-b^2*a)^{1/3}+1/b^2*(-b^2*a)^{2/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*b/(-b^2*a)^{1/3}*EllipticF(((1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*x/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2},((3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*(1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(3/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})^{1/2}+(1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*EllipticE(((1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*x/(-1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(x-1/b*(-b^2*a)^{1/3})^{1/2},((3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*(1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})/(3/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})^{1/2})*b/(-b^2*a)^{1/3})/(b*x*(x-1/b*(-b^2*a)^{1/3})*(x+1/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})*(x+1/2/b*(-b^2*a)^{1/3}-1/2*I*3^{1/2})/b*(-b^2*a)^{1/3})^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}x^4}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**4+a*x)**(1/2),x)

[Out] Integral(x**5/sqrt(x*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)

$$3.101 \quad \int \frac{x^2}{\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=474

$$\frac{(1 - \sqrt{3}) \sqrt[3]{ax} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \text{EllipticF} \left(\cos^{-1} \left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}} \right), \frac{1}{4} (2 + \sqrt{3}) \right) \sqrt[4]{3} \sqrt[3]{ax} (\sqrt[3]{a} + \sqrt[3]{bx})}{2 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}}$$

[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rubi [A] time = 0.390082, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2032, 329, 308, 225, 1881}

$$\frac{(1 - \sqrt{3}) \sqrt[3]{ax} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F \left(\cos^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}} \right) \right) \frac{1}{4} (2 + \sqrt{3}) \sqrt[4]{3} \sqrt[3]{ax} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx}}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}}}{2 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4} \quad b^{2/3} \sqrt{\frac{\sqrt[3]{a} \sqrt[3]{bx}}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x + b*x^4],x]

[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

$$\left. \right] * b^{(1/3)*x}], (2 + \text{Sqrt}[3])/4) / (b^{(2/3)} * \text{Sqrt}[(b^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)^2] * \text{Sqrt}[a * x + b * x^4]) - ((1 - \text{Sqrt}[3]) * a^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x) * \text{Sqrt}[(a^{(2/3)} - a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)^2] * \text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) * b^{(1/3)} * x) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)], (2 + \text{Sqrt}[3])/4]) / (2 * 3^{(1/4)} * b^{(2/3)} * \text{Sqrt}[(b^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)^2] * \text{Sqrt}[a * x + b * x^4])$$

Rule 2032

$$\text{Int}[\left((c_{.}) * (x_{.}) \right)^{m_{.}} * \left((a_{.}) * (x_{.})^{j_{.}} + (b_{.}) * (x_{.})^{n_{.}} \right)^{p_{.}}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]} * (c * x)^{\text{FracPart}[m]} * (a * x^j + b * x^n)^{\text{FracPart}[p]}) / (x^{\text{FracPart}[m] + j * \text{FracPart}[p]} * (a + b * x^{(n - j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m + j * p)} * (a + b * x^{(n - j)})^p, x], x] /; \text{FreeQ}[\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$$

Rule 329

$$\text{Int}[\left((c_{.}) * (x_{.}) \right)^{m_{.}} * \left((a_{.}) + (b_{.}) * (x_{.})^{n_{.}} \right)^{p_{.}}, x_{\text{Symbol}}] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k * (m + 1) - 1)} * (a + (b * x^{(k * n)}) / c^n)^p, x], x, (c * x)^{(1/k)}], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

Rule 308

$$\text{Int}[(x_{.})^4 / \text{Sqrt}[(a_{.}) + (b_{.}) * (x_{.})^6], x_{\text{Symbol}}] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[\left((\text{Sqrt}[3] - 1) * s^2 \right) / (2 * r^2), \text{Int}[1 / \text{Sqrt}[a + b * x^6], x], x] - \text{Dist}[1 / (2 * r^2), \text{Int}[\left((\text{Sqrt}[3] - 1) * s^2 - 2 * r^2 * x^4 \right) / \text{Sqrt}[a + b * x^6], x], x]] /; \text{FreeQ}[\{a, b\}, x]$$

Rule 225

$$\text{Int}[1 / \text{Sqrt}[(a_{.}) + (b_{.}) * (x_{.})^6], x_{\text{Symbol}}] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(x * (s + r * x^2) * \text{Sqrt}[(s^2 - r * s * x^2 + r^2 * x^4) / (s + (1 + \text{Sqrt}[3]) * r * x^2)^2] * \text{EllipticF}[\text{ArcCos}[(s + (1 - \text{Sqrt}[3]) * r * x^2) / (s + (1 + \text{Sqrt}[3]) * r * x^2)], (2 + \text{Sqrt}[3])/4]) / (2 * 3^{(1/4)} * s * \text{Sqrt}[a + b * x^6] * \text{Sqrt}[(r * x^2 * (s + r * x^2)) / (s + (1 + \text{Sqrt}[3]) * r * x^2)^2]), x]] /; \text{FreeQ}[\{a, b\}, x]$$

Rule 1881

$$\text{Int}[\left((c_{.}) + (d_{.}) * (x_{.})^4 \right) / \text{Sqrt}[(a_{.}) + (b_{.}) * (x_{.})^6], x_{\text{Symbol}}] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[\left((1 + \text{Sqrt}[3]) * d * s^3 * x * \text{Sqrt}[a + b * x^6] \right) / (2 * a * r^2 * (s + (1 + \text{Sqrt}[3]) * r * x^2)), x] - \text{Simp}[\left(3^{(1/4)} * d * s * x * (s + r * x^2) * \text{Sqrt}[(s^2 - r * s * x^2 + r^2 * x^4) / (s + (1 + \text{Sqrt}[3]) * r * x^2)^2] * \text{El}$$

```

lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4)]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax + bx^4}} dx &= \frac{(\sqrt{x}\sqrt{a + bx^3}) \int \frac{x^{3/2}}{\sqrt{a+bx^3}} dx}{\sqrt{ax + bx^4}} \\
&= \frac{(2\sqrt{x}\sqrt{a + bx^3}) \text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{\sqrt{ax + bx^4}} \\
&= -\frac{(\sqrt{x}\sqrt{a + bx^3}) \text{Subst}\left(\int \frac{(-1+\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax + bx^4}} - \frac{\left((1 - \sqrt{3}) a^{2/3} \sqrt{x}\sqrt{a + bx^3}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax + bx^4}} \\
&= \frac{(1 + \sqrt{3}) x (a + bx^3)}{b^{2/3} (\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}) \sqrt{ax + bx^4}} - \frac{\sqrt[4]{3} \sqrt[3]{ax} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} E\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}}\right)\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0117792, size = 53, normalized size = 0.11

$$\frac{2x^3 \sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^3}{a}\right)}{5\sqrt{x(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x + b*x^4], x]

[Out] (2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]) / (5*Sqrt[x*(a + b*x^3)])

Maple [C] time = 0.015, size = 1054, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^4+a*x)^(1/2),x)`

[Out] $(x*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})+(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(x-1/b*(-b^2*a)^{(1/3)})^2*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/((-1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)}+1/b^2*(-b^2*a)^{(2/3)})/((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*b/(-b^2*a)^{(1/3)})*EllipticF(((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)},((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(3/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)},((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(3/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})*b/(-b^2*a)^{(1/3)})))/(b*x*(x-1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/sqrt(b*x^4 + a*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + axx}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**4+a*x)**(1/2),x)

[Out] Integral(x**2/sqrt(x*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)

$$3.102 \quad \int \frac{1}{x\sqrt{ax+bx^4}} dx$$

Optimal. Leaf size=497

$$(1 - \sqrt{3}) \sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \text{EllipticF} \left(\cos^{-1} \left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}} \right), \frac{1}{4} (2 + \sqrt{3}) \right) - 2 \sqrt[4]{3} \sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{\sqrt[3]{3} a^{2/3}}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}$$

[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

Rubi [A] time = 0.472092, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$(1 - \sqrt{3}) \sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F \left(\cos^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}} \right), \frac{1}{4} (2 + \sqrt{3}) \right) - 2 \sqrt[4]{3} \sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax + bx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a*x + b*x^4]), x]

[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])

$$\begin{aligned} & (1/3) + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x^2 * \text{EllipticE}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) * b^{(1/3)} * x) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)], (2 + \text{Sqrt}[3])/4)] / (a^{(2/3)} * \text{Sqrt}[(b^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)]^2 * \text{Sqrt}[a * x + b * x^4]) - ((1 - \text{Sqrt}[3]) * b^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x) * \text{Sqrt}[(a^{(2/3)} - a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)]^2 * \text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) * b^{(1/3)} * x) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)], (2 + \text{Sqrt}[3])/4)] / (3^{(1/4)} * a^{(2/3)} * \text{Sqrt}[(b^{(1/3)} * x * (a^{(1/3)} + b^{(1/3)} * x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)} * x)]^2 * \text{Sqrt}[a * x + b * x^4]) \end{aligned}$$

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqrt[a + b*x^6], x], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4]]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqrt[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4]]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt{ax+bx^4}} dx &= -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{(2b) \int \frac{x^2}{\sqrt{ax+bx^4}} dx}{a} \\
 &= -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{(2b\sqrt{x}\sqrt{a+bx^3}) \int \frac{x^{3/2}}{\sqrt{a+bx^3}} dx}{a\sqrt{ax+bx^4}} \\
 &= -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{(4b\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^4}} \\
 &= -\frac{2\sqrt{ax+bx^4}}{ax} - \frac{(2\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{(-1+\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^4}} - \frac{(2(1-\sqrt{3})\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{(-1-\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^4}} \\
 &= \frac{2(1+\sqrt{3})\sqrt[3]{bx}(a+bx^3)}{a(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax} - \frac{2\sqrt[3]{3}\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{a^{2/3}\sqrt{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}} E\left(\cos^{-1}\left(\frac{\sqrt{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}}{\sqrt{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\right)\right) \\
 &\quad - \frac{2\sqrt[3]{3}\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{a^{2/3}\sqrt{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}} E\left(\cos^{-1}\left(\frac{\sqrt{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}}{\sqrt{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\right)\right)
 \end{aligned}$$

Mathematica [C] time = 0.0114298, size = 48, normalized size = 0.1

$$\frac{2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; -\frac{bx^3}{a}\right)}{\sqrt{x(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x + b*x^4]),x]

[Out] $(-2*\sqrt{1 + (b*x^3)/a}*\text{Hypergeometric2F1}[-1/6, 1/2, 5/6, -((b*x^3)/a)])/\sqrt{x*(a + b*x^3)}$

Maple [C] time = 0.015, size = 1083, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^4+a*x)^(1/2),x)

[Out]
$$\begin{aligned} & -2*(b*x^3+a)/a/((b*x^3+a)*x)^{(1/2)}+2*b/a*(x*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})+(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(x-1/b*(-b^2*a)^{(1/3)})^2*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(1/b*(-b^2*a)^{(1/3)}*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)}*((-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)}+1/b^2*(-b^2*a)^{(2/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*b/(-b^2*a)^{(1/3)}*\text{EllipticF}(((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)},((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)})/((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)})/((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)}))^{(1/2)}+(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*x/(-1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(x-1/b*(-b^2*a)^{(1/3)}))^{(1/2)},((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)})/((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)})/((3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/b*(-b^2*a)^{(1/3)}))^{(1/2)})*b/(-b^2*a)^{(1/3)})/(b*x*(x-1/b*(-b^2*a)^{(1/3)})*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}}{bx^5 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a*x)/(b*x^5 + a*x^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**4+a*x)**(1/2),x)

[Out] Integral(1/(x*sqrt(x*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + axx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

$$3.103 \quad \int \frac{x^2}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=174

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3} - \frac{63b^5 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}} - \frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2} + \frac{2x^2\sqrt{ax+b\sqrt{x}}}{5a}$$

[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))

Rubi [A] time = 0.151043, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 670, 640, 620, 206}

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3} - \frac{63b^5 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}} - \frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2} + \frac{2x^2\sqrt{ax+b\sqrt{x}}}{5a}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 670


```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[
((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c),
Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt{x} + ax}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^5}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right) \\
&= \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} - \frac{(9b) \operatorname{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{5a} \\
&= -\frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} + \frac{(63b^2) \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{40a^2} \\
&= \frac{21b^2 x \sqrt{b\sqrt{x} + ax}}{40a^3} - \frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} - \frac{(21b^3) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{16a^3} \\
&= -\frac{21b^3 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{21b^2 x \sqrt{b\sqrt{x} + ax}}{40a^3} - \frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} + \frac{(63b^4) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{16a^3} \\
&= \frac{63b^4 \sqrt{b\sqrt{x} + ax}}{64a^5} - \frac{21b^3 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{21b^2 x \sqrt{b\sqrt{x} + ax}}{40a^3} - \frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} \\
&= \frac{63b^4 \sqrt{b\sqrt{x} + ax}}{64a^5} - \frac{21b^3 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{21b^2 x \sqrt{b\sqrt{x} + ax}}{40a^3} - \frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a} \\
&= \frac{63b^4 \sqrt{b\sqrt{x} + ax}}{64a^5} - \frac{21b^3 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{21b^2 x \sqrt{b\sqrt{x} + ax}}{40a^3} - \frac{9bx^{3/2} \sqrt{b\sqrt{x} + ax}}{20a^2} + \frac{2x^2 \sqrt{b\sqrt{x} + ax}}{5a}
\end{aligned}$$

Mathematica [A] time = 0.170274, size = 151, normalized size = 0.87

$$\frac{(a\sqrt{x} + b) \left(\sqrt{a}\sqrt{x} \sqrt{\frac{a\sqrt{x}}{b}} + 1 (168a^2b^2x - 144a^3bx^{3/2} + 128a^4x^2 - 210ab^3\sqrt{x} + 315b^4) - 315b^{9/2} \sqrt[4]{x} \sinh^{-1} \left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}} \right) \right)}{320a^{11/2} \sqrt{\frac{a\sqrt{x}}{b}} + 1 \sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x], x]

[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a^3*b*x^(3/2) + 128*a^4*x^2) - 315*b^(9/2)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(320*a^(11/2)*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[b*Sqrt[x] + a*x])

Maple [A] time = 0.06, size = 223, normalized size = 1.3

$$\frac{1}{640} \sqrt{b\sqrt{x} + ax} \left(-544 \sqrt{x} a^{7/2} (b\sqrt{x} + ax)^{3/2} b + 256 x (b\sqrt{x} + ax)^{3/2} a^{9/2} - 1300 \sqrt{x} a^{5/2} \sqrt{b\sqrt{x} + ax} b^3 + 880 a^{5/2} (b\sqrt{x} + ax)^{3/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^(1/2)+a*x)^(1/2),x)`

[Out] $\frac{1}{640} (b\sqrt{x} + ax)^{1/2} (-544 x^{1/2} a^{7/2} (b\sqrt{x} + ax)^{3/2} b + 256 x (b\sqrt{x} + ax)^{3/2} a^{9/2} - 1300 \sqrt{x} a^{5/2} \sqrt{b\sqrt{x} + ax} b^3 + 880 a^{5/2} (b\sqrt{x} + ax)^{3/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/sqrt(a*x + b*sqrt(x)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(x**2/sqrt(a*x + b*sqrt(x)), x)

Giac [A] time = 1.39757, size = 150, normalized size = 0.86

$$\frac{1}{320} \sqrt{ax + b\sqrt{x}} \left(2 \left(4 \left(2 \sqrt{x} \left(\frac{8\sqrt{x}}{a} - \frac{9b}{a^2} \right) + \frac{21b^2}{a^3} \right) \sqrt{x} - \frac{105b^3}{a^4} \right) \sqrt{x} + \frac{315b^4}{a^5} \right) + \frac{63b^5 \log \left(\left| -2\sqrt{a} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) \right| \right)}{128a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x))*(8*sqrt(x)/a - 9*b/a^2) + 21*b^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2)

$$3.104 \quad \int \frac{x}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=116

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3} - \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}} - \frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2} + \frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))

Rubi [A] time = 0.0868686, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3} - \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}} - \frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2} + \frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]

[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))

Rule 2018

Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 670

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(m + p)*(2*c*d - b*e)/(c*(m + 2*p + 1)), Int[(d + e*x)^(

$m - 1) * (a + b * x + c * x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \ \text{EqQ}[c * d^2 - b * d * e + a * e^2, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2 * p + 1, 0] \ \&\& \ \text{IntegerQ}[2 * p]$

Rule 640

$\text{Int}[\{(d _.) + (e _.) * (x _.) * ((a _.) + (b _.) * (x _.) + (c _.) * (x _.)^2)^{(p _.)}, x_Symbol] \rightarrow \text{Simp}[(e * (a + b * x + c * x^2)^{(p + 1)}) / (2 * c * (p + 1)), x] + \text{Dist}[(2 * c * d - b * e) / (2 * c), \text{Int}[(a + b * x + c * x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[2 * c * d - b * e, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 620

$\text{Int}[1 / \text{Sqrt}[(b _.) * (x _.) + (c _.) * (x _.)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1 / (1 - c * x^2), x], x, x / \text{Sqrt}[b * x + c * x^2]], x] /; \text{FreeQ}[\{b, c\}, x]$

Rule 206

$\text{Int}[\{(a _.) + (b _.) * (x _.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2])] / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a / b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
 \int \frac{x}{\sqrt{b\sqrt{x} + ax}} dx &= 2 \text{Subst} \left(\int \frac{x^3}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right) \\
 &= \frac{2x\sqrt{b\sqrt{x} + ax}}{3a} - \frac{(5b) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{3a} \\
 &= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x} + ax}}{3a} + \frac{(5b^2) \text{Subst} \left(\int \frac{x}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{4a^2} \\
 &= \frac{5b^2\sqrt{b\sqrt{x} + ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x} + ax}}{3a} - \frac{(5b^3) \text{Subst} \left(\int \frac{1}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{8a^3} \\
 &= \frac{5b^2\sqrt{b\sqrt{x} + ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x} + ax}}{3a} - \frac{(5b^3) \text{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{4a^3} \\
 &= \frac{5b^2\sqrt{b\sqrt{x} + ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x} + ax}}{3a} - \frac{5b^3 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{4a^{7/2}}
 \end{aligned}$$

Mathematica [A] time = 0.106052, size = 129, normalized size = 1.11

$$\frac{5b^4 \left(\frac{a\sqrt{x}}{b} + 1 \right) \left(\frac{16a^3x^{3/2}}{15b^3} - \frac{4a^2x}{3b^2} + \frac{2a\sqrt{x}}{b} - \frac{2\sqrt{a}\sqrt[4]{x} \sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}} \right)}{8a^4\sqrt{\sqrt{x}(a\sqrt{x}+b)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (5*b^4*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3*x^(3/2))/(15*b^3) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(8*a^4*Sqrt[(b + a*Sqrt[x])*Sqrt[x]])

Maple [B] time = 0.007, size = 181, normalized size = 1.6

$$\frac{1}{24}\sqrt{b\sqrt{x}+ax}\left(16(b\sqrt{x}+ax)^{3/2}a^{5/2}-36\sqrt{b\sqrt{x}+ax}a^{5/2}\sqrt{xb}-18\sqrt{b\sqrt{x}+ax}a^{3/2}b^2+48a^{3/2}\sqrt{\sqrt{x}(b+a\sqrt{x})}b^2-24a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^(1/2)+a*x)^(1/2), x)

[Out] 1/24*(b*x^(1/2)+a*x)^(1/2)/a^(9/2)*(16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)-36*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b-18*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^2+48*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2-24*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3+9*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(x/sqrt(a*x + b*sqrt(x)), x)

Giac [A] time = 1.36586, size = 112, normalized size = 0.97

$$\frac{1}{12} \sqrt{ax + b\sqrt{x}} \left(2\sqrt{x} \left(\frac{4\sqrt{x}}{a} - \frac{5b}{a^2} \right) + \frac{15b^2}{a^3} \right) + \frac{5b^3 \log \left(\left| -2\sqrt{a} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")


```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
+ 5/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b
))/a^(7/2)
```

$$3.105 \quad \int \frac{1}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=56

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)

Rubi [A] time = 0.059945, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2010, 2013, 620, 206}

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Simp[(-2*Sqrt[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n - 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] && LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && EqQ[Simplify[m - n + 1], 0]
```

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{b\sqrt{x} + ax}} dx &= \frac{2\sqrt{b\sqrt{x} + ax}}{a} - \frac{b \int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x} + ax}} dx}{2a} \\
 &= \frac{2\sqrt{b\sqrt{x} + ax}}{a} - \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x}\right)}{a} \\
 &= \frac{2\sqrt{b\sqrt{x} + ax}}{a} - \frac{(2b) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}}\right)}{a} \\
 &= \frac{2\sqrt{b\sqrt{x} + ax}}{a} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}}\right)}{a^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0479315, size = 88, normalized size = 1.57

$$\frac{2\sqrt{a}\sqrt{x}(a\sqrt{x} + b) - 2b^{3/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (2*Sqrt[a]*(b + a*Sqrt[x])*Sqrt[x] - 2*b^(3/2)*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])

Maple [A] time = 0.003, size = 83, normalized size = 1.5

$$-\sqrt{b\sqrt{x} + ax} \left(b \ln \left(\frac{1}{2} \left(2\sqrt{\sqrt{x}(b + a\sqrt{x})}\sqrt{a} + 2a\sqrt{x} + b \right) \frac{1}{\sqrt{a}} \right) - 2\sqrt{\sqrt{x}(b + a\sqrt{x})}\sqrt{a} \right) \frac{1}{\sqrt{\sqrt{x}(b + a\sqrt{x})}} a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^(1/2)+a*x)^(1/2),x)

[Out] -(b*x^(1/2)+a*x)^(1/2)*(b*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))-2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/sqrt(a*x + b*sqrt(x)), x)

Giac [A] time = 1.3343, size = 73, normalized size = 1.3

$$\frac{b \log \left(\left| -2\sqrt{a} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{ax + b\sqrt{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x))) - b))/a^(3/2) + 2*sqrt(ax + b*sqrt(x))/a

$$3.106 \quad \int \frac{1}{x\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=25

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(b*\text{Sqrt}[x])$

Rubi [A] time = 0.0379828, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]),x]$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(b*\text{Sqrt}[x])$

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}} dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Mathematica [A] time = 0.0080369, size = 25, normalized size = 1.

$$\frac{4\sqrt{ax + b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])

Maple [C] time = 0.011, size = 160, normalized size = 6.4

$$-\frac{1}{b^2x}\sqrt{b\sqrt{x} + ax}\left(4(b\sqrt{x} + ax)^{3/2}\sqrt{a} - 2\sqrt{b\sqrt{x} + ax}a^{3/2}x - \ln\left(\frac{1}{2}\left(2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b\right)\frac{1}{\sqrt{a}}\right)\right)xab - 2a^{3/2}\sqrt{b\sqrt{x} + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^(1/2)+a*x)^(1/2),x)

[Out] $-(b*x^{(1/2)}+a*x)^{(1/2)}*(4*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(1/2)}-2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(3/2)}*x-\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)}))^{(1/2)}*x*a*b-2*a^{(3/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*x+\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)}))^{(1/2)}*x*a*b)/(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}/b^2/x/a^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{xx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)

Fricas [A] time = 2.43133, size = 51, normalized size = 2.04

$$-\frac{4\sqrt{ax + b\sqrt{x}}}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.22164, size = 34, normalized size = 1.36

$$\frac{4}{\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))

$$3.107 \quad \int \frac{1}{x^2 \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=84

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}} + \frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x} - \frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(5*b*x^{(3/2)}) + (16*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(15*b^2*x) - (32*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(15*b^3*\text{Sqrt}[x])$

Rubi [A] time = 0.11658, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}} + \frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x} - \frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]),x]$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(5*b*x^{(3/2)}) + (16*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(15*b^2*x) - (32*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(15*b^3*\text{Sqrt}[x])$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx &= -\frac{4\sqrt{b\sqrt{x} + ax}}{5bx^{3/2}} - \frac{(4a) \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx}{5b} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{5bx^{3/2}} + \frac{16a\sqrt{b\sqrt{x} + ax}}{15b^2x} + \frac{(8a^2) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{15b^2} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{5bx^{3/2}} + \frac{16a\sqrt{b\sqrt{x} + ax}}{15b^2x} - \frac{32a^2\sqrt{b\sqrt{x} + ax}}{15b^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0481187, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{ax + b\sqrt{x}}(8a^2x - 4ab\sqrt{x} + 3b^2)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2))

Maple [C] time = 0.01, size = 218, normalized size = 2.6

$$-\frac{1}{15b^4}\sqrt{b\sqrt{x} + ax}\left(60(b\sqrt{x} + ax)^{3/2}a^{5/2}x^{5/2} - 30\sqrt{b\sqrt{x} + ax}a^{7/2}x^{7/2} - 15\ln\left(\frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b}{\sqrt{a}}\right)x^{7/2}a^3b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x)

[Out] -1/15*(b*x^(1/2)+a*x)^(1/2)*(60*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(5/2)-30*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x^(7/2)-15*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(7/2)*a^3*b-30*a^(7/2)*x^(7/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)+15*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2))

$(/2)+b)/a^{(1/2)}) * x^{(7/2)} * a^3 * b + 12 * (b * x^{(1/2)} + a * x)^{(3/2)} * a^{(1/2)} * x^{(3/2)} * b^2 - 28 * a^{(3/2)} * (b * x^{(1/2)} + a * x)^{(3/2)} * b * x^2) / (x^{(1/2)} * (b + a * x^{(1/2)}))^{(1/2)} / b^4 / a^{(1/2)} / x^{(7/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b}\sqrt{xx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)

Fricas [A] time = 2.31516, size = 103, normalized size = 1.23

$$\frac{4(4abx - (8a^2x + 3b^2)\sqrt{x})\sqrt{ax + b\sqrt{x}}}{15b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{ax + b}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.25171, size = 113, normalized size = 1.35

$$\frac{4 \left(20 a \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^2 + 15 \sqrt{ab} \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + 3 b^2 \right)}{15 \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5

$$3.108 \quad \int \frac{1}{x^3 \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=142

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}} - \frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}} + \frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x} + \frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2} - \frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(9*b*x^{(5/2)}) + (32*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^2*x^2) - (64*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(105*b^3*x^{(3/2)}) + (256*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(315*b^4*x) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(315*b^5*\text{Sqrt}[x])$

Rubi [A] time = 0.203379, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}} - \frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}} + \frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x} + \frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2} - \frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]),x]$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(9*b*x^{(5/2)}) + (32*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^2*x^2) - (64*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(105*b^3*x^{(3/2)}) + (256*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(315*b^4*x) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(315*b^5*\text{Sqrt}[x])$

Rule 2016

$\text{Int}[(c*(x_1))^{(m_1)}*((a_1)*(x_1)^{(j_1)} + (b_1)*(x_1)^{(n_1)})^{(p_1)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[m+j*p+1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{b\sqrt{x} + ax}} dx &= -\frac{4\sqrt{b\sqrt{x} + ax}}{9bx^{5/2}} - \frac{(8a) \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x} + ax}} dx}{9b} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{63b^2x^2} + \frac{(16a^2) \int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx}{21b^2} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x} + ax}}{105b^3x^{3/2}} - \frac{(64a^3) \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx}{105b^3} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x} + ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x} + ax}}{315b^4x} + \frac{(128a^4) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{315b^4} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x} + ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x} + ax}}{315b^4x} - \frac{512a^4\sqrt{b\sqrt{x} + ax}}{315b^5\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.0528787, size = 72, normalized size = 0.51

$$\frac{4\sqrt{ax + b\sqrt{x}}(48a^2b^2x - 64a^3bx^{3/2} + 128a^4x^2 - 40ab^3\sqrt{x} + 35b^4)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

Maple [C] time = 0.013, size = 262, normalized size = 1.9

$$-\frac{1}{315b^6} \sqrt{b\sqrt{x} + ax} \left(1260 (b\sqrt{x} + ax)^{3/2} a^{9/2} x^{9/2} - 630 \sqrt{b\sqrt{x} + ax} a^{11/2} x^{11/2} - 315 \ln \left(\frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x)`

[Out]
$$-1/315*(b*x^{(1/2)+a*x})^{(1/2)}*(1260*(b*x^{(1/2)+a*x})^{(3/2)}*a^{(9/2)}*x^{(9/2)}-630*(b*x^{(1/2)+a*x})^{(1/2)}*a^{(11/2)}*x^{(11/2)}-315*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)+a*x})^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^{(11/2)}*a^{5*b}-630*a^{(11/2)}*x^{(11/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}+315*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^{(11/2)}*a^{5*b}+492*(b*x^{(1/2)+a*x})^{(3/2)}*a^{(5/2)}*x^{(7/2)}*b^2+140*(b*x^{(1/2)+a*x})^{(3/2)}*a^{(1/2)}*x^{(5/2)}*b^4-748*a^{(7/2)}*(b*x^{(1/2)+a*x})^{(3/2)}*b*x^4-300*(b*x^{(1/2)+a*x})^{(3/2)}*a^{(3/2)}*x^3*b^3)/(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}/b^6/a^{(1/2)}/x^{(11/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b}\sqrt{xx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)`

Fricas [A] time = 2.33588, size = 155, normalized size = 1.09

$$\frac{4(64a^3bx^2 + 40ab^3x - (128a^4x^2 + 48a^2b^2x + 35b^4)\sqrt{x})\sqrt{ax + b\sqrt{x}}}{315b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")`

[Out]
$$4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*\sqrt{x})*\sqrt{a*x + b*\sqrt{x}}/(b^5*x^3)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.16857, size = 197, normalized size = 1.39

$$\frac{4 \left(1008 a^2 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^4 + 1680 a^{\frac{3}{2}} b \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^3 + 1080 a b^2 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^2 + 315 \sqrt{a} b^3 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) \right)}{315 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9

$$3.109 \quad \int \frac{1}{x^4 \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=200

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}} + \frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2} - \frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}} - \frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}} + \frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x} + \frac{48a\sqrt{ax+b\sqrt{x}}}{143b^2}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(13*b*x^{(7/2)}) + (48*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(143*b^2*x^3) - (160*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(429*b^3*x^{(5/2)}) + (1280*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^4*x^2) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(1001*b^5*x^{(3/2)}) + (2048*a^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^6*x) - (4096*a^6*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^7*\text{Sqrt}[x])$

Rubi [A] time = 0.296884, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}} + \frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2} - \frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}} - \frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}} + \frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x} + \frac{48a\sqrt{ax+b\sqrt{x}}}{143b^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(13*b*x^{(7/2)}) + (48*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(143*b^2*x^3) - (160*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(429*b^3*x^{(5/2)}) + (1280*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^4*x^2) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(1001*b^5*x^{(3/2)}) + (2048*a^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^6*x) - (4096*a^6*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3003*b^7*\text{Sqrt}[x])$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{b\sqrt{x} + ax}} dx &= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} - \frac{(12a) \int \frac{1}{x^{7/2} \sqrt{b\sqrt{x} + ax}} dx}{13b} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} + \frac{(120a^2) \int \frac{1}{x^3 \sqrt{b\sqrt{x} + ax}} dx}{143b^2} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x} + ax}}{429b^3x^{5/2}} - \frac{(320a^3) \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x} + ax}} dx}{429b^3} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x} + ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x} + ax}}{3003b^4x^2} + \frac{(640a^4) \int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx}{1001b^4} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x} + ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x} + ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x} + ax}}{1001b^5x^{3/2}} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x} + ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x} + ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x} + ax}}{1001b^5x^{3/2}} \\
&= -\frac{4\sqrt{b\sqrt{x} + ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x} + ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x} + ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x} + ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x} + ax}}{1001b^5x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.059094, size = 96, normalized size = 0.48

$$\frac{4\sqrt{ax + b\sqrt{x}}(384a^4b^2x^2 - 320a^3b^3x^{3/2} + 280a^2b^4x - 512a^5bx^{5/2} + 1024a^6x^3 - 252ab^5\sqrt{x} + 231b^6)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]), x]
```

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]*(231*b^6 - 252*a*b^5*\text{Sqrt}[x] + 280*a^2*b^4*x - 320*a^3*b^3*x^{3/2} + 384*a^4*b^2*x^2 - 512*a^5*b*x^{5/2} + 1024*a^6*x^3))/(3003*b^7*x^{7/2})$

Maple [C] time = 0.013, size = 306, normalized size = 1.5

$$-\frac{1}{3003 b^8} \sqrt{b\sqrt{x} + ax} \left(12012 (b\sqrt{x} + ax)^{3/2} a^{13/2} x^{13/2} - 6006 \sqrt{b\sqrt{x} + ax} a^{15/2} x^{15/2} - 3003 \ln \left(\frac{1}{2} \frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x)`

[Out] $-1/3003*(b*x^{1/2}+a*x)^{1/2}*(12012*(b*x^{1/2}+a*x)^{3/2}*a^{13/2}*x^{13/2})-6006*(b*x^{1/2}+a*x)^{1/2}*a^{15/2}*x^{15/2}-3003*\ln(1/2*(2*a*x^{1/2}+2*(b*x^{1/2}+a*x)^{1/2})*a^{1/2}+b)/a^{1/2})*x^{15/2}*a^7*b-6006*a^{15/2}*x^{15/2}*(x^{1/2}*(b+a*x^{1/2}))^{1/2}+3003*\ln(1/2*(2*(x^{1/2}*(b+a*x^{1/2}))^{1/2})*a^{1/2}+2*a*x^{1/2}+b)/a^{1/2})*x^{15/2}*a^7*b+5868*(b*x^{1/2}+a*x)^{3/2}*a^{9/2}*x^{11/2}*b^2+3052*(b*x^{1/2}+a*x)^{3/2}*a^{5/2}*x^{9/2}*b^4-7916*a^{11/2}*(b*x^{1/2}+a*x)^{3/2}*b*x^6+924*(b*x^{1/2}+a*x)^{3/2}*a^{1/2}*x^{7/2}*b^6-4332*(b*x^{1/2}+a*x)^{3/2}*a^{7/2}*x^5*b^3-1932*(b*x^{1/2}+a*x)^{3/2}*a^{3/2}*x^4*b^5)/(x^{1/2}*(b+a*x^{1/2}))^{1/2}/b^8/a^{1/2}/x^{15/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{xx^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)`

Fricas [A] time = 2.41402, size = 212, normalized size = 1.06

$$\frac{4 \left(512 a^5 b x^3 + 320 a^3 b^3 x^2 + 252 a b^5 x - (1024 a^6 x^3 + 384 a^4 b^2 x^2 + 280 a^2 b^4 x + 231 b^6) \sqrt{x} \right) \sqrt{ax + b\sqrt{x}}}{3003 b^7 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7*x^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.27784, size = 281, normalized size = 1.4

$$\frac{4 \left(27456 a^3 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^6 + 72072 a^{\frac{5}{2}} b \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^5 + 80080 a^2 b^2 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^4 + 48048 a \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^3 + 14412 a^{\frac{3}{2}} b \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^2 + 14412 a b^2 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + 48048 a^{\frac{3}{2}} \right)}{3003 \left(\sqrt{a}\sqrt{x} - \sqrt{ax} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x)))^6 + 72072*a^(5/2)*b*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x)))^4 + 48048*a*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x)))^3 + 14412*a^(3/2)*b*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x)))^2 + 14412*a*b^2*(sqrt(a)*sqrt(x) - sqrt(ax + b*sqrt(x))) + 48048*a^(3/2))

$$\begin{aligned} & \sqrt{x} - \sqrt{ax + b\sqrt{x}})^4 + 48048a^{3/2}b^3(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}})^3 + 16380ab^4(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}})^2 \\ & + 3003\sqrt{a}b^5(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}) + 231b^6 / (\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}})^{13} \end{aligned}$$

$$3.110 \quad \int \frac{x^3}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=197

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5} + \frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4} - \frac{693b^5 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}} - \frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3} + \frac{22x^2}{5a^2}$$

[Out] $(-4*x^3)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (693*b^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(64*a^6) - (231*b^3*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^5) + (231*b^2*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(40*a^4) - (99*b*x^{(3/2)}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(20*a^3) + (22*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(5*a^2) - (693*b^5*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(64*a^{(13/2)})$

Rubi [A] time = 0.175458, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5} + \frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4} - \frac{693b^5 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}} - \frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3} + \frac{22x^2}{5a^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*x^3)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (693*b^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(64*a^6) - (231*b^3*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^5) + (231*b^2*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(40*a^4) - (99*b*x^{(3/2)}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(20*a^3) + (22*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(5*a^2) - (693*b^5*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(64*a^{(13/2)})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 668

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]

Rule 670

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^7}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{22 \operatorname{Subst} \left(\int \frac{x^5}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{a} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{22x^2\sqrt{b\sqrt{x} + ax}}{5a^2} - \frac{(99b) \operatorname{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{5a^2} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x} + ax}}{5a^2} + \frac{(693b^2) \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{40a^3} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{231b^2x\sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x} + ax}}{5a^2} - \frac{(231b^3) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{40a^3} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x} + ax}}{5a^2} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{693b^4\sqrt{b\sqrt{x} + ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{693b^4\sqrt{b\sqrt{x} + ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3} \\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} + \frac{693b^4\sqrt{b\sqrt{x} + ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x} + ax}}{20a^3}
\end{aligned}$$

Mathematica [C] time = 0.059961, size = 64, normalized size = 0.32

$$\frac{4x^{7/2}\sqrt{\frac{a\sqrt{x}}{b}} + {}_2F_1\left(\frac{3}{2}, \frac{13}{2}; \frac{15}{2}; -\frac{a\sqrt{x}}{b}\right)}{13b\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]

[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(7/2)*Hypergeometric2F1[3/2, 13/2, 15/2, -((a*Sqrt[x])/b)])/(13*b*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.012, size = 549, normalized size = 2.8

$$\frac{1}{640} \sqrt{b\sqrt{x} + ax} \left(256 (b\sqrt{x} + ax)^{3/2} a^{13/2} x^2 - 352 (b\sqrt{x} + ax)^{3/2} a^{11/2} x^{3/2} b - 4060 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{3/2} b^3 + 528 (b\sqrt{x} + ax)^{3/2} a^{7/2} x b^3 - 10150 (b\sqrt{x} + ax)^{3/2} a^{5/2} x^2 b^3 + 8960 (b\sqrt{x} + ax)^{3/2} a^{3/2} x^3 b^3 - 2030 (b\sqrt{x} + ax)^{3/2} a^{1/2} x^4 b^3 + 2030 \ln(1/2 * (2*a*x^{1/2} + 2*(b*x^{1/2} + a*x)^{1/2} * a^{1/2} + b) / a^{1/2}) * x^{1/2} * a^2 * b^6 + 1015 * \ln(1/2 * (2*a*x^{1/2} + 2*(b*x^{1/2} + a*x)^{1/2} * a^{1/2} + b) / a^{1/2}) * x * a^3 * b^5 - 8960 * a^2 * \ln(1/2 * (x^{1/2} * (b + a*x^{1/2}))^{1/2} * (b + a*x^{1/2}))^{1/2} * b^6 - 4480 * a^3 * \ln(1/2 * (x^{1/2} * (b + a*x^{1/2}))^{1/2} * (b + a*x^{1/2}))^{1/2} * a * b^7 - 4480 * a * \ln(1/2 * (x^{1/2} * (b + a*x^{1/2}))^{1/2} * (b + a*x^{1/2}))^{1/2} / (b + a*x^{1/2})^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^(1/2)+a*x)^(3/2),x)

[Out] 1/640*(b*x^(1/2)+a*x)^(1/2)*(256*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^2-352*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(3/2)*b-4060*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(3/2)*b^3+528*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x*b^2+3136*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(1/2)*b^3-10150*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x*b^4+8960*a^(7/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^4+2000*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^4-8120*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^5+17920*a^(5/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^5-2560*a^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*b^4-2030*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^6+8960*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^6+2030*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(1/2)*a^2*b^6+1015*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x*a^3*b^5-8960*a^2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2)*b^6-4480*a^3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*b^5+1015*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^7-4480*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^7)/a^(15/2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.111 \quad \int \frac{x^2}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=139

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4} - \frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}} - \frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3} + \frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2} - \frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

[Out] $(-4*x^2)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (35*b^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(4*a^4) - (35*b*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(6*a^3) + (14*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3*a^2) - (35*b^3*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(4*a^{9/2})$

Rubi [A] time = 0.126971, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4} - \frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}} - \frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3} + \frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2} - \frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*x^2)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (35*b^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(4*a^4) - (35*b*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(6*a^3) + (14*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3*a^2) - (35*b^3*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(4*a^{9/2})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^5}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} + \frac{14 \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{a} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} + \frac{14x\sqrt{b\sqrt{x} + ax}}{3a^2} - \frac{(35b) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{3a^2} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x} + ax}}{3a^2} + \frac{(35b^2) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{4a^3} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} + \frac{35b^2\sqrt{b\sqrt{x} + ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x} + ax}}{3a^2} - \frac{(35b^3) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{8a^3} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} + \frac{35b^2\sqrt{b\sqrt{x} + ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x} + ax}}{3a^2} - \frac{(35b^3) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{4a^3} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} + \frac{35b^2\sqrt{b\sqrt{x} + ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x} + ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x} + ax}}{3a^2} - \frac{35b^3 \tanh^{-1} \left(\frac{\sqrt{a}}{\sqrt{b\sqrt{x} + ax}} \right)}{4a^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.0430175, size = 64, normalized size = 0.46

$$\frac{4x^{5/2} \sqrt{\frac{a\sqrt{x}}{b}} + {}_2F_1 \left(\frac{3}{2}, \frac{9}{2}; \frac{11}{2}; -\frac{a\sqrt{x}}{b} \right)}{9b\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2), x]

[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(5/2)*Hypergeometric2F1[3/2, 9/2, 11/2, -(a*Sqrt[x])/b])/(9*b*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.01, size = 503, normalized size = 3.6

$$\frac{1}{24} \sqrt{b\sqrt{x} + ax} \left(16x (b\sqrt{x} + ax)^{3/2} a^{9/2} - 60 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{3/2} b + 32 \sqrt{xa}^{7/2} (b\sqrt{x} + ax)^{3/2} b - 150 \sqrt{b\sqrt{x} + ax} a^{7/2} x b^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^(1/2)+a*x)^(3/2), x)`

[Out] $\frac{1}{24} (b\sqrt{x} + ax)^{1/2} / a^{11/2} \left(16x (b\sqrt{x} + ax)^{3/2} a^{9/2} - 60 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{3/2} b + 32 \sqrt{xa}^{7/2} (b\sqrt{x} + ax)^{3/2} b - 150 \sqrt{b\sqrt{x} + ax} a^{7/2} x b^2 + \dots \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax + b\sqrt{x})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/2)+a*x)^(3/2), x, algorithm="maxima")`

[Out] `integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)`

[Out] `Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)`

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.112 \quad \int \frac{x}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=77

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2} - \frac{6b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}} - \frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

[Out] $(-4*x)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (6*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a^2 - (6*b*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[b*\text{Sqrt}[x] + a*x])])/a^{(5/2)}$

Rubi [A] time = 0.0742663, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2018, 668, 640, 620, 206}

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2} - \frac{6b \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}} - \frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*x)/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (6*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a^2 - (6*b*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[b*\text{Sqrt}[x] + a*x])])/a^{(5/2)}$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 668

$\text{Int}[(d_. + (e_.)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(p+1)), x] - \text{Dist}[(e^2*(m+p))/(c*(p+1)), \text{Int}[(d + e*x)^{(m-2)}*(a + b*x + c*x^2)^{(p+1)}, x]]$

$2)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{EqQ}[c^2d^2 - b^2de + a^2e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{IntegerQ}[2p]$

Rule 640

$\text{Int}[(d_.) + (e_.)*(x_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(e*(a + b*x + c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 620

$\text{Int}[1/\text{Sqrt}[(b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(1 - c*x^2), x], x, x/\text{Sqrt}[b*x + c*x^2]], x] /; \text{FreeQ}[\{b, c\}, x]$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \text{Subst} \left(\int \frac{x^3}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\ &= -\frac{4x}{a\sqrt{b\sqrt{x} + ax}} + \frac{6 \text{Subst} \left(\int \frac{x}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{a} \\ &= -\frac{4x}{a\sqrt{b\sqrt{x} + ax}} + \frac{6\sqrt{b\sqrt{x} + ax}}{a^2} - \frac{(3b) \text{Subst} \left(\int \frac{1}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{a^2} \\ &= -\frac{4x}{a\sqrt{b\sqrt{x} + ax}} + \frac{6\sqrt{b\sqrt{x} + ax}}{a^2} - \frac{(6b) \text{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{a^2} \\ &= -\frac{4x}{a\sqrt{b\sqrt{x} + ax}} + \frac{6\sqrt{b\sqrt{x} + ax}}{a^2} - \frac{6b \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{a^{5/2}} \end{aligned}$$

Mathematica [C] time = 0.0378811, size = 64, normalized size = 0.83

$$\frac{4x^{3/2}\sqrt{\frac{a\sqrt{x}}{b}} + {}_2F_1\left(\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, -\frac{a\sqrt{x}}{b}\right)}{5b\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2), x]

[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*Sqrt[x])/b])/(5*b*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.006, size = 237, normalized size = 3.1

$$-\sqrt{b\sqrt{x} + ax} \left(3 \ln \left(\frac{2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + 2a\sqrt{x} + b}{\sqrt{a}} \right) \right) xa^2b - 6a^{5/2}x\sqrt{\sqrt{x}(b+a\sqrt{x})} + 6 \ln \left(\frac{2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a}}{\sqrt{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^(1/2)+a*x)^(3/2), x)

[Out] -(b*x^(1/2)+a*x)^(1/2)/a^(5/2)*(3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*a^2*b-6*a^(5/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(1/2)+6*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2)*a*b^2-12*a^(3/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b+4*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)+3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3-6*a^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.113 \quad \int \frac{1}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=25

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])

Rubi [A] time = 0.0051318, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2000}

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]

[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int \frac{1}{(b\sqrt{x} + ax)^{3/2}} dx = \frac{4\sqrt{x}}{b\sqrt{b\sqrt{x} + ax}}$$

Mathematica [A] time = 0.0164931, size = 25, normalized size = 1.

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]

[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])

Maple [C] time = 0.008, size = 404, normalized size = 16.2

$$\frac{1}{b^2} \sqrt{b\sqrt{x} + ax} \left(2 \sqrt{b\sqrt{x} + ax} a^{5/2} x + \ln \left(\frac{1}{2} \left(2 a \sqrt{x} + 2 \sqrt{b\sqrt{x} + ax} \sqrt{a} + b \right) \frac{1}{\sqrt{a}} \right) x a^2 b + 2 a^{5/2} x \sqrt{\sqrt{x} (b + a \sqrt{x})} - \ln \left(\frac{1}{2} \left(\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^(1/2)+a*x)^(3/2),x)

[Out] (b*x^(1/2)+a*x)^(1/2)*(2*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x+ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x*a^2*b+2*a^(5/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(1/2)-ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*a^2*b+4*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*x^(1/2)*b+2*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(1/2)*a*b^2+4*a^(3/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b-4*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)-2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2)*a*b^2+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)*b^2+ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*b^3+2*a^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2-ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^2/(b+a*x^(1/2))^2/a^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)

Fricas [A] time = 2.28203, size = 77, normalized size = 3.08

$$\frac{4\sqrt{ax + b\sqrt{x}}(a\sqrt{x} - b)}{a^2bx - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a²*b*x - b³)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x)

[Out] Integral((a*x + b*sqrt(x))^(-3/2), x)

Giac [A] time = 1.14704, size = 46, normalized size = 1.84

$$\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))

$$3.114 \quad \int \frac{1}{x(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=79

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}} - \frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x} + \frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])

Rubi [A] time = 0.122479, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 2014}

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}} - \frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x} + \frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x} + ax}} + \frac{4 \int \frac{1}{x^{3/2}\sqrt{b\sqrt{x} + ax}} dx}{b} \\ &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x} + ax}} - \frac{16\sqrt{b\sqrt{x} + ax}}{3b^2x} - \frac{(8a) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{3b^2} \\ &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x} + ax}} - \frac{16\sqrt{b\sqrt{x} + ax}}{3b^2x} + \frac{32a\sqrt{b\sqrt{x} + ax}}{3b^3\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.0524482, size = 48, normalized size = 0.61

$$\frac{4(8a^2x + 4ab\sqrt{x} - b^2)}{3b^3\sqrt{x}\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)), x]
```

```
[Out] (4*(-b^2 + 4*a*b*Sqrt[x] + 8*a^2*x))/(3*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
```


Maple [C] time = 0.011, size = 524, normalized size = 6.6

$$\frac{1}{3b^4} \sqrt{b\sqrt{x} + ax} \left(24 (b\sqrt{x} + ax)^{3/2} a^{7/2} x^{5/2} - 6 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{7/2} - 3 \ln \left(\frac{1}{2} \frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b}{\sqrt{a}} \right) x^{7/2} a^4 b - 6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^(1/2)+a*x)^(3/2), x)

[Out] $\frac{1}{3} (b\sqrt{x} + ax)^{1/2} \left(24 (b\sqrt{x} + ax)^{3/2} a^{7/2} x^{5/2} - 6 (b\sqrt{x} + ax)^{1/2} a^{9/2} x^{7/2} - 3 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) / a^{1/2} \right) x^{7/2} a^4 b - 6 (b\sqrt{x} + ax)^{1/2} a^{9/2} x^{7/2} - 3 \ln \left(\frac{1}{2} (2(x^{1/2}(b+a\sqrt{x})))^{1/2} + 3 \ln \left(\frac{1}{2} (2(x^{1/2}(b+a\sqrt{x})))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) / a^{1/2} \right) x^{7/2} a^4 b + 44 (b\sqrt{x} + ax)^{3/2} a^{5/2} x^{2b} - 12 (b\sqrt{x} + ax)^{1/2} a^{7/2} x^{3b} - 6 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) / a^{1/2} \right) x^{3a^3 b^2} - 12 a^{7/2} x^3 (x^{1/2}(b+a\sqrt{x}))^{1/2} b - 12 a^{7/2} x^{5/2} (x^{1/2}(b+a\sqrt{x}))^{3/2} + 6 \ln \left(\frac{1}{2} (2(x^{1/2}(b+a\sqrt{x})))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) / a^{1/2} \right) x^3 a^3 b^2 + 16 (b\sqrt{x} + ax)^{3/2} a^{3/2} x^{3/2} b^2 - 6 (b\sqrt{x} + ax)^{1/2} a^{5/2} x^{5/2} b^2 - 3 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) / a^{1/2} \right) x^{5/2} a^2 b^3 - 6 a^{5/2} x^{5/2} (x^{1/2}(b+a\sqrt{x}))^{1/2} b^2 + 3 \ln \left(\frac{1}{2} (2(x^{1/2}(b+a\sqrt{x})))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) / a^{1/2} \right) x^{5/2} a^2 b^3 - 4 (b\sqrt{x} + ax)^{3/2} a^{1/2} x^3 b^3 / (x^{1/2}(b+a\sqrt{x}))^{1/2} / b^4 a^{1/2} / (b+a\sqrt{x} + ax)^{2/2} x^{5/2} \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)

Fricas [A] time = 1.87056, size = 132, normalized size = 1.67

$$\frac{4(4a^2bx - b^3 - (8a^3x - 5ab^2)\sqrt{x})\sqrt{ax + b\sqrt{x}}}{3(a^2b^3x^2 - b^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^3*x^2 - b^5*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)

$$3.115 \quad \int \frac{1}{x^2(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=137

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}} - \frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x} + \frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}} - \frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2} + \frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

[Out] $4/(b*x^{(3/2)}*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^{(3/2)}) - (256*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])$

Rubi [A] time = 0.201879, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}} - \frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x} + \frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}} - \frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2} + \frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)), x]

[Out] $4/(b*x^{(3/2)}*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^{(3/2)}) - (256*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])$

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 (b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x} + ax}} + \frac{8 \int \frac{1}{x^{5/2}\sqrt{b\sqrt{x} + ax}} dx}{b} \\
 &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x} + ax}} - \frac{32\sqrt{b\sqrt{x} + ax}}{7b^2x^2} - \frac{(48a) \int \frac{1}{x^2\sqrt{b\sqrt{x} + ax}} dx}{7b^2} \\
 &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x} + ax}} - \frac{32\sqrt{b\sqrt{x} + ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x} + ax}}{35b^3x^{3/2}} + \frac{(192a^2) \int \frac{1}{x^{3/2}\sqrt{b\sqrt{x} + ax}} dx}{35b^3} \\
 &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x} + ax}} - \frac{32\sqrt{b\sqrt{x} + ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x} + ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x} + ax}}{35b^4x} - \frac{(128a^3) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{35b^4} \\
 &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x} + ax}} - \frac{32\sqrt{b\sqrt{x} + ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x} + ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x} + ax}}{35b^4x} + \frac{512a^3\sqrt{b\sqrt{x} + ax}}{35b^5\sqrt{x}}
 \end{aligned}$$

Mathematica [A] time = 0.0594126, size = 72, normalized size = 0.53

$$\frac{4(-16a^2b^2x + 64a^3bx^{3/2} + 128a^4x^2 + 8ab^3\sqrt{x} - 5b^4)}{35b^5x^{3/2}\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] (4*(-5*b^4 + 8*a*b^3*Sqrt[x] - 16*a^2*b^2*x + 64*a^3*b*x^(3/2) + 128*a^4*x^2))/(35*b^5*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])

Maple [C] time = 0.01, size = 570, normalized size = 4.2

$$\frac{1}{35b^6} \sqrt{b\sqrt{x} + ax} \left(560 (b\sqrt{x} + ax)^{3/2} a^{11/2} x^{9/2} - 210 \sqrt{b\sqrt{x} + ax} a^{13/2} x^{11/2} - 105 \ln \left(\frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a+b}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x)

[Out] 1/35*(b*x^(1/2)+a*x)^(1/2)*(560*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(9/2)-210*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^(11/2)-105*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(11/2)*a^6*b-210*a^(13/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x^(11/2)+105*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(11/2)*a^6*b+256*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(7/2)*b^2+932*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^4*b-420*(b*x^(1/2)+a*x)^(1/2)*a^(11/2)*x^5*b-210*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^5*a^5*b^2-140*a^(11/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*x^(9/2)-420*a^(11/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x^5*b+210*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^5*a^5*b^2-64*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^3*b^3-210*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(9/2)*b^2-105*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(9/2)*a^4*b^3-210*a^(9/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x^(9/2)*b^2+105*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(9/2)*a^4*b^3+32*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(5/2)*b^4-20*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^2*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^6/a^(1/2)/x^(9/2)/(b+a*x^(1/2))^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)

Fricas [A] time = 1.78855, size = 190, normalized size = 1.39

$$\frac{4 \left(64 a^4 b x^2 - 24 a^2 b^3 x - 5 b^5 - (128 a^5 x^2 - 80 a^3 b^2 x - 13 a b^4) \sqrt{x} \right) \sqrt{a x + b \sqrt{x}}}{35 (a^2 b^5 x^3 - b^7 x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x - 13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

$$3.116 \quad \int \frac{1}{x^3(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=195

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}} - \frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2} + \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}} - \frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x} + \frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}} - \frac{48\sqrt{ax+b\sqrt{x}}}{11b^2x^3}$$

[Out] $4/(b*x^{(5/2)}*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^{(5/2)}) - (1280*a^2*Sqrt[b*Sqrt[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^{(3/2)}) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x] + a*x])/(231*b^7*Sqrt[x])$

Rubi [A] time = 0.299999, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}} - \frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2} + \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}} - \frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x} + \frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}} - \frac{48\sqrt{ax+b\sqrt{x}}}{11b^2x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] $4/(b*x^{(5/2)}*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^{(5/2)}) - (1280*a^2*Sqrt[b*Sqrt[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^{(3/2)}) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x] + a*x])/(231*b^7*Sqrt[x])$

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

```

Rule 2014

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} + \frac{12 \int \frac{1}{x^{7/2}\sqrt{b\sqrt{x} + ax}} dx}{b} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} - \frac{(120a) \int \frac{1}{x^3\sqrt{b\sqrt{x} + ax}} dx}{11b^2} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x} + ax}}{33b^3x^{5/2}} + \frac{(320a^2) \int \frac{1}{x^{5/2}\sqrt{b\sqrt{x} + ax}} dx}{33b^3} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x} + ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x} + ax}}{231b^4x^2} - \frac{(640a^3) \int \frac{1}{x^2\sqrt{b\sqrt{x} + ax}} dx}{77b^4} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x} + ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x} + ax}}{231b^4x^2} + \frac{512a^3\sqrt{b\sqrt{x} + ax}}{77b^5x^{3/2}} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x} + ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x} + ax}}{231b^4x^2} + \frac{512a^3\sqrt{b\sqrt{x} + ax}}{77b^5x^{3/2}} \\
&= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x} + ax}} - \frac{48\sqrt{b\sqrt{x} + ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x} + ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x} + ax}}{231b^4x^2} + \frac{512a^3\sqrt{b\sqrt{x} + ax}}{77b^5x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0627457, size = 96, normalized size = 0.49

$$\frac{4 \left(-128a^4b^2x^2 + 64a^3b^3x^{3/2} - 40a^2b^4x + 512a^5bx^{5/2} + 1024a^6x^3 + 28ab^5\sqrt{x} - 21b^6 \right)}{231b^7x^{5/2}\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] (4*(-21*b^6 + 28*a*b^5*Sqrt[x] - 40*a^2*b^4*x + 64*a^3*b^3*x^(3/2) - 128*a^4*b^2*x^2 + 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(231*b^7*x^(5/2)*Sqrt[b*Sqrt[x] + a*x])

Maple [C] time = 0.012, size = 614, normalized size = 3.2

$$\frac{1}{231 b^8} \sqrt{b\sqrt{x} + ax} \left(1155 \ln \left(\frac{1}{2} \frac{2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + 2a\sqrt{x} + b}{\sqrt{a}} \right) x^{15/2} a^8 b - 2310 \ln \left(\frac{1}{2} \frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(1/2)+a*x)^(3/2), x)

[Out] $\frac{1}{231} (b\sqrt{x} + ax)^{1/2} \left(1155 \ln \left(\frac{1}{2} (2\sqrt{x}(b+a\sqrt{x}))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) x^{15/2} a^8 b - 2310 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) a^{1/2} \right) x^{15/2} a^8 b - 924 a^{15/2} x^{13/2} (b\sqrt{x} + ax)^{3/2} + 2310 \ln \left(\frac{1}{2} (2\sqrt{x}(b+a\sqrt{x}))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) x^{13/2} a^6 b^3 + 256 (b\sqrt{x} + ax)^{3/2} a^{7/2} x^{9/2} b^4 - 1155 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) a^{1/2} \right) x^{13/2} a^6 b^3 + 1155 \ln \left(\frac{1}{2} (2\sqrt{x}(b+a\sqrt{x}))^{1/2} a^{1/2} + 2a\sqrt{x} + b \right) x^{13/2} a^6 b^3 - 160 (b\sqrt{x} + ax)^{3/2} a^{5/2} x^4 b^5 + 112 (b\sqrt{x} + ax)^{3/2} a^{3/2} x^{7/2} b^6 - 84 (b\sqrt{x} + ax)^{3/2} a^{1/2} x^3 b^7 - 512 (b\sqrt{x} + ax)^{3/2} a^{9/2} x^5 b^3 + 2048 (b\sqrt{x} + ax)^{3/2} a^{11/2} x^{11/2} b^2 - 4620 a^{15/2} x^7 (b\sqrt{x} + ax)^{1/2} b + 8716 (b\sqrt{x} + ax)^{3/2} a^{13/2} x^6 b - 4620 (b\sqrt{x} + ax)^{1/2} a^{15/2} x^7 b - 2310 (b\sqrt{x} + ax)^{1/2} a^{13/2} x^{13/2} b^2 - 2310 a^{13/2} x^{13/2} (b\sqrt{x} + ax)^{1/2} b^2 + 5544 (b\sqrt{x} + ax)^{3/2} a^{15/2} x^{13/2} - 2310 (b\sqrt{x} + ax)^{1/2} a^{17/2} x^{15/2} - 2310 a^{17/2} x^{15/2} (b\sqrt{x} + ax)^{1/2} - 1155 \ln \left(\frac{1}{2} (2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b) a^{1/2} \right) x^{15/2} a^8 b \right) / (x^{1/2} (b\sqrt{x} + ax)^{1/2})^2 / b^8 / a^{1/2} / x^{13/2} / (b\sqrt{x} + ax)^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)

Fricas [A] time = 1.89979, size = 246, normalized size = 1.26

$$\frac{4 \left(512 a^6 b x^3 - 192 a^4 b^3 x^2 - 68 a^2 b^5 x - 21 b^7 - (1024 a^7 x^3 - 640 a^5 b^2 x^2 - 104 a^3 b^4 x - 49 a b^6) \sqrt{x} \right) \sqrt{a x + b \sqrt{x}}}{231 (a^2 b^7 x^4 - b^9 x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^7*x^4 - b^9*x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)

$$3.117 \quad \int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=204

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3} - \frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5} - \frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4} + \frac{231b^6 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}} - \frac{11bx}{11bx}$$

[Out] $(-231*b^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(256*a^6) + (77*b^4*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(128*a^5) - (77*b^3*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(160*a^4) + (33*b^2*x^{3/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(80*a^3) - (11*b*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(30*a^2) + (x^{5/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3*a) + (231*b^6*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(256*a^{13/2})$

Rubi [A] time = 0.17155, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2018, 670, 640, 620, 206}

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3} - \frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5} - \frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4} + \frac{231b^6 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}} - \frac{11bx}{11bx}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{5/2}/\text{Sqrt}[b*\text{Sqrt}[x] + a*x], x]$

[Out] $(-231*b^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(256*a^6) + (77*b^4*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(128*a^5) - (77*b^3*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(160*a^4) + (33*b^2*x^{3/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(80*a^3) - (11*b*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(30*a^2) + (x^{5/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(3*a) + (231*b^6*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(256*a^{13/2})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{b\sqrt{x} + ax}} dx &= 2 \text{Subst} \left(\int \frac{x^6}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right) \\
&= \frac{x^{5/2} \sqrt{b\sqrt{x} + ax}}{3a} - \frac{(11b) \text{Subst} \left(\int \frac{x^5}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{6a} \\
&= -\frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} + \frac{x^{5/2} \sqrt{b\sqrt{x} + ax}}{3a} + \frac{(33b^2) \text{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{20a^2} \\
&= \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} + \frac{x^{5/2} \sqrt{b\sqrt{x} + ax}}{3a} - \frac{(231b^3) \text{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{160a^3} \\
&= -\frac{77b^3 x \sqrt{b\sqrt{x} + ax}}{160a^4} + \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} + \frac{x^{5/2} \sqrt{b\sqrt{x} + ax}}{3a} + \frac{(77b^4) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{128a^5} \\
&= \frac{77b^4 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{128a^5} - \frac{77b^3 x \sqrt{b\sqrt{x} + ax}}{160a^4} + \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} + \frac{x^{5/2} \sqrt{b\sqrt{x} + ax}}{3a} \\
&= -\frac{231b^5 \sqrt{b\sqrt{x} + ax}}{256a^6} + \frac{77b^4 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{128a^5} - \frac{77b^3 x \sqrt{b\sqrt{x} + ax}}{160a^4} + \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} \\
&= -\frac{231b^5 \sqrt{b\sqrt{x} + ax}}{256a^6} + \frac{77b^4 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{128a^5} - \frac{77b^3 x \sqrt{b\sqrt{x} + ax}}{160a^4} + \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2} \\
&= -\frac{231b^5 \sqrt{b\sqrt{x} + ax}}{256a^6} + \frac{77b^4 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{128a^5} - \frac{77b^3 x \sqrt{b\sqrt{x} + ax}}{160a^4} + \frac{33b^2 x^{3/2} \sqrt{b\sqrt{x} + ax}}{80a^3} - \frac{11bx^2 \sqrt{b\sqrt{x} + ax}}{30a^2}
\end{aligned}$$

Mathematica [A] time = 0.179312, size = 164, normalized size = 0.8

$$\frac{(a\sqrt{x} + b) \left(\sqrt{a}\sqrt{x} \sqrt{\frac{a\sqrt{x}}{b}} + 1 (1584a^3b^2x^{3/2} - 1848a^2b^3x - 1408a^4bx^2 + 1280a^5x^{5/2} + 2310ab^4\sqrt{x} - 3465b^5) + 3465b^{11/2} \right)}{3840a^{13/2} \sqrt{\frac{a\sqrt{x}}{b}} + 1 \sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x], x]

[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1

$$280*a^5*x^{(5/2)} + 3465*b^{(11/2)}*x^{(1/4)}*ArcSinh[(Sqrt[a]*x^{(1/4)})/Sqrt[b]] \\))/(3840*a^{(13/2)}*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[b*Sqrt[x] + a*x])$$

Maple [A] time = 0.01, size = 245, normalized size = 1.2

$$\frac{1}{7680} \sqrt{b\sqrt{x} + ax} \left(2560 x^{3/2} (b\sqrt{x} + ax)^{3/2} a^{11/2} + 8544 (b\sqrt{x} + ax)^{3/2} a^{7/2} \sqrt{x} b^2 - 5376 (b\sqrt{x} + ax)^{3/2} a^{9/2} x b - 12240 (b\sqrt{x} + ax)^{3/2} a^{5/2} b^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)

[Out] 1/7680*(b*x^(1/2)+a*x)^(1/2)*(2560*x^(3/2)*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)+8544*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(1/2)*b^2-5376*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x*b-12240*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^2+16860*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^4+8430*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^5-15360*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^5+7680*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^6-4215*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2))*a^(1/2)+b)/a^(1/2))*a*b^6)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a^(15/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)`

[Out] `Integral(x**(5/2)/sqrt(a*x + b*sqrt(x)), x)`

Giac [A] time = 1.32523, size = 169, normalized size = 0.83

$$\frac{1}{3840} \sqrt{ax + b\sqrt{x}} \left(2 \left(4 \left(2 \left(8 \sqrt{x} \left(\frac{10\sqrt{x}}{a} - \frac{11b}{a^2} \right) + \frac{99b^2}{a^3} \right) \sqrt{x} - \frac{231b^3}{a^4} \right) \sqrt{x} + \frac{1155b^4}{a^5} \right) \sqrt{x} - \frac{3465b^5}{a^6} \right) - \frac{231b^6 \log \left(\left| -2\sqrt{a} \sqrt{ax + b\sqrt{x}} - \sqrt{a} \sqrt{x} \right| \right)}{a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")`

[Out] `1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2) + 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465*b^5/a^6) - 231/512*b^6*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(13/2)`

$$3.118 \quad \int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=146

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3} + \frac{35b^4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}} - \frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2} + \frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

[Out] $(-35*b^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^4) + (35*b^2*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(48*a^3) - (7*b*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(12*a^2) + (x^{3/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a) + (35*b^4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(32*a^{9/2})$

Rubi [A] time = 0.119912, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2018, 670, 640, 620, 206}

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3} + \frac{35b^4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}} - \frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2} + \frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{3/2}/\text{Sqrt}[b*\text{Sqrt}[x] + a*x], x]$

[Out] $(-35*b^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^4) + (35*b^2*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(48*a^3) - (7*b*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(12*a^2) + (x^{3/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a) + (35*b^4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(32*a^{9/2})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, j, m, n, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{IntegerQ}[\text{Simplify}[j/n]] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ \text{NeQ}[n^2, 1]$

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[
((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c),
Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{b\sqrt{x} + ax}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^4}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right) \\
&= \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} - \frac{(7b) \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{4a} \\
&= -\frac{7bx \sqrt{b\sqrt{x} + ax}}{12a^2} + \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} + \frac{(35b^2) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{24a^2} \\
&= \frac{35b^2 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{48a^3} - \frac{7bx \sqrt{b\sqrt{x} + ax}}{12a^2} + \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} - \frac{(35b^3) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{32a^3} \\
&= -\frac{35b^3 \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{35b^2 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{48a^3} - \frac{7bx \sqrt{b\sqrt{x} + ax}}{12a^2} + \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} + \frac{(35b^4) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{64a^4} \\
&= -\frac{35b^3 \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{35b^2 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{48a^3} - \frac{7bx \sqrt{b\sqrt{x} + ax}}{12a^2} + \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} + \frac{(35b^4) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{32a^4} \\
&= -\frac{35b^3 \sqrt{b\sqrt{x} + ax}}{32a^4} + \frac{35b^2 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{48a^3} - \frac{7bx \sqrt{b\sqrt{x} + ax}}{12a^2} + \frac{x^{3/2} \sqrt{b\sqrt{x} + ax}}{2a} + \frac{35b^4 \tanh^{-1} \left(\frac{\sqrt{a}}{\sqrt{b\sqrt{x} + ax}} \right)}{32a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.137471, size = 142, normalized size = 0.97

$$\frac{35b^5 \left(\frac{a\sqrt{x}}{b} + 1 \right) \left(-\frac{32a^4 x^2}{35b^4} + \frac{16a^3 x^{3/2}}{15b^3} - \frac{4a^2 x}{3b^2} + \frac{2a\sqrt{x}}{b} - \frac{2\sqrt{a} \sqrt[4]{x} \sinh^{-1} \left(\frac{\sqrt{a} \sqrt[4]{x}}{\sqrt{b}} \right)}{\sqrt{b} \sqrt{\frac{a\sqrt{x}}{b} + 1}} \right)}{64a^5 \sqrt{\sqrt{x} (a\sqrt{x} + b)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (-35*b^5*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3*x^(3/2))/(15*b^3) - (32*a^4*x^2)/(35*b^4) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(64*a^5*Sqrt[(b + a*Sqrt[x])*Sqrt[x]])

Maple [A] time = 0.006, size = 203, normalized size = 1.4

$$\frac{1}{192} \sqrt{b\sqrt{x} + ax} \left(96 \sqrt{x} (b\sqrt{x} + ax)^{3/2} a^{7/2} - 208 (b\sqrt{x} + ax)^{3/2} a^{5/2} b + 348 \sqrt{b\sqrt{x} + ax} a^{5/2} \sqrt{x} b^2 + 174 \sqrt{b\sqrt{x} + ax} a^{3/2} b^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2), x)

[Out] 1/192*(b*x^(1/2)+a*x)^(1/2)*(96*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)-208*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b+348*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^2+174*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^3-384*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^3+192*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^4-87*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^4)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a^(11/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(x**(3/2)/sqrt(a*x + b*sqrt(x)), x)

Giac [A] time = 1.31092, size = 131, normalized size = 0.9

$$\frac{1}{96} \sqrt{ax + b\sqrt{x}} \left(2 \left(4 \sqrt{x} \left(\frac{6\sqrt{x}}{a} - \frac{7b}{a^2} \right) + \frac{35b^2}{a^3} \right) \sqrt{x} - \frac{105b^3}{a^4} \right) - \frac{35b^4 \log \left(\left| -2\sqrt{a} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(9/2)

$$3.119 \quad \int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=87

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}} - \frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2} + \frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

[Out] $(-3*b*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^2) + (\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a + (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(2*a^{(5/2)})$

Rubi [A] time = 0.0788138, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}} - \frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2} + \frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[x]/\text{Sqrt}[b*\text{Sqrt}[x] + a*x], x]$

[Out] $(-3*b*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^2) + (\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a + (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(2*a^{(5/2)})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n] + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 670

$\text{Int}[(d_. + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(m+2*p+1)), x] + \text{Dist}[(m+p)*(2*c*d - b*e)/(c*(m+2*p+1)), \text{Int}[(d + e*x)^{(m-1)}*(a + b*x + c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m+2*p

+ 1, 0] && IntegerQ[2*p]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right) \\
 &= \frac{\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a} - \frac{(3b) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{2a} \\
 &= -\frac{3b\sqrt{b\sqrt{x} + ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a} + \frac{(3b^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{4a^2} \\
 &= -\frac{3b\sqrt{b\sqrt{x} + ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a} + \frac{(3b^2) \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{2a^2} \\
 &= -\frac{3b\sqrt{b\sqrt{x} + ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a} + \frac{3b^2 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{2a^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0799227, size = 102, normalized size = 1.17

$$\frac{\sqrt{a}\sqrt{x}(2a^2x - ab\sqrt{x} - 3b^2) + 3b^{5/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{2a^{5/2}\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x], x]

[Out] (Sqrt[a]*Sqrt[x]*(-3*b^2 - a*b*Sqrt[x] + 2*a^2*x) + 3*b^(5/2)*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(2*a^(5/2)*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.007, size = 160, normalized size = 1.8

$$\frac{1}{4}\sqrt{b\sqrt{x} + ax} \left(4\sqrt{b\sqrt{x} + ax}a^{5/2}\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}a^{3/2}b - 8a^{3/2}\sqrt{\sqrt{x}(b + a\sqrt{x})}b + 4a \ln \left(\frac{2\sqrt{\sqrt{x}(b + a\sqrt{x})}\sqrt{a} + 2}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2), x)

[Out] 1/4*(b*x^(1/2)+a*x)^(1/2)*(4*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b-8*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b+4*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^2-b^2*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a^(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)

Giac [A] time = 1.34643, size = 93, normalized size = 1.07

$$\frac{1}{2} \sqrt{ax + b\sqrt{x}} \left(\frac{2\sqrt{x}}{a} - \frac{3b}{a^2} \right) - \frac{3b^2 \log \left(\left| -2\sqrt{a} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5/2)

$$3.120 \quad \int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=34

$$\frac{4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]

Rubi [A] time = 0.0490875, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2013, 620, 206}

$$\frac{4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]

Rule 2013

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p), x_Symbol] :> Dist[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && EqQ[Simplify[m - n + 1], 0]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}} dx &= 2 \text{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right) \\ &= 4 \text{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} \right) \\ &= \frac{4 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} \right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0415114, size = 65, normalized size = 1.91

$$\frac{4\sqrt{b}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1} \sinh^{-1} \left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}} \right)}{\sqrt{a}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (4*Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.009, size = 133, normalized size = 3.9

$$\frac{1}{b}\sqrt{b\sqrt{x}+ax} \left(2\sqrt{b\sqrt{x}+ax}\sqrt{a} + b \ln \left(\frac{1}{2} \left(2a\sqrt{x} + 2\sqrt{b\sqrt{x}+ax}\sqrt{a} + b \right) \frac{1}{\sqrt{a}} \right) - 2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + b \ln \left(\frac{1}{2} \left(2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + b \right) \frac{1}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x)

[Out] $(b*x^{(1/2)}+a*x)^{(1/2)}*(2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})-2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+b*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})))/(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}/b/a^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{ax + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)`

[Out] `Integral(1/(sqrt(x)*sqrt(a*x + b*sqrt(x))), x)`

Giac [A] time = 1.29107, size = 50, normalized size = 1.47

$$\frac{2 \log \left(\left| -2 \sqrt{a} \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] -2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/sqrt(a)

$$3.121 \quad \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=54

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}} - \frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])

Rubi [A] time = 0.073614, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}} - \frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegerQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx = -\frac{4\sqrt{b\sqrt{x} + ax}}{3bx} - \frac{(2a) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{3b}$$

$$= -\frac{4\sqrt{b\sqrt{x} + ax}}{3bx} + \frac{8a\sqrt{b\sqrt{x} + ax}}{3b^2\sqrt{x}}$$

Mathematica [A] time = 0.0466093, size = 37, normalized size = 0.69

$$\frac{4(2a\sqrt{x} - b)\sqrt{ax + b\sqrt{x}}}{3b^2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (4*(-b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x))/(3*b^2*x)

Maple [C] time = 0.009, size = 194, normalized size = 3.6

$$\frac{1}{3b^3} \sqrt{b\sqrt{x} + ax} \left(12 (b\sqrt{x} + ax)^{3/2} a^{3/2} x^{3/2} - 6 \sqrt{b\sqrt{x} + ax} a^{5/2} x^{5/2} - 3 \ln \left(1/2 \frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + b}{\sqrt{a}} \right) \right) x^{5/2} a^2 b - 6 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x)

[Out] 1/3*(b*x^(1/2)+a*x)^(1/2)*(12*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(3/2)-6*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(5/2)-3*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(5/2)*a^2*b-6*a^(5/2)*x^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)+3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(5/2)*a^2*b-4*(b*x^(1/2)+a*x)^(3/2)*b*a^(1/2)*x/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^3/a^(1/2)/x^(5/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{xx^{\frac{3}{2}}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)

Fricas [A] time = 2.30499, size = 72, normalized size = 1.33

$$\frac{4\sqrt{ax + b\sqrt{x}}(2a\sqrt{x} - b)}{3b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.27765, size = 72, normalized size = 1.33

$$\frac{4 \left(3 \sqrt{a} \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + b \right)}{3 \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3

$$3.122 \quad \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=112

$$\frac{64a^3 \sqrt{ax+b\sqrt{x}}}{35b^4 \sqrt{x}} - \frac{32a^2 \sqrt{ax+b\sqrt{x}}}{35b^3 x} + \frac{24a \sqrt{ax+b\sqrt{x}}}{35b^2 x^{3/2}} - \frac{4 \sqrt{ax+b\sqrt{x}}}{7bx^2}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(7*b*x^2) + (24*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^2*x^{(3/2)}) - (32*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^3*x) + (64*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^4*\text{Sqrt}[x])$

Rubi [A] time = 0.153552, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{64a^3 \sqrt{ax+b\sqrt{x}}}{35b^4 \sqrt{x}} - \frac{32a^2 \sqrt{ax+b\sqrt{x}}}{35b^3 x} + \frac{24a \sqrt{ax+b\sqrt{x}}}{35b^2 x^{3/2}} - \frac{4 \sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(7*b*x^2) + (24*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^2*x^{(3/2)}) - (32*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^3*x) + (64*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(35*b^4*\text{Sqrt}[x])$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)

*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x} + ax}} dx &= -\frac{4\sqrt{b\sqrt{x} + ax}}{7bx^2} - \frac{(6a) \int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx}{7b} \\ &= -\frac{4\sqrt{b\sqrt{x} + ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x} + ax}}{35b^2x^{3/2}} + \frac{(24a^2) \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx}{35b^2} \\ &= -\frac{4\sqrt{b\sqrt{x} + ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x} + ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x} + ax}}{35b^3x} - \frac{(16a^3) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{35b^3} \\ &= -\frac{4\sqrt{b\sqrt{x} + ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x} + ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x} + ax}}{35b^3x} + \frac{64a^3\sqrt{b\sqrt{x} + ax}}{35b^4\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.0499462, size = 59, normalized size = 0.53

$$\frac{4\sqrt{ax + b\sqrt{x}}(-8a^2bx + 16a^3x^{3/2} + 6ab^2\sqrt{x} - 5b^3)}{35b^4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^3 + 6*a*b^2*Sqrt[x] - 8*a^2*b*x + 16*a^3*x^(3/2)))/(35*b^4*x^2)

Maple [C] time = 0.008, size = 240, normalized size = 2.1

$$\frac{1}{35b^5} \sqrt{b\sqrt{x} + ax} \left(140 (b\sqrt{x} + ax)^{3/2} a^{7/2} x^{7/2} - 70 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{9/2} - 35 \ln \left(\frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a+b}}{\sqrt{a}} \right) \right) x^{9/2} a^4 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)`

[Out] $\frac{1}{35}(b\sqrt{x}+ax)^{1/2}(140(b\sqrt{x}+ax)^{3/2}a^{7/2}x^{7/2}-70(b\sqrt{x}+ax)^{1/2}a^{9/2}x^{9/2}-35\ln(1/2(2ax^{1/2}+2(b\sqrt{x}+ax)^{1/2}a^{1/2}+b)/a^{1/2})x^{9/2}a^{4b-70}a^{9/2}x^{9/2}(x^{1/2}(b+ax^{1/2}))^{1/2}+35\ln(1/2(2(x^{1/2}(b+ax^{1/2})))^{1/2}a^{1/2}+2ax^{1/2}+b)/a^{1/2})x^{9/2}a^{4b+44}(b\sqrt{x}+ax)^{3/2}a^{3/2}x^{5/2}b^2-76a^{5/2}(b\sqrt{x}+ax)^{3/2}b^3x^3-20(b\sqrt{x}+ax)^{3/2}a^{1/2}x^2b^3)/(x^{1/2}(b+ax^{1/2}))^{1/2}b^5/a^{1/2}/x^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{xx^2}}^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)`

Fricas [A] time = 2.36806, size = 123, normalized size = 1.1

$$\frac{4(8a^2bx + 5b^3 - 2(8a^3x + 3ab^2)\sqrt{x})\sqrt{ax + b\sqrt{x}}}{35b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")`

[Out] $-4/35(8a^2bx + 5b^3 - 2(8a^3x + 3ab^2)\sqrt{x})\sqrt{ax + b\sqrt{x}}/(b^4x^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{ax + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)

Giac [A] time = 1.22808, size = 155, normalized size = 1.38

$$\frac{4 \left(70 a^{\frac{3}{2}} \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^3 + 84 ab \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^2 + 35 \sqrt{ab^2} \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + 5 b^3 \right)}{35 \left(\sqrt{a} \sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7

$$3.123 \quad \int \frac{1}{x^{7/2} \sqrt{b\sqrt{x}+ax}} dx$$

Optimal. Leaf size=170

$$\frac{128a^3 \sqrt{ax+b\sqrt{x}}}{231b^4 x^{3/2}} - \frac{320a^2 \sqrt{ax+b\sqrt{x}}}{693b^3 x^2} + \frac{1024a^5 \sqrt{ax+b\sqrt{x}}}{693b^6 \sqrt{x}} - \frac{512a^4 \sqrt{ax+b\sqrt{x}}}{693b^5 x} + \frac{40a \sqrt{ax+b\sqrt{x}}}{99b^2 x^{5/2}} - \frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(11*b*x^3) + (40*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(99*b^2*x^{(5/2)}) - (320*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^3*x^2) + (128*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(231*b^4*x^{(3/2)}) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^5*x) + (1024*a^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^6*\text{Sqrt}[x])$

Rubi [A] time = 0.244644, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{128a^3 \sqrt{ax+b\sqrt{x}}}{231b^4 x^{3/2}} - \frac{320a^2 \sqrt{ax+b\sqrt{x}}}{693b^3 x^2} + \frac{1024a^5 \sqrt{ax+b\sqrt{x}}}{693b^6 \sqrt{x}} - \frac{512a^4 \sqrt{ax+b\sqrt{x}}}{693b^5 x} + \frac{40a \sqrt{ax+b\sqrt{x}}}{99b^2 x^{5/2}} - \frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(7/2)}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]),x]$

[Out] $(-4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(11*b*x^3) + (40*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(99*b^2*x^{(5/2)}) - (320*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^3*x^2) + (128*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(231*b^4*x^{(3/2)}) - (512*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^5*x) + (1024*a^5*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(693*b^6*\text{Sqrt}[x])$

Rule 2016

$\text{Int}[(c*(x_))^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[m+j*p+1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}} dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} - \frac{(10a) \int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}} dx}{11b} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} + \frac{(80a^2) \int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}} dx}{99b^2} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} - \frac{(160a^3) \int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}} dx}{231b^3} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} + \frac{(128a^4) \int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}} dx}{231b^4} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}
\end{aligned}$$

Mathematica [A] time = 0.0548261, size = 83, normalized size = 0.49

$$\frac{4\sqrt{ax+b\sqrt{x}}(96a^3b^2x^{3/2}-80a^2b^3x-128a^4bx^2+256a^5x^{5/2}+70ab^4\sqrt{x}-63b^5)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-63*b^5 + 70*a*b^4*Sqrt[x] - 80*a^2*b^3*x + 96*a^3*b^2*x^(3/2) - 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(693*b^6*x^3)
```


Maple [C] time = 0.01, size = 284, normalized size = 1.7

$$\frac{1}{693 b^7} \sqrt{b\sqrt{x} + ax} \left(2772 (b\sqrt{x} + ax)^{3/2} a^{11/2} x^{11/2} - 1386 \sqrt{b\sqrt{x} + ax} a^{13/2} x^{13/2} - 693 \ln \left(\frac{1}{2} \frac{2a\sqrt{x} + 2\sqrt{b\sqrt{x} + ax}\sqrt{a} + \dots}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2), x)

[Out] 1/693*(b*x^(1/2)+a*x)^(1/2)*(2772*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(11/2)-1386*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^(13/2)-693*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(13/2)*a^6*b-1386*a^(13/2)*x^(13/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)+693*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(13/2)*a^6*b+1236*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(9/2)*b^2+532*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(7/2)*b^4-1748*a^(9/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^5-852*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^4*b^3-252*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^3*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^7/a^(1/2)/x^(13/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{xx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)

Fricas [A] time = 2.13488, size = 178, normalized size = 1.05

$$\frac{4 \left(128 a^4 b x^2 + 80 a^2 b^3 x + 63 b^5 - 2 \left(128 a^5 x^2 + 48 a^3 b^2 x + 35 a b^4 \right) \sqrt{x} \right) \sqrt{ax + b\sqrt{x}}}{693 b^6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

[Out]
$$-4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2*x + 35*a*b^4)*\sqrt{x})*\sqrt{a*x + b*\sqrt{x}}/(b^6*x^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.18865, size = 239, normalized size = 1.41

$$\frac{4 \left(3696 a^{\frac{5}{2}} \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^5 + 7920 a^2 b \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^4 + 6930 a^{\frac{3}{2}} b^2 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^3 + 3080 ab^3 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^2 + 693 b^4 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + 63 b^5 \right)}{693 \left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")

[Out]
$$4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11$$

$$3.124 \quad \int \frac{x^{5/2}}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=171

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4} + \frac{315b^4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}} + \frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2} - \frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3} - \frac{4x}{a\sqrt{ax}}$$

[Out] $(-4*x^{(5/2)})/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (315*b^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^5) + (105*b^2*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(16*a^4) - (21*b*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(4*a^3) + (9*x^{(3/2)}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^2) + (315*b^4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(32*a^{(11/2)})$

Rubi [A] time = 0.146974, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2018, 668, 670, 640, 620, 206}

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4} + \frac{315b^4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}} + \frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2} - \frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3} - \frac{4x}{a\sqrt{ax}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(5/2)}/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*x^{(5/2)})/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (315*b^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(32*a^5) + (105*b^2*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(16*a^4) - (21*b*x*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(4*a^3) + (9*x^{(3/2)}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^2) + (315*b^4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/(32*a^{(11/2)})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, j, m, n, p\}, x \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& \text{NeQ}[n^2, 1]$

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \text{Subst} \left(\int \frac{x^6}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} + \frac{18 \text{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{a} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2} - \frac{(63b) \text{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{4a^2} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{21bx\sqrt{b\sqrt{x} + ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2} + \frac{(105b^2) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{8a^3} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x} + ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x} + ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2} - \frac{(315b^3) \text{Subst} \left(\int \frac{x}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{8a^3} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{315b^3\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x} + ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x} + ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{315b^3\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x} + ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x} + ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2} \\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{315b^3\sqrt{b\sqrt{x} + ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x} + ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x} + ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x} + ax}}{2a^2}
\end{aligned}$$

Mathematica [C] time = 0.0517102, size = 62, normalized size = 0.36

$$\frac{4x^3 \sqrt{\frac{a\sqrt{x}}{b}} + {}_2F_1\left(\frac{3}{2}, \frac{11}{2}; \frac{13}{2}; -\frac{a\sqrt{x}}{b}\right)}{11b\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2), x]

[Out] $(4\sqrt{1 + (a\sqrt{x})/b} * x^3 \text{Hypergeometric2F1}[3/2, 11/2, 13/2, -((a\sqrt{x})/b)]) / (11 * b\sqrt{b\sqrt{x} + ax})$

Maple [B] time = 0.009, size = 527, normalized size = 3.1

$$\frac{1}{64} \sqrt{b\sqrt{x} + ax} \left(32 x^{3/2} (b\sqrt{x} + ax)^{3/2} a^{11/2} - 48 (b\sqrt{x} + ax)^{3/2} a^{9/2} x b + 276 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{3/2} b^2 - 192 (b\sqrt{x} + ax)^{3/2} a^{7/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{5/2}/(b*x^{1/2}+a*x)^{3/2}, x)$

[Out] $\frac{1}{64} * (b*x^{1/2}+a*x)^{1/2} / a^{13/2} * (32*x^{3/2} * (b*x^{1/2}+a*x)^{3/2} * a^{11/2} - 48 * (b*x^{1/2}+a*x)^{3/2} * a^{9/2} * x * b + 276 * (b*x^{1/2}+a*x)^{1/2} * a^{9/2} * x^{3/2} * b^2 - 192 * (b*x^{1/2}+a*x)^{3/2} * a^{7/2} * x^{1/2} * b^2 + 690 * (b*x^{1/2}+a*x)^{1/2} * a^{7/2} * x * b^3 - 768 * a^{7/2} * x * (x^{1/2} * (b+a*x^{1/2}))^{1/2} * b^3 + 384 * a^3 * \ln(1/2 * (2 * (x^{1/2} * (b+a*x^{1/2})))^{1/2} * a^{1/2} + 2 * a * x^{1/2} + b) / a^{1/2}) * x * b^4 - 112 * (b*x^{1/2}+a*x)^{3/2} * a^{5/2} * b^3 + 552 * (b*x^{1/2}+a*x)^{1/2} * a^{5/2} * x^{1/2} * b^4 - 1536 * a^{5/2} * x^{1/2} * (x^{1/2} * (b+a*x^{1/2}))^{1/2} * b^4 + 256 * a^{5/2} * (x^{1/2} * (b+a*x^{1/2}))^{3/2} * b^3 + 768 * a^2 * \ln(1/2 * (2 * (x^{1/2} * (b+a*x^{1/2})))^{1/2} * a^{1/2} + 2 * a * x^{1/2} + b) / a^{1/2}) * x^{1/2} * b^5 - 69 * \ln(1/2 * (2 * a * x^{1/2} + 2 * (b*x^{1/2}+a*x)^{1/2} * a^{1/2} + b) / a^{1/2}) * x * a^3 * b^4 + 138 * (b*x^{1/2}+a*x)^{1/2} * a^{3/2} * b^5 - 768 * a^{3/2} * (x^{1/2} * (b+a*x^{1/2}))^{1/2} * b^5 + 384 * a * \ln(1/2 * (2 * (x^{1/2} * (b+a*x^{1/2})))^{1/2} * a^{1/2} + 2 * a * x^{1/2} + b) / a^{1/2}) * b^6 - 138 * \ln(1/2 * (2 * a * x^{1/2} + 2 * (b*x^{1/2}+a*x)^{1/2} * a^{1/2} + b) / a^{1/2}) * x^{1/2} * a^2 * b^5 - 69 * \ln(1/2 * (2 * a * x^{1/2} + 2 * (b*x^{1/2}+a*x)^{1/2} * a^{1/2} + b) / a^{1/2}) * a * b^6) / (x^{1/2} * (b+a*x^{1/2}))^{1/2} / (b+a*x^{1/2})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{5/2}/(b*x^{1/2}+a*x)^{3/2}, x, \text{algorithm}="maxima")$

[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.125 \quad \int \frac{x^{3/2}}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=113

$$\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}} - \frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3} + \frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2} - \frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

[Out] $(-4*x^{(3/2)})/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (15*b*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^3) + (5*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a^2 + (15*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[b*\text{Sqrt}[x] + a*x])])/(2*a^{(7/2)})$

Rubi [A] time = 0.102718, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}} - \frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3} + \frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2} - \frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(3/2)}/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*x^{(3/2)})/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (15*b*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(2*a^3) + (5*\text{Sqrt}[x]*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/a^2 + (15*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/(\text{Sqrt}[b*\text{Sqrt}[x] + a*x])])/(2*a^{(7/2)})$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 668

$\text{Int}[(d_. + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(p+1)),$

$x] - \text{Dist}[(e^{2(m+p)})/(c(p+1)), \text{Int}[(d+e*x)^{(m-2)}*(a+b*x+c*x^2)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]

Rule 670

$\text{Int}[(d_.) + (e_.)*(x_))^{(m_)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_)}, x_Symbol] := \text{Simp}[(e*(d+e*x)^{(m-1)}*(a+b*x+c*x^2)^{(p+1)})/(c*(m+2*p+1)), x] + \text{Dist}[(m+p)*(2*c*d-b*e)/(c*(m+2*p+1)), \text{Int}[(d+e*x)^{(m-1)}*(a+b*x+c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 640

$\text{Int}[(d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_)}, x_Symbol] := \text{Simp}[(e*(a+b*x+c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \text{Dist}[(2*c*d-b*e)/(2*c), \text{Int}[(a+b*x+c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 620

$\text{Int}[1/\text{Sqrt}[(b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] := \text{Dist}[2, \text{Subst}[\text{Int}[1/(1-c*x^2), x], x, x/\text{Sqrt}[b*x+c*x^2]], x] /;$ FreeQ[{b, c}, x]

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \operatorname{Subst} \left(\int \frac{x^4}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} + \frac{10 \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{a} \\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} + \frac{5\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a^2} - \frac{(15b) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{2a^2} \\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{15b\sqrt{b\sqrt{x} + ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a^2} + \frac{(15b^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \sqrt{x} \right)}{4a^3} \\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{15b\sqrt{b\sqrt{x} + ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a^2} + \frac{(15b^2) \operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} \right)}{2a^3} \\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{15b\sqrt{b\sqrt{x} + ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a^2} + \frac{15b^2 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} \right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.049401, size = 62, normalized size = 0.55

$$\frac{4x^2 \sqrt{\frac{a\sqrt{x}}{b}} + {}_2F_1 \left(\frac{3}{2}, \frac{7}{2}; \frac{9}{2}; -\frac{a\sqrt{x}}{b} \right)}{7b\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]

[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^2*Hypergeometric2F1[3/2, 7/2, 9/2, -(a*Sqrt[x])/b])/(7*b*Sqrt[b*Sqrt[x] + a*x])

Maple [B] time = 0.008, size = 440, normalized size = 3.9

$$\frac{1}{4} \sqrt{b\sqrt{x} + ax} \left(4 \sqrt{b\sqrt{x} + ax} a^{9/2} x^{3/2} + 10 \sqrt{b\sqrt{x} + ax} a^{7/2} x b - 32 a^{7/2} x \sqrt{\sqrt{x} (b + a\sqrt{x})} b + 16 a^3 \ln \left(\frac{2 \sqrt{\sqrt{x} (b + a\sqrt{x})}}{\sqrt{\dots}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2), x)

[Out] $\frac{1}{4} (b\sqrt{x} + ax)^{1/2} a^{-9/2} (4 (b\sqrt{x} + ax)^{1/2} a^{9/2} x^{3/2} + 10 (b\sqrt{x} + ax)^{1/2} a^{7/2} x b - 32 a^{7/2} x (x^{1/2} (b + a\sqrt{x}))^{1/2} (b + a\sqrt{x})^{1/2} + 16 a^3 \ln(1/2 (2 (x^{1/2} (b + a\sqrt{x}))^{1/2} a^{1/2} + 2 a x^{1/2} + b) / a^{1/2}) x b^2 + 8 (b\sqrt{x} + ax)^{1/2} a^{5/2} x^{1/2} b^2 - 64 a^{5/2} x^{1/2} (x^{1/2} (b + a\sqrt{x}))^{1/2} b^2 + 16 a^{5/2} (x^{1/2} (b + a\sqrt{x}))^{1/2} (3/2) b + 32 a^2 \ln(1/2 (2 (x^{1/2} (b + a\sqrt{x}))^{1/2} a^{1/2} + 2 a x^{1/2} + b) / a^{1/2}) x^{1/2} b^3 - \ln(1/2 (2 a x^{1/2} + 2 (b\sqrt{x} + a x)^{1/2} a^{1/2} + b) / a^{1/2}) x a^3 b^2 + 2 (b\sqrt{x} + ax)^{1/2} a^{3/2} b^3 - 32 a^{3/2} (x^{1/2} (b + a\sqrt{x}))^{1/2} b^3 + 16 a \ln(1/2 (2 (x^{1/2} (b + a\sqrt{x}))^{1/2} a^{1/2} + 2 a x^{1/2} + b) / a^{1/2}) b^4 - 2 \ln(1/2 (2 a x^{1/2} + 2 (b\sqrt{x} + a x)^{1/2} a^{1/2} + b) / a^{1/2}) x^{1/2} a^2 b^3 - \ln(1/2 (2 a x^{1/2} + 2 (b\sqrt{x} + a x)^{1/2} a^{1/2} + b) / a^{1/2}) a b^4) / (x^{1/2} (b + a\sqrt{x}))^{1/2} / (b + a\sqrt{x})^{1/2})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.126 \quad \int \frac{\sqrt{x}}{(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=60

$$\frac{4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}} - \frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

[Out] $(-4*\text{Sqrt}[x])/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/a^{(3/2)}$

Rubi [A] time = 0.0671288, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2018, 652, 620, 206}

$$\frac{4 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}} - \frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[x]/(b*\text{Sqrt}[x] + a*x)^{(3/2)}, x]$

[Out] $(-4*\text{Sqrt}[x])/(a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) + (4*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[b*\text{Sqrt}[x] + a*x]])/a^{(3/2)}$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1}*(a*x^{\text{Simplify}[j/n] + b*x})^p, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 652

$\text{Int}[(d_. + (e_.)*(x_))^{2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)*(a + b*x + c*x^2)^{(p+1)})/(c*(p+1)), x] - \text{Dist}[(e^2*(p+2))/(c*(p+1)), \text{Int}[(a + b*x + c*x^2)^{(p+1)}, x], x] /;$ FreeQ[{

$a, b, c, d, e, p, x]$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $\text{EqQ}[c*d^2 - b*d*e + a*e^2, 0]$ && $\text{IntegerQ}[p]$ && $\text{LtQ}[p, -1]$

Rule 620

$\text{Int}[1/\text{Sqrt}[(b_)*(x_)+(c_)*(x_)^2], x_Symbol] \text{ :> } \text{Dist}[2, \text{Subst}[\text{Int}[1/(1 - c*x^2), x], x, x/\text{Sqrt}[b*x + c*x^2]], x] \text{ /; } \text{FreeQ}[\{b, c\}, x]$

Rule 206

$\text{Int}[(a_)+(b_)*(x_)^2]^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \text{ \&\& } \text{NegQ}[a/b] \text{ \&\& } (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{(b\sqrt{x} + ax)^{3/2}} dx &= 2 \text{Subst} \left(\int \frac{x^2}{(bx + ax^2)^{3/2}} dx, x, \sqrt{x} \right) \\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x} + ax}} + \frac{2 \text{Subst} \left(\int \frac{1}{\sqrt{bx + ax^2}} dx, x, \sqrt{x} \right)}{a} \\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x} + ax}} + \frac{4 \text{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{a} \\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x} + ax}} + \frac{4 \tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.105056, size = 79, normalized size = 1.32

$$\frac{4\sqrt[4]{x} \left(\sqrt{b} \sqrt{\frac{a\sqrt{x}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}} \right) - \sqrt{a}\sqrt[4]{x} \right)}{a^{3/2} \sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]

[Out] $(4*x^{1/4}*(-(\text{Sqrt}[a]*x^{1/4}) + \text{Sqrt}[b]*\text{Sqrt}[1 + (a*\text{Sqrt}[x])/b])* \text{ArcSinh}[(\text{Sqrt}[a]*x^{1/4})/\text{Sqrt}[b]])/(a^{3/2}*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])$

Maple [B] time = 0.006, size = 238, normalized size = 4.

$$2 \frac{\sqrt{b\sqrt{x} + ax}}{a^{3/2} \sqrt{\sqrt{x}(b + a\sqrt{x})} b (b + a\sqrt{x})^2} \left(\ln \left(\frac{1}{2} \frac{2 \sqrt{\sqrt{x}(b + a\sqrt{x})} \sqrt{a} + 2 a \sqrt{x} + b}{\sqrt{a}} \right) x a^2 b - 2 a^{5/2} x \sqrt{\sqrt{x}(b + a\sqrt{x})} + 2 \ln \left(\frac{1}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)

[Out] $2*(b*x^{1/2}+a*x)^{1/2}/a^{3/2}*(\ln(1/2*(2*(x^{1/2}*(b+a*x^{1/2})))^{1/2}*a^{1/2}+2*a*x^{1/2}+b)/a^{1/2})*x*a^{2*b-2*a^{5/2}}*x*(x^{1/2}*(b+a*x^{1/2}))^{(1/2)+2*\ln(1/2*(2*(x^{1/2}*(b+a*x^{1/2})))^{1/2}*a^{1/2}+2*a*x^{1/2}+b)/a^{1/2})*x^{1/2}*a*b^{2-4*a^{3/2}}*x^{1/2}*(x^{1/2}*(b+a*x^{1/2}))^{1/2}*b+2*a^{3/2}*(x^{1/2}*(b+a*x^{1/2}))^{3/2}+\ln(1/2*(2*(x^{1/2}*(b+a*x^{1/2})))^{1/2}*a^{1/2}+2*a*x^{1/2}+b)/a^{1/2})*b^{3-2*a^{1/2}}*(x^{1/2}*(b+a*x^{1/2}))^{1/2}*b^2/(x^{1/2}*(b+a*x^{1/2}))^{1/2}/b/(b+a*x^{1/2})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.127 \quad \int \frac{1}{\sqrt{x}(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=30

$$-\frac{4(2a\sqrt{x}+b)}{b^2\sqrt{ax+b\sqrt{x}}}$$

[Out] $(-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])$

Rubi [A] time = 0.0471669, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2013, 613}

$$-\frac{4(2a\sqrt{x}+b)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[x]*(b*\text{Sqrt}[x] + a*x)^{(3/2)}), x]$

[Out] $(-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])$

Rule 2013

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(a*x^{\text{Simplify}[j/n]} + b*x)^p, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, j, m, n, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{EqQ}[\text{Simplify}[m - n + 1], 0]$

Rule 613

$\text{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-3/2}, x_Symbol] \rightarrow \text{Simp}[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]), x] /;$ $\text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\int \frac{1}{\sqrt{x}(b\sqrt{x}+ax)^{3/2}} dx = 2 \text{Subst} \left(\int \frac{1}{(bx+ax^2)^{3/2}} dx, x, \sqrt{x} \right)$$

$$= -\frac{4(b+2a\sqrt{x})}{b^2\sqrt{b\sqrt{x}+ax}}$$

Mathematica [A] time = 0.0373334, size = 45, normalized size = 1.5

$$\frac{4(2a\sqrt{x}+b)\sqrt{ax+b\sqrt{x}}}{ab^2x+b^3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^3*Sqrt[x] + a*b^2*x)

Maple [B] time = 0.01, size = 111, normalized size = 3.7

$$-4 \frac{\sqrt{b\sqrt{x}+ax} \left((b\sqrt{x}+ax)^{3/2} xa^2 + 2(b\sqrt{x}+ax)^{3/2} \sqrt{x}ab - (\sqrt{x}(b+a\sqrt{x}))^{3/2} xa^2 + (b\sqrt{x}+ax)^{3/2} b^2 \right)}{\sqrt{\sqrt{x}(b+a\sqrt{x})} b^3 (b+a\sqrt{x})^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)

[Out] -4*(b*x^(1/2)+a*x)^(1/2)*((b*x^(1/2)+a*x)^(3/2)*x*a^2+2*(b*x^(1/2)+a*x)^(3/2)*x^(1/2)*a*b-(x^(1/2)*(b+a*x^(1/2)))^(3/2)*x*a^2+(b*x^(1/2)+a*x)^(3/2)*b^2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^3/(b+a*x^(1/2))^2/x

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax+b\sqrt{x})^{\frac{3}{2}}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)`

Fricas [B] time = 2.2923, size = 109, normalized size = 3.63

$$\frac{4 \left(abx - \left(2a^2x - b^2 \right) \sqrt{x} \right) \sqrt{ax + b\sqrt{x}}}{a^2b^2x^2 - b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^4*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x} (ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)`

[Out] `Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)`

Giac [A] time = 1.21724, size = 35, normalized size = 1.17

$$-\frac{4 \left(\frac{2a\sqrt{x}}{b^2} + \frac{1}{b} \right)}{\sqrt{ax + b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

$$3.128 \quad \int \frac{1}{x^{3/2}(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=107

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}} - \frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}} + \frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x} + \frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(5*b^4*Sqrt[x])

Rubi [A] time = 0.156028, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2015, 2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}} - \frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}} + \frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x} + \frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(5*b^4*Sqrt[x])

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^{3/2} (b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{bx\sqrt{b\sqrt{x} + ax}} + \frac{6 \int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx}{b} \\ &= \frac{4}{bx\sqrt{b\sqrt{x} + ax}} - \frac{24\sqrt{b\sqrt{x} + ax}}{5b^2x^{3/2}} - \frac{(24a) \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx}{5b^2} \\ &= \frac{4}{bx\sqrt{b\sqrt{x} + ax}} - \frac{24\sqrt{b\sqrt{x} + ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{5b^3x} + \frac{(16a^2) \int \frac{1}{x\sqrt{b\sqrt{x} + ax}} dx}{5b^3} \\ &= \frac{4}{bx\sqrt{b\sqrt{x} + ax}} - \frac{24\sqrt{b\sqrt{x} + ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x} + ax}}{5b^3x} - \frac{64a^2\sqrt{b\sqrt{x} + ax}}{5b^4\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.0506263, size = 57, normalized size = 0.53

$$\frac{4(8a^2bx + 16a^3x^{3/2} - 2ab^2\sqrt{x} + b^3)}{5b^4x\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/2)))/(5*b^4*x*Sqrt[b*Sqrt[x] + a*x])
```

Maple [C] time = 0.011, size = 548, normalized size = 5.1

$$-\frac{2}{5b^5}\sqrt{b\sqrt{x}+ax}\left(30(b\sqrt{x}+ax)^{3/2}a^{9/2}x^{7/2}-10\sqrt{b\sqrt{x}+ax}a^{11/2}x^{9/2}-5\ln\left(\frac{1}{2}\frac{2a\sqrt{x}+2\sqrt{b\sqrt{x}+ax}\sqrt{a+b}}{\sqrt{a}}\right)\right)x^{9/2}a^5b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2), x)

[Out] $-2/5*(b*x^{(1/2)}+a*x)^{(1/2)}*(30*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(9/2)}*x^{(7/2)}-10*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(11/2)}*x^{(9/2)}-5*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^{(9/2)}*a^5*b-10*a^{(11/2)}*x^{(9/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}+5*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^{(9/2)}*a^5*b+16*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(5/2)}*x^{(5/2)}*b^2+5*2*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(7/2)}*x^3*b-20*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(9/2)}*x^4*b-10*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^4*a^4*b^2-20*a^{(9/2)}*x^4*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*b-10*a^{(9/2)}*x^{(7/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(3/2)}+10*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^4*a^4*b^2-4*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(3/2)}*x^2*b^3-10*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(7/2)}*x^{(7/2)}*b^2-5*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^{(7/2)}*a^3*b^3-10*a^{(7/2)}*x^{(7/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*b^2+5*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^{(7/2)}*a^3*b^3+2*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(1/2)}*x^{(3/2)}*b^4)/(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}/b^5/a^{(1/2)}/x^{(7/2)}/(b+a*x^{(1/2)})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)

Fricas [A] time = 2.34268, size = 163, normalized size = 1.52

$$\frac{4 \left(8 a^3 b x^2 - 3 a b^3 x - (16 a^4 x^2 - 10 a^2 b^2 x - b^4) \sqrt{x} \right) \sqrt{a x + b \sqrt{x}}}{5 \left(a^2 b^4 x^3 - b^6 x^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} (a x + b \sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a x + b \sqrt{x})^{\frac{3}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)

$$3.129 \quad \int \frac{1}{x^{5/2}(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=165

$$-\frac{128a^2\sqrt{ax+b}\sqrt{x}}{21b^4x^{3/2}} - \frac{1024a^4\sqrt{ax+b}\sqrt{x}}{63b^6\sqrt{x}} + \frac{512a^3\sqrt{ax+b}\sqrt{x}}{63b^5x} + \frac{320a\sqrt{ax+b}\sqrt{x}}{63b^3x^2} - \frac{40\sqrt{ax+b}\sqrt{x}}{9b^2x^{5/2}} + \frac{4}{bx^2\sqrt{ax+b}\sqrt{x}}$$

[Out] $4/(b*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (40*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(9*b^2*x^(5/2)) + (320*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^3*x^2) - (128*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(21*b^4*x^(3/2)) + (512*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^5*x) - (1024*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^6*\text{Sqrt}[x])$

Rubi [A] time = 0.257146, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2015, 2016, 2014}

$$-\frac{128a^2\sqrt{ax+b}\sqrt{x}}{21b^4x^{3/2}} - \frac{1024a^4\sqrt{ax+b}\sqrt{x}}{63b^6\sqrt{x}} + \frac{512a^3\sqrt{ax+b}\sqrt{x}}{63b^5x} + \frac{320a\sqrt{ax+b}\sqrt{x}}{63b^3x^2} - \frac{40\sqrt{ax+b}\sqrt{x}}{9b^2x^{5/2}} + \frac{4}{bx^2\sqrt{ax+b}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] $4/(b*x^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x]) - (40*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(9*b^2*x^(5/2)) + (320*a*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^3*x^2) - (128*a^2*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(21*b^4*x^(3/2)) + (512*a^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^5*x) - (1024*a^4*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])/(63*b^6*\text{Sqrt}[x])$

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2} (b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} + \frac{10 \int \frac{1}{x^3 \sqrt{b\sqrt{x} + ax}} dx}{b} \\
&= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} - \frac{40 \sqrt{b\sqrt{x} + ax}}{9b^2 x^{5/2}} - \frac{(80a) \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x} + ax}} dx}{9b^2} \\
&= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} - \frac{40 \sqrt{b\sqrt{x} + ax}}{9b^2 x^{5/2}} + \frac{320a \sqrt{b\sqrt{x} + ax}}{63b^3 x^2} + \frac{(160a^2) \int \frac{1}{x^2 \sqrt{b\sqrt{x} + ax}} dx}{21b^3} \\
&= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} - \frac{40 \sqrt{b\sqrt{x} + ax}}{9b^2 x^{5/2}} + \frac{320a \sqrt{b\sqrt{x} + ax}}{63b^3 x^2} - \frac{128a^2 \sqrt{b\sqrt{x} + ax}}{21b^4 x^{3/2}} - \frac{(128a^3) \int \frac{1}{x^{3/2} \sqrt{b\sqrt{x} + ax}} dx}{21b^4} \\
&= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} - \frac{40 \sqrt{b\sqrt{x} + ax}}{9b^2 x^{5/2}} + \frac{320a \sqrt{b\sqrt{x} + ax}}{63b^3 x^2} - \frac{128a^2 \sqrt{b\sqrt{x} + ax}}{21b^4 x^{3/2}} + \frac{512a^3 \sqrt{b\sqrt{x} + ax}}{63b^5 x} \\
&= \frac{4}{bx^2 \sqrt{b\sqrt{x} + ax}} - \frac{40 \sqrt{b\sqrt{x} + ax}}{9b^2 x^{5/2}} + \frac{320a \sqrt{b\sqrt{x} + ax}}{63b^3 x^2} - \frac{128a^2 \sqrt{b\sqrt{x} + ax}}{21b^4 x^{3/2}} + \frac{512a^3 \sqrt{b\sqrt{x} + ax}}{63b^5 x}
\end{aligned}$$

Mathematica [A] time = 0.0522184, size = 83, normalized size = 0.5

$$\frac{4(-32a^3b^2x^{3/2} + 16a^2b^3x + 128a^4bx^2 + 256a^5x^{5/2} - 10ab^4\sqrt{x} + 7b^5)}{63b^6x^2\sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)), x]

[Out] (-4*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*x^2*Sqrt[b*Sqrt[x] + a*x])

Maple [C] time = 0.01, size = 592, normalized size = 3.6

$$-\frac{4}{63b^7}\sqrt{b\sqrt{x} + ax}\left(63\ln\left(\frac{1}{2}\frac{2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + 2a\sqrt{x} + b}{\sqrt{a}}\right)\right)x^{11/2}a^5b^3 - 10(b\sqrt{x} + ax)^{3/2}a^{3/2}x^3b^5 + 63\ln\left(\frac{1}{2}\frac{2\sqrt{\sqrt{x}(b+a\sqrt{x})}\sqrt{a} + 2a\sqrt{x} + b}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2), x)

[Out] -4/63*(b*x^(1/2)+a*x)^(1/2)*(63*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(11/2)*a^5*b^3-10*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^3*b^5+63*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(13/2)*a^7*b-126*(b*x^(1/2)+a*x)^(1/2)*a^(15/2)*x^(13/2)-63*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2))*a^(1/2)+b)/a^(1/2))*x^(13/2)*a^7*b-126*a^(15/2)*x^(13/2)*(x^(1/2)*(b+a*x^(1/2))))^(1/2)-126*(b*x^(1/2)+a*x)^(1/2)*a^(11/2)*x^(11/2)*b^2+128*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^(9/2)*b^2+508*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^5*b-252*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^6*b-252*a^(13/2)*x^6*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*b-32*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^4*b^3+7*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^(5/2)*b^6+315*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^(11/2)-126*a^(11/2)*x^(11/2)*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*b^2-126*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2))*a^(1/2)+b)/a^(1/2))*x^6*a^6*b^2-63*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2))*a^(1/2)+b)/a^(1/2))*x^(11/2)*a^5*b^3-63*a^(13/2)*x^(11/2)*(x^(1/2)*(b+a*x^(1/2))))^(3/2)+126*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2))))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^6*a^6*b^2+16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(7/2)*b^4)/(x^(1/2)*(b+a*x^(1/2))))^(1/2)/b^7/a^(1/2)/x^(11/2)/(b+a*x^(1/2))^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)

Fricas [A] time = 2.39902, size = 220, normalized size = 1.33

$$\frac{4(128a^5bx^3 - 48a^3b^3x^2 - 17ab^5x - (256a^6x^3 - 160a^4b^2x^2 - 26a^2b^4x - 7b^6)\sqrt{x})\sqrt{ax + b\sqrt{x}}}{63(a^2b^6x^4 - b^8x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4 - b^8*x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}} (ax + b\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

$$3.130 \quad \int \frac{1}{x^{7/2}(b\sqrt{x}+ax)^{3/2}} dx$$

Optimal. Leaf size=223

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}} + \frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2} - \frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}} - \frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}} + \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x} + \frac{672a\sqrt{ax+b\sqrt{x}}}{143b^3x^3}$$

[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2) - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sqrt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[x])

Rubi [A] time = 0.353146, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2015, 2016, 2014}

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}} + \frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2} - \frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}} - \frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}} + \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x} + \frac{672a\sqrt{ax+b\sqrt{x}}}{143b^3x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)), x]

[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2) - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sqrt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[x])

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]

```
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2} (b\sqrt{x} + ax)^{3/2}} dx &= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} + \frac{14 \int \frac{1}{x^4 \sqrt{b\sqrt{x} + ax}} dx}{b} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} - \frac{(168a) \int \frac{1}{x^{7/2} \sqrt{b\sqrt{x} + ax}} dx}{13b^2} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} + \frac{672a\sqrt{b\sqrt{x} + ax}}{143b^3 x^3} + \frac{(1680a^2) \int \frac{1}{x^3 \sqrt{b\sqrt{x} + ax}} dx}{143b^3} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} + \frac{672a\sqrt{b\sqrt{x} + ax}}{143b^3 x^3} - \frac{2240a^2 \sqrt{b\sqrt{x} + ax}}{429b^4 x^{5/2}} - \frac{(4480a^3) \int \frac{1}{x^{5/2} \sqrt{b\sqrt{x} + ax}} dx}{429b^4} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} + \frac{672a\sqrt{b\sqrt{x} + ax}}{143b^3 x^3} - \frac{2240a^2 \sqrt{b\sqrt{x} + ax}}{429b^4 x^{5/2}} + \frac{2560a^3 \sqrt{b\sqrt{x} + ax}}{429b^5 x^2} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} + \frac{672a\sqrt{b\sqrt{x} + ax}}{143b^3 x^3} - \frac{2240a^2 \sqrt{b\sqrt{x} + ax}}{429b^4 x^{5/2}} + \frac{2560a^3 \sqrt{b\sqrt{x} + ax}}{429b^5 x^2} \\
&= \frac{4}{bx^3 \sqrt{b\sqrt{x} + ax}} - \frac{56\sqrt{b\sqrt{x} + ax}}{13b^2 x^{7/2}} + \frac{672a\sqrt{b\sqrt{x} + ax}}{143b^3 x^3} - \frac{2240a^2 \sqrt{b\sqrt{x} + ax}}{429b^4 x^{5/2}} + \frac{2560a^3 \sqrt{b\sqrt{x} + ax}}{429b^5 x^2}
\end{aligned}$$

Mathematica [A] time = 0.0629794, size = 107, normalized size = 0.48

$$\frac{4(-256a^5 b^2 x^{5/2} + 128a^4 b^3 x^2 - 80a^3 b^4 x^{3/2} + 56a^2 b^5 x + 1024a^6 b x^3 + 2048a^7 x^{7/2} - 42ab^6 \sqrt{x} + 33b^7)}{429b^8 x^3 \sqrt{ax + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]

[Out] $(-4*(33*b^7 - 42*a*b^6*\text{Sqrt}[x] + 56*a^2*b^5*x - 80*a^3*b^4*x^{(3/2)} + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^{(5/2)} + 1024*a^6*b*x^3 + 2048*a^7*x^{(7/2)}))/(429*b^8*x^3*\text{Sqrt}[b*\text{Sqrt}[x] + a*x])$

Maple [C] time = 0.011, size = 636, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^{(7/2)}/(b*x^{(1/2)}+a*x)^{(3/2)}, x)$

[Out] $-2/429*(b*x^{(1/2)}+a*x)^{(1/2)}*(2048*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(13/2)}*x^{(13/2)}*b^2+9244*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(15/2)}*x^7*b+66*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(1/2)}*x^{(7/2)}*b^8-160*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(7/2)}*x^5*b^5-2574*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(15/2)}*x^{(15/2)}*b^2-2574*a^{(15/2)}*x^{(15/2)}*(x^{(1/2)}*(b+a*x^{(1/2)})))^{(1/2)}*b^2-1287*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^{(17/2)}*a^9*b-2574*a^{(19/2)}*x^{(17/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}+1287*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)})))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^{(17/2)}*a^9*b-2574*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^8*a^8*b^2-858*a^{(17/2)}*x^{(15/2)}*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(3/2)}+2574*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)})))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^8*a^8*b^2-1287*\ln(1/2*(2*a*x^{(1/2)}+2*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(1/2)}+b)/a^{(1/2)})*x^{(15/2)}*a^7*b^3+1287*\ln(1/2*(2*(x^{(1/2)}*(b+a*x^{(1/2)})))^{(1/2)}*a^{(1/2)}+2*a*x^{(1/2)}+b)/a^{(1/2)})*x^{(15/2)}*a^7*b^3+112*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(5/2)}*x^{(9/2)}*b^6-84*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(3/2)}*x^4*b^7+6006*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(17/2)}*x^{(15/2)}-2574*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(19/2)}*x^{(17/2)}-5148*(b*x^{(1/2)}+a*x)^{(1/2)}*a^{(17/2)}*x^8*b-5148*a^{(17/2)}*x^8*(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}*b+256*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(9/2)}*x^{(11/2)}*b^4-512*(b*x^{(1/2)}+a*x)^{(3/2)}*a^{(11/2)}*x^6*b^3)/(x^{(1/2)}*(b+a*x^{(1/2)}))^{(1/2)}/b^9/a^{(1/2)}/x^{(15/2)}/(b+a*x^{(1/2)})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}} x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)

Fricas [A] time = 2.36309, size = 278, normalized size = 1.25

$$\frac{4(1024a^7bx^4 - 384a^5b^3x^3 - 136a^3b^5x^2 - 75ab^7x - (2048a^8x^4 - 1280a^6b^2x^3 - 208a^4b^4x^2 - 98a^2b^6x - 33b^8)\sqrt{x})\sqrt{ax}}{429(a^2b^8x^5 - b^{10}x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt{x})^{\frac{3}{2}}x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

3.131 $\int x^3 \sqrt{b\sqrt[3]{x} + ax} dx$

Optimal. Leaf size=301

$$\frac{442b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4}$$

[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.509048, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4} + \frac{476b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^2} + \frac{442b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[b*x^(1/3) + a*x], x]

[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x],
x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{b\sqrt[3]{x} + ax} dx &= 3 \operatorname{Subst} \left(\int x^{11} \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} + \frac{1}{9} (2b) \operatorname{Subst} \left(\int \frac{x^{12}}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(14b^2) \operatorname{Subst} \left(\int \frac{x^{10}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{69a} \\
&= -\frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{4bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} + \frac{(238b^3) \operatorname{Subst} \left(\int \frac{x^8}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{4bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(3094b^4) \operatorname{Subst} \left(\int \frac{x^6}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= -\frac{6188b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{4bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(3994b^5) \operatorname{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= \frac{884b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{24035a^5} - \frac{6188b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(4994b^6) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= -\frac{884b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^6} + \frac{884b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{24035a^5} - \frac{6188b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(5994b^7) \operatorname{Subst} \left(\int \frac{x^0}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= -\frac{884b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^6} + \frac{884b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{24035a^5} - \frac{6188b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(6994b^8) \operatorname{Subst} \left(\int \frac{x^{-2}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2} \\
&= -\frac{884b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^6} + \frac{884b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{24035a^5} - \frac{6188b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^2} + \frac{2}{9} x^4 \sqrt{b\sqrt[3]{x} + ax} - \frac{(7994b^9) \operatorname{Subst} \left(\int \frac{x^{-4}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^2}
\end{aligned}$$

Mathematica [C] time = 0.160714, size = 155, normalized size = 0.51

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(\sqrt{\frac{ax^{2/3}}{b}} + 1 \left(-2310a^4 b^2 x^{8/3} + 2618a^3 b^3 x^2 - 3094a^2 b^4 x^{4/3} + 2090a^5 b x^{10/3} + 24035a^6 x^4 + 3978ab^5 x^{2/3} - 994b^6 \right) \right)}{216315a^6 \sqrt{\frac{ax^{2/3}}{b}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*x^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) + 2090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1/4, 5/4, -(a*x^(2/3))/b]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])

Maple [A] time = 0.033, size = 264, normalized size = 0.9

$$\frac{2x^4}{9}\sqrt{b^3x+ax} + \frac{4b}{207a}x^{\frac{10}{3}}\sqrt{b^3x+ax} - \frac{28b^2}{1311a^2}x^{\frac{8}{3}}\sqrt{b^3x+ax} + \frac{476b^3x^2}{19665a^3}\sqrt{b^3x+ax} - \frac{6188b^4}{216315a^4}x^{\frac{4}{3}}\sqrt{b^3x+ax} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^(1/3)+a*x)^(1/2), x)

[Out] 2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+442/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*x + b*x^(1/3))*x^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{ax + b\sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)`

[Out] `Integral(x**3*sqrt(a*x + b*x**(1/3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(a*x + b*x^(1/3))*x^3, x)`

3.132 $\int x^2 \sqrt{b\sqrt[3]{x} + ax} dx$

Optimal. Leaf size=411

$$\frac{22b^{21/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{221a^{19/4} \sqrt{ax + b\sqrt[3]{x}}} + \frac{44b^5 \sqrt[3]{x} (ax^{2/3} + b)}{221a^{9/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b\sqrt[3]{x}}} - \frac{60b^2 x^{5/3}}{1547a^2}$$

[Out] $(44*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(9/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^4) + (220*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^3) - (60*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^2) + (4*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a) + (2*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/7 - (44*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(19/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (22*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(19/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.577651, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{44b^5 \sqrt[3]{x} (ax^{2/3} + b)}{221a^{9/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b\sqrt[3]{x}}} - \frac{60b^2 x^{5/3} \sqrt{ax + b\sqrt[3]{x}}}{1547a^2} + \frac{22b^{21/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{221a^{19/4} \sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{Sqrt}[b*x^{(1/3)} + a*x], x]$

[Out] $(44*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(9/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^4) + (220*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^3) - (60*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^2) + (4*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a) + (2*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/7 - (44*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(19/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (22*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(19/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

$$*b^{21/4}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})*\text{Sqrt}[(b + a*x^{2/3})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})^2]*x^{1/6}*\text{EllipticF}[2*\text{ArcTan}[(a^{1/4}*x^{1/6})/b^{1/4}], 1/2] / (221*a^{19/4}*\text{Sqrt}[b*x^{1/3} + a*x])$$

Rule 2018

$$\text{Int}[(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& \text{NeQ}[n^2, 1]$$

Rule 2021

$$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p / (c*(m+n*p+1)), x] + \text{Dist}[(a*(n-j)*p) / (c^j*(m+n*p+1)), \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \|\| \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[m+n*p+1, 0]$$

Rule 2024

$$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n-j)}*(m+j*p-n+j+1)) / (b*(m+n*p+1)), \text{Int}[(c*x)^{(m-(n-j))}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \|\| \text{GtQ}[c, 0]) \&\& \text{GtQ}[m+j*p+1-n+j, 0] \&\& \text{NeQ}[m+n*p+1, 0]$$

Rule 2032

$$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]}) / (x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n-j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m+j*p)}*(a + b*x^{(n-j)})^p, x], x] /; \text{FreeQ}[\{a, b, c, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n-j]$$

Rule 329

$$\text{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1) - 1)}*(a + (b*x^{(k*n)})) / c^n]^p, x], x, (c*x)^{(1/k)}], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{b\sqrt[3]{x} + ax} dx &= 3 \operatorname{Subst} \left(\int x^8 \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} + \frac{1}{7} (2b) \operatorname{Subst} \left(\int \frac{x^9}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} - \frac{(30b^2) \operatorname{Subst} \left(\int \frac{x^7}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{119a} \\
&= -\frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} + \frac{(330b^3) \operatorname{Subst} \left(\int \frac{x^5}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1547a^2} \\
&= \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} - \frac{(110b^4) \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1547a^2} \\
&= -\frac{44b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^4} + \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} \\
&= -\frac{44b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^4} + \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} \\
&= -\frac{44b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^4} + \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} \\
&= -\frac{44b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^4} + \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax} \\
&= \frac{44b^5 (b + ax^{2/3}) \sqrt[3]{x}}{221a^{9/2} (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} - \frac{44b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^4} + \frac{220b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^3} - \frac{60b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^2} + \frac{4bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a} + \frac{2}{7} x^3 \sqrt{b\sqrt[3]{x} + ax}
\end{aligned}$$

Mathematica [C] time = 0.113741, size = 136, normalized size = 0.33

$$\frac{2\sqrt[3]{x} \sqrt{ax + b\sqrt[3]{x}} \left(\sqrt{\frac{ax^{2/3}}{b}} + 1 \left(-90a^2 b^2 x^{4/3} + 78a^3 b x^2 + 663a^4 x^{8/3} + 110ab^3 x^{2/3} - 385b^4 \right) + 385b^4 {}_2F_1 \left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{ax^{2/3}}{b} \right) \right)}{4641a^4 \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*sqrt[b*x^(1/3) + a*x], x]

[Out] $(2x^{1/3}\sqrt{bx^{1/3} + ax}(\sqrt{1 + (ax^{2/3})/b})(-385b^4 + 110ab^3x^{2/3} - 90a^2b^2x^{4/3} + 78a^3bx^2 + 663a^4x^{8/3}) + 385b^4\text{Hypergeometric2F1}[-1/2, 3/4, 7/4, -((ax^{2/3})/b)])/(4641a^4\sqrt{1 + (ax^{2/3})/b})$

Maple [A] time = 0.011, size = 273, normalized size = 0.7

$$\frac{2x^3}{7}\sqrt{b^3\sqrt{x} + ax} + \frac{4b}{119a}x^{\frac{7}{3}}\sqrt{b^3\sqrt{x} + ax} - \frac{60b^2}{1547a^2}x^{\frac{5}{3}}\sqrt{b^3\sqrt{x} + ax} + \frac{220b^3x}{4641a^3}\sqrt{b^3\sqrt{x} + ax} - \frac{44b^4}{663a^4}\sqrt[3]{x}\sqrt{b^3\sqrt{x} + ax} + \frac{22b^5}{221a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^(1/3)+a*x)^(1/2), x)`

[Out] $2/7*x^3*(b*x^{1/3}+a*x)^{1/2}+4/119*b*x^{7/3}*(b*x^{1/3}+a*x)^{1/2}/a-60/1547*b^2*x^{5/3}*(b*x^{1/3}+a*x)^{1/2}/a^2+220/4641*b^3*x*(b*x^{1/3}+a*x)^{1/2}/a^3-44/663*b^4*x^{1/3}*(b*x^{1/3}+a*x)^{1/2}/a^4+22/221*b^5/a^5*(-a*b)^{1/2}*((x^{1/3}+1/a*(-a*b)^{1/2})*a/(-a*b)^{1/2})^{1/2}*(-2*(x^{1/3}-1/a*(-a*b)^{1/2})*a/(-a*b)^{1/2})^{1/2}*(-x^{1/3}*a/(-a*b)^{1/2})^{1/2}/(b*x^{1/3}+a*x)^{1/2}*(-2/a*(-a*b)^{1/2}*\text{EllipticE}(((x^{1/3}+1/a*(-a*b)^{1/2})*a/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2}))+1/a*(-a*b)^{1/2}*\text{EllipticF}(((x^{1/3}+1/a*(-a*b)^{1/2})*a/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^(1/3)+a*x)^(1/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^(1/3))*x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{ax + b\sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

3.133 $\int x \sqrt{b \sqrt[3]{x} + ax} dx$

Optimal. Leaf size=213

$$\frac{6b^{15/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77a^{13/4} \sqrt{ax + b \sqrt[3]{x}}} - \frac{36b^2 x^{2/3} \sqrt{ax + b \sqrt[3]{x}}}{385a^2} + \frac{12b^3 \sqrt{ax + b \sqrt[3]{x}}}{77a^3} + \frac{4bx^4}{55a}$$

[Out] $(12*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^3) - (36*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a^2) + (4*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*a) + (2*x^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/5 - (6*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(13/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.299155, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$\frac{36b^2 x^{2/3} \sqrt{ax + b \sqrt[3]{x}}}{385a^2} - \frac{6b^{15/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{77a^{13/4} \sqrt{ax + b \sqrt[3]{x}}} + \frac{12b^3 \sqrt{ax + b \sqrt[3]{x}}}{77a^3} + \frac{4bx^{4/3} \sqrt{ax + b \sqrt[3]{x}}}{55a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sqrt}[b*x^{(1/3)} + a*x], x]$

[Out] $(12*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^3) - (36*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a^2) + (4*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*a) + (2*x^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/5 - (6*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(13/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]], Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{b\sqrt[3]{x}+ax} dx &= 3 \text{Subst} \left(\int x^5\sqrt{bx+ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{5}(2b) \text{Subst} \left(\int \frac{x^6}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(18b^2) \text{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{55a} \\
&= -\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{(18b^3) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a^2} \\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(6b^4) \text{Subst} \left(\int \frac{x}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a^2} \\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(6b^4\sqrt{b+ax^{2/3}})}{77a^2} \\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(12b^4\sqrt{b+ax^{2/3}})}{77a^2} \\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{6b^{15/4}(\sqrt{b} + \sqrt{ax^{2/3}})}{77a^2}
\end{aligned}$$

Mathematica [C] time = 0.098597, size = 118, normalized size = 0.55

$$\frac{2\sqrt{ax+b\sqrt[3]{x}} \left(\sqrt{\frac{ax^{2/3}}{b}+1} (14a^2bx^{4/3} + 77a^3x^2 - 18ab^2x^{2/3} + 45b^3) - 45b^3 {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{ax^{2/3}}{b}\right) \right)}{385a^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/3) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4, 5/4, -(a*x^(2/3))/b]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])

Maple [A] time = 0.013, size = 198, normalized size = 0.9

$$\frac{2x^2}{5}\sqrt{b\sqrt[3]{x}+ax} + \frac{4b}{55a}x^{\frac{4}{3}}\sqrt{b\sqrt[3]{x}+ax} - \frac{36b^2}{385a^2}x^{\frac{2}{3}}\sqrt{b\sqrt[3]{x}+ax} + \frac{12b^3}{77a^3}\sqrt{b\sqrt[3]{x}+ax} - \frac{6b^4}{77a^4}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x} + \frac{1}{a}\sqrt{-ab}\right)}\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^(1/3)+a*x)^(1/2),x)`

[Out] `2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-6/77/a^4*b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^(1/3))*x, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*x + b*x^(1/3))*x, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{ax + b\sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

3.134 $\int \sqrt{b\sqrt[3]{x} + ax} dx$

Optimal. Leaf size=323

$$\frac{2b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{4b^2\sqrt[3]{x}(ax^{2/3}+b)}{5a^{3/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{4b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $(-4*b^2*(b + a*x^{(2/3)})*x^{(1/3)})/(5*a^{(3/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4*b*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(15*a) + (2*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/3 + (4*b^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (2*b^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.325712, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2004, 2018, 2024, 2032, 329, 305, 220, 1196}

$$\frac{4b^2\sqrt[3]{x}(ax^{2/3}+b)}{5a^{3/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} - \frac{2b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{4b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x], x]

[Out] $(-4*b^2*(b + a*x^{(2/3)})*x^{(1/3)})/(5*a^{(3/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4*b*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(15*a) + (2*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/3 + (4*b^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (2*b^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*a^{(7/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j
+ b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \sqrt{b\sqrt[3]{x} + ax} dx &= \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} + \frac{1}{9}(2b) \int \frac{\sqrt[3]{x}}{\sqrt{b\sqrt[3]{x} + ax}} dx \\
 &= \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} + \frac{1}{3}(2b) \text{Subst}\left(\int \frac{x^3}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x}\right) \\
 &= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a} + \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} - \frac{(2b^2) \text{Subst}\left(\int \frac{x}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x}\right)}{5a} \\
 &= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a} + \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} - \frac{(2b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b + ax^2}} dx, x, \sqrt[3]{x}\right)}{5a\sqrt{b\sqrt[3]{x} + ax}} \\
 &= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a} + \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} - \frac{(4b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst}\left(\int \frac{x^2}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x}\right)}{5a\sqrt{b\sqrt[3]{x} + ax}} \\
 &= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a} + \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} - \frac{(4b^{5/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst}\left(\int \frac{1}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x} + ax}} + \frac{(4b^{5/2}\sqrt{b - ax^{2/3}}\sqrt[6]{x}) \text{Subst}\left(\int \frac{1}{\sqrt{b - ax^4}} dx, x, \sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x} + ax}} \\
 &= -\frac{4b^2(b + ax^{2/3})\sqrt[3]{x}}{5a^{3/2}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} + \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a} + \frac{2}{3}x\sqrt{b\sqrt[3]{x} + ax} + \frac{4b^{9/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{\frac{b}{\sqrt{b} + \sqrt{a}\sqrt[3]{x}}}}{5a^{7/4}}
 \end{aligned}$$

Mathematica [C] time = 0.0537649, size = 94, normalized size = 0.29

$$\frac{2\sqrt[3]{x}\sqrt{ax+b}\sqrt[3]{x}\left((ax^{2/3}+b)\sqrt{\frac{ax^{2/3}}{b}+1}-b{}_2F_1\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b] - b*Hypergeometric2F1[-1/2, 3/4, 7/4, -(a*x^(2/3))/b]))/(3*a*Sqrt[1 + (a*x^(2/3))/b])

Maple [A] time = 0.012, size = 207, normalized size = 0.6

$$\frac{2x}{3}\sqrt{b\sqrt[3]{x}+ax}+\frac{4b}{15a}\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}-\frac{2b^2}{5a^2}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-2\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}-\frac{\sqrt{-ab}}{a}\right)}\sqrt{-a\sqrt[3]{x}\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(1/2), x)

[Out] 2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5/a^2*b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2), x, algorithm="maxima")

[Out] `integrate(sqrt(a*x + b*x^(1/3)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*x + b*x^(1/3)), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + b\sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**(1/3)+a*x)**(1/2),x)`

[Out] `Integral(sqrt(a*x + b*x**(1/3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(a*x + b*x^(1/3)), x)`

$$3.135 \quad \int \frac{\sqrt{b\sqrt[3]{x+ax}}}{x} dx$$

Optimal. Leaf size=123

$$\frac{2b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}} + 2\sqrt{ax+b\sqrt[3]{x}}$$

[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.146223, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 2021, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}} + 2\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x]/x, x]

[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a

$(n - j)p / (c^j(m + np + 1))$, $\text{Int}[(c*x)^{(m + j)}*(a*x^j + b*x^n)^{(p - 1)}, x]$, x /; $\text{FreeQ}\{a, b, c, m\}, x$ && $\text{!IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$ && $\text{GtQ}[p, 0]$ && $\text{NeQ}[m + np + 1, 0]$

Rule 2011

$\text{Int}[(a_.*x_.)^{(j_.*)} + (b_.*x_.)^{(n_.*)}]^{(p_*)}$, x_Symbol] \rightarrow $\text{Dist}[(a*x^j + b*x^n)^{\text{FracPart}[p]} / (x^{(j*\text{FracPart}[p])}*(a + b*x^{(n - j)})^{\text{FracPart}[p]})$, $\text{Int}[x^{(j*p)}*(a + b*x^{(n - j)})^p, x]$, x /; $\text{FreeQ}\{a, b, j, n, p\}, x$ && $\text{!IntegerQ}[p]$ && $\text{NeQ}[n, j]$ && $\text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_.*x_.)^{(m_*)}*(a_*) + (b_.*x_.)^{(n_*)}]^{(p_*)}$, x_Symbol] \rightarrow $\text{With}\{k = \text{Denominator}[m]\}$, $\text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}]$, x] /; $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{FractionQ}[m]$ && $\text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 220

$\text{Int}[1/\text{Sqrt}[a_*) + (b_.*x_.)^4]$, x_Symbol] \rightarrow $\text{With}\{q = \text{Rt}[b/a, 4]\}$, $\text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^4]/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x, 1/2]]/(2*q*\text{Sqrt}[a + b*x^4])$, x] /; $\text{FreeQ}\{a, b\}, x$ && $\text{PosQ}[b/a]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x} dx &= 3 \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x} dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{b\sqrt[3]{x} + ax} + (2b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{b\sqrt[3]{x} + ax} + \frac{(2b\sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{\sqrt{b\sqrt[3]{x} + ax}} \\
&= 2\sqrt{b\sqrt[3]{x} + ax} + \frac{(4b\sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{\sqrt{b\sqrt[3]{x} + ax}} \\
&= 2\sqrt{b\sqrt[3]{x} + ax} + \frac{2b^{3/4} (\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}} \sqrt[6]{x} F \left(2 \tan^{-1} \left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt{b}} \right) \middle| \frac{1}{2} \right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0439971, size = 54, normalized size = 0.44

$$\frac{6\sqrt{ax + b\sqrt[3]{x}} {}_2F_1 \left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{ax^{2/3}}{b} \right)}{\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]

[Out] (6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -(a*x^(2/3))/b])/Sqrt[1 + (a*x^(2/3))/b]

Maple [A] time = 0.012, size = 132, normalized size = 1.1

$$2\sqrt{b\sqrt[3]{x} + ax} + 2 \frac{b\sqrt{-ab}}{a\sqrt{b\sqrt[3]{x} + ax}} \sqrt{\frac{a}{\sqrt{-ab}} \left(\sqrt[3]{x} + \frac{\sqrt{-ab}}{a} \right)} \sqrt{-2 \frac{a}{\sqrt{-ab}} \left(\sqrt[3]{x} - \frac{\sqrt{-ab}}{a} \right)} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a}{\sqrt{-ab}} \left(\sqrt[3]{x} + \frac{\sqrt{-ab}}{a} \right)} \middle| \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(1/3)+a*x)^(1/2)/x,x)`

[Out] $2*(b*x^{(1/3)}+a*x)^{(1/2)}+2*b/a*(-a*b)^{(1/2)}*((x^{(1/3)}+1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x^{(1/3)}-1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}/(b*x^{(1/3)}+a*x)^{(1/2)}*EllipticF(((x^{(1/3)}+1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^(1/3))/x, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")`

[Out] `integral(sqrt(a*x + b*x^(1/3))/x, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

$$3.136 \quad \int \frac{\sqrt{b\sqrt[3]{x+ax}}}{x^2} dx$$

Optimal. Leaf size=325

$$\frac{6a^{5/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{12a^{5/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3) + a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.352673, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{6a^{5/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{12a^{5/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]

[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3) + a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^2} dx &= 3 \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^4} dx, x, \sqrt[3]{x} \right) \\
 &= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} + \frac{1}{5}(6a) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
 &= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} + \frac{(6a^2) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{5b} \\
 &= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} + \frac{(6a^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{5b\sqrt{b\sqrt[3]{x} + ax}} \\
 &= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} + \frac{(12a^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{5b\sqrt{b\sqrt[3]{x} + ax}} \\
 &= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} + \frac{(12a^{3/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}} - \frac{(12a^{3/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x})}{5\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}} \\
 &= \frac{12a^{3/2}(b + ax^{2/3})\sqrt[3]{x}}{5b(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{6\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{12a^{5/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})}}}{5b^{3/4}\sqrt{b\sqrt[3]{x} + ax}}
 \end{aligned}$$

Mathematica [C] time = 0.046491, size = 59, normalized size = 0.18

$$\frac{6\sqrt{ax + b}\sqrt[3]{x} {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2}; -\frac{1}{4}; -\frac{ax^{2/3}}{b}\right)}{5x\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x]*\text{Hypergeometric2F1}[-5/4, -1/2, -1/4, -(a*x^{(2/3)})/b])/ (5*\text{Sqrt}[1 + (a*x^{(2/3)})/b]*x)$

Maple [A] time = 0.018, size = 213, normalized size = 0.7

$$-\frac{6}{5x}\sqrt{b\sqrt[3]{x} + ax} - \frac{12a}{5b}\left(b + ax^{\frac{2}{3}}\right) \frac{1}{\sqrt{\sqrt[3]{x}\left(b + ax^{\frac{2}{3}}\right)}} + \frac{6a}{5b}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x} + \frac{1}{a}\sqrt{-ab}\right)} \frac{1}{\sqrt{-ab}} \sqrt{-2\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x} - \frac{\sqrt{-ab}}{a}\right)} \sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(1/2)/x^2,x)

[Out] $-6/5*(b*x^{(1/3)+a*x})^{(1/2)}/x - 12/5*(b+a*x^{(2/3)})/b*a/(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)} + 6/5/b*a*(-a*b)^{(1/2)}*((x^{(1/3)}+1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}*(-2*(x^{(1/3)}-1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}/(b*x^{(1/3)+a*x})^{(1/2)}*(-2/a*(-a*b)^{(1/2)}*\text{EllipticE}(((x^{(1/3)}+1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})+1/a*(-a*b)^{(1/2)}*\text{EllipticF}(((x^{(1/3)}+1/a*(-a*b)^{(1/2)})*a/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(a*x + b*x^(1/3))/x^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)

[Out] Integral(sqrt(a*x + b*x**(1/3))/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)

$$3.137 \quad \int \frac{\sqrt{b\sqrt[3]{x+ax}}}{x^3} dx$$

Optimal. Leaf size=188

$$\frac{10a^{11/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}} - \frac{6\sqrt{ax+b\sqrt[3]{x}}}{11x^2}$$

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(11*x^2) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b*x^{(4/3)}) + (20*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^2*x^{(2/3)}) + (10*a^{(11/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*b^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.243934, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}} + \frac{10a^{11/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}} - \frac{6\sqrt{ax+b\sqrt[3]{x}}}{11x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x]/x^3, x]

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(11*x^2) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b*x^{(4/3)}) + (20*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^2*x^{(2/3)}) + (10*a^{(11/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*b^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]

&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^3} dx &= 3 \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^7} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} + \frac{1}{11} (6a) \operatorname{Subst} \left(\int \frac{1}{x^4 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{77bx^{4/3}} - \frac{(30a^2) \operatorname{Subst} \left(\int \frac{1}{x^2 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{77b} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} + \frac{(10a^3) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{77b^2} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} + \frac{(10a^3\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{77b^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} + \frac{(20a^3\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[3]{x} \right)}{77b^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} + \frac{10a^{11/4} (\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}} \sqrt[6]{x} F_2 \left(2, \frac{1}{2}, \frac{1}{2}, -\frac{ax^{2/3}}{b} \right)}{77b^{9/4}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0501204, size = 59, normalized size = 0.31

$$-\frac{6\sqrt{ax + b\sqrt[3]{x}} {}_2F_1 \left(-\frac{11}{4}, -\frac{1}{2}; -\frac{7}{4}; -\frac{ax^{2/3}}{b} \right)}{11x^2 \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]

[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3))/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)

Maple [A] time = 0.017, size = 179, normalized size = 1.

$$-\frac{6}{11x^2}\sqrt{b\sqrt[3]{x}+ax}-\frac{12a}{77b}\sqrt{b\sqrt[3]{x}+axx^{-\frac{4}{3}}}+\frac{20a^2}{77b^2}\sqrt{b\sqrt[3]{x}+axx^{-\frac{2}{3}}}+\frac{10a^2}{77b^2}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(1/2)/x^3,x)

[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(a*x + b*x^(1/3))/x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)

[Out] Integral(sqrt(a*x + b*x**(1/3))/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)

$$3.138 \quad \int \frac{\sqrt{b\sqrt[3]{x+ax}}}{x^4} dx$$

Optimal. Leaf size=413

$$\frac{154a^{17/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{308a^{9/2}\sqrt[3]{x}(ax^{2/3}+b)}{1105b^4(\sqrt{a}\sqrt[3]{x}+\sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2}$$

[Out] $(-308*a^{(9/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(17*x^3) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(221*b*x^{(7/3)}) + (44*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*b^2*x^{(5/3)}) - (308*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*b^3*x) + (308*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1105*b^4*x^{(1/3)}) + (308*a^{(17/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)}))*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (154*a^{(17/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.543533, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{308a^{9/2}\sqrt[3]{x}(ax^{2/3}+b)}{1105b^4(\sqrt{a}\sqrt[3]{x}+\sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2x^{5/3}} - \frac{154a^{17/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x]/x^4, x]

[Out] $(-308*a^{(9/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(17*x^3) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(221*b*x^{(7/3)}) + (44*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*b^2*x^{(5/3)}) - (308*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*b^3*x) + (308*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1105*b^4*x^{(1/3)}) + (308*a^{(17/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)}))*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a$

$*x]) - (154*a^{(17/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})]/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

$\text{Int}[(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, j, m, n, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& \text{NeQ}[n^2, 1]$

Rule 2020

$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m + 1)}*(a*x^j + b*x^n)^p/(c*(m + j*p + 1)), x] - \text{Dist}[(b*p*(n - j))/(c^n*(m + j*p + 1)), \text{Int}[(c*x)^{(m + n)}*(a*x^j + b*x^n)^{(p - 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m + j*p + 1, 0]$

Rule 2025

$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j - 1)}*(c*x)^{(m - j + 1)}*(a*x^j + b*x^n)^{(p + 1)})/(a*(m + j*p + 1)), x] - \text{Dist}[(b*(m + n*p + n - j + 1))/(a*c^{(n - j)}*(m + j*p + 1)), \text{Int}[(c*x)^{(m + n - j)}*(a*x^j + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, m, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[m + j*p + 1, 0]$

Rule 2032

$\text{Int}[(c_)*(x_)^{(m_)}*((a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]})/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n - j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m + j*p)}*(a + b*x^{(n - j)})^p, x], x] /;$ $\text{FreeQ}\{a, b, c, j, m, n, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^4} dx &= 3 \text{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^{10}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} + \frac{1}{17}(6a) \text{Subst} \left(\int \frac{1}{x^7\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} - \frac{(66a^2) \text{Subst} \left(\int \frac{1}{x^5\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{221b} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} + \frac{(154a^3) \text{Subst} \left(\int \frac{1}{x^3\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{663b^2} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} - \frac{(154a^4) \text{Subst} \left(\int \frac{1}{x\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1105b^4} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{308a^{9/2}(b + ax^{2/3})\sqrt[3]{x}}{1105b^4(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{6\sqrt{b\sqrt[3]{x} + ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x} + ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x}
\end{aligned}$$

Mathematica [C] time = 0.0470283, size = 59, normalized size = 0.14

$$-\frac{6\sqrt{ax + b\sqrt[3]{x}} {}_2F_1\left(-\frac{17}{4}, -\frac{1}{2}; -\frac{13}{4}; -\frac{ax^{2/3}}{b}\right)}{17x^3\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]

[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/3))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)

Maple [A] time = 0.018, size = 281, normalized size = 0.7

$$-\frac{6}{17x^3}\sqrt{b\sqrt[3]{x}+ax}-\frac{12a}{221b}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{7}{3}}+\frac{44a^2}{663b^2}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{5}{3}}-\frac{308a^3}{3315b^3x}\sqrt{b\sqrt[3]{x}+ax}+\frac{308a^4}{1105b^4}\left(b+ax^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{x}\left(b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(1/2)/x^4,x)

[Out] -6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-154/1105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")

[Out] integral(sqrt(a*x + b*x^(1/3))/x^4, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)

[Out] Integral(sqrt(a*x + b*x**(1/3))/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)

$$3.139 \quad \int \frac{\sqrt{b\sqrt[3]{x+ax}}}{x^5} dx$$

Optimal. Leaf size=276

$$\frac{1326a^{23/4}\sqrt{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}} + \frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}$$

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(23*x^4) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(437*b*x^{(10/3)}) + (68*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(2185*b^2*x^{(8/3)}) - (884*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(24035*b^3*x^2) + (7956*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(168245*b^4*x^{(4/3)}) - (2652*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(33649*b^5*x^{(2/3)}) - (1326*a^{(23/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(33649*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.412899, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}} + \frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}} - \frac{884a^3\sqrt{ax+b\sqrt[3]{x}}}{24035b^3x^2} + \frac{68a^2\sqrt{ax+b\sqrt[3]{x}}}{2185b^2x^{8/3}} - \frac{1326a^{23/4}\sqrt{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax}{(\sqrt{a}\sqrt[3]{x} + \sqrt{b})^2}}}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(1/3) + a*x]/x^5, x]

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(23*x^4) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(437*b*x^{(10/3)}) + (68*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(2185*b^2*x^{(8/3)}) - (884*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(24035*b^3*x^2) + (7956*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(168245*b^4*x^{(4/3)}) - (2652*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(33649*b^5*x^{(2/3)}) - (1326*a^{(23/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(33649*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x} + ax}}{x^5} dx &= 3 \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^{13}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} + \frac{1}{23} (6a) \operatorname{Subst} \left(\int \frac{1}{x^{10}\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} - \frac{(102a^2) \operatorname{Subst} \left(\int \frac{1}{x^8\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{437b} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} + \frac{(442a^3) \operatorname{Subst} \left(\int \frac{1}{x^6\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2185b^2} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x} + ax}}{24035b^3x^2} - \frac{(3978a^4) \operatorname{Subst} \left(\int \frac{1}{x^4\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{24035b^2} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x} + ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} + \dots \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x} + ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} - \dots \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x} + ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} - \dots \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x} + ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x} + ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.0546913, size = 59, normalized size = 0.21

$$\frac{6\sqrt{ax + b\sqrt[3]{x}} {}_2F_1 \left(-\frac{23}{4}, -\frac{1}{2}; -\frac{19}{4}; -\frac{ax^{2/3}}{b} \right)}{23x^4\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^5, x]

[Out] $(-6\sqrt{b^{1/3}x + ax} \text{Hypergeometric2F1}[-23/4, -1/2, -19/4, -((a^{2/3}x^{2/3})/b)]) / (23\sqrt{1 + (a^{2/3}x^{2/3})/b})x^4$

Maple [A] time = 0.019, size = 245, normalized size = 0.9

$$-\frac{6}{23x^4}\sqrt{b\sqrt[3]{x}+ax}-\frac{12a}{437b}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{10}{3}}+\frac{68a^2}{2185b^2}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{8}{3}}-\frac{884a^3}{24035b^3x^2}\sqrt{b\sqrt[3]{x}+ax}+\frac{7956a^4}{168245b^4}\sqrt{b\sqrt[3]{x}+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b^{1/3}x+a^{1/3})^{1/2}/x^5, x)$

[Out] $-6/23*(b^{1/3}x+a^{1/3})^{1/2}/x^4-12/437*a*(b^{1/3}x+a^{1/3})^{1/2}/b/x^{10/3}+6/2185*a^2*(b^{1/3}x+a^{1/3})^{1/2}/b^2/x^{8/3}-884/24035*a^3*(b^{1/3}x+a^{1/3})^{1/2}/b^3/x^2+7956/168245*a^4*(b^{1/3}x+a^{1/3})^{1/2}/b^4/x^{4/3}-2652/33649*a^5*(b^{1/3}x+a^{1/3})^{1/2}/b^5/x^{2/3}-1326/33649*a^5/b^5*(-a^{1/3})^{1/2}*((x^{1/3}+1/a^{1/3})^{1/2})^{1/2}*(-a^{1/3})^{1/2}*(-2*(x^{1/3}-1/a^{1/3})^{1/2})^{1/2}*(-a^{1/3})^{1/2}*(-x^{1/3})^{1/2}*(-a^{1/3})^{1/2}/(b^{1/3}x+a^{1/3})^{1/2})^{1/2}*\text{EllipticF}(((x^{1/3}+1/a^{1/3})^{1/2})^{1/2}, 1/2*2^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{1/3}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b^{1/3}x+a^{1/3})^{1/2}/x^5, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\text{sqrt}(a^{1/3}x + b^{1/3}x^{1/3})/x^5, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax + bx^{1/3}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")

[Out] integral(sqrt(a*x + b*x^(1/3))/x^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")

[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)

3.140 $\int x^2 (b\sqrt[3]{x} + ax)^{3/2} dx$

Optimal. Leaf size=298

$$\frac{884b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{100947a^{21/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{1768b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^3}$$

[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 + (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.50061, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$-\frac{1768b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^3} - \frac{136b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^2} - \frac{884b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{100947a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b*x^(1/3) + a*x)^(3/2), x]

[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 + (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int x^2 (b\sqrt[3]{x} + ax)^{3/2} dx &= 3 \operatorname{Subst} \left(\int x^8 (bx + ax^3)^{3/2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{9} x^3 (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{3} (2b) \operatorname{Subst} \left(\int x^9 \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4}{69} bx^{10/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{9} x^3 (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{69} (4b^2) \operatorname{Subst} \left(\int \frac{x^{10}}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{8b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a} + \frac{4}{69} bx^{10/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{9} x^3 (b\sqrt[3]{x} + ax)^{3/2} - \frac{(68b^3) \operatorname{Subst} \left(\int \frac{x^8}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a} \\
&= -\frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \frac{8b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a} + \frac{4}{69} bx^{10/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{9} x^3 (b\sqrt[3]{x} + ax)^{3/2} + \dots \\
&= \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \frac{8b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a} + \frac{4}{69} bx^{10/3} \sqrt{b\sqrt[3]{x} + ax} + \dots \\
&= -\frac{1768b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{168245a^4} + \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \frac{8b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a} + \dots \\
&= \frac{1768b^6 \sqrt{b\sqrt[3]{x} + ax}}{100947a^5} - \frac{1768b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{168245a^4} + \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \dots \\
&= \frac{1768b^6 \sqrt{b\sqrt[3]{x} + ax}}{100947a^5} - \frac{1768b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{168245a^4} + \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \dots \\
&= \frac{1768b^6 \sqrt{b\sqrt[3]{x} + ax}}{100947a^5} - \frac{1768b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{168245a^4} + \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \dots \\
&= \frac{1768b^6 \sqrt{b\sqrt[3]{x} + ax}}{100947a^5} - \frac{1768b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{168245a^4} + \frac{1768b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{216315a^3} - \frac{136b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{19665a^2} + \dots
\end{aligned}$$

Mathematica [C] time = 0.137646, size = 142, normalized size = 0.48

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left((ax^{2/3} + b)^2 \sqrt{\frac{ax^{2/3}}{b} + 1} (12155a^2 b^2 x^{4/3} - 17765a^3 bx^2 + 24035a^4 x^{8/3} - 7293ab^3 x^{2/3} + 3315b^4) - 3315b^6 \sqrt[2]{\frac{ax^{2/3}}{b} + 1} \right)}{216315a^5 \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b*x^(1/3) + a*x)^(3/2), x]

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*b^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035*a^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)])/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] time = 0.027, size = 197, normalized size = 0.7

$$-\frac{2}{1514205 a^6} \left(-216755 x^{11/3} a^6 b^2 - 380380 x^{13/3} a^7 b + 616 x^3 a^5 b^3 + 6630 b^7 \sqrt{-ab} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -2/1514205*(-216755*x^(11/3)*a^6*b^2-380380*x^(13/3)*a^7*b+616*x^3*a^5*b^3+6630*b^7*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1768*x^(5/3)*a^3*b^5-952*x^(7/3)*a^4*b^4-168245*x^5*a^8-5304*x*a^2*b^6-13260*x^(1/3)*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(ax^3 + bx^{\frac{7}{3}} \right) \sqrt{ax + bx^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)`

3.141 $\int x (b\sqrt[3]{x} + ax)^{3/2} dx$

Optimal. Leaf size=408

$$\frac{44b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{88b^5\sqrt[3]{x}(ax^{2/3}+b)}{1105a^{7/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{88b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $(-88*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*a^{(7/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})$
 $*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (88*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*a^3)$
 $- (88*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^2) + (24*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)}$
 $+ a*x])/(1547*a) + (12*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/119 + (2*x^2*($
 $b*x^{(1/3)} + a*x)^{(3/2)}/7 + (88*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[($
 $b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a$
 $^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*a^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44$
 $*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[$
 $a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]$
 $/ (1105*a^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.554065, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{88b^5\sqrt[3]{x}(ax^{2/3}+b)}{1105a^{7/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} - \frac{44b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{88b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(b*x^{(1/3)} + a*x)^{(3/2)}, x]$

[Out] $(-88*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*a^{(7/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})$
 $*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (88*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*a^3)$
 $- (88*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^2) + (24*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)}$
 $+ a*x])/(1547*a) + (12*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/119 + (2*x^2*($
 $b*x^{(1/3)} + a*x)^{(3/2)}/7 + (88*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[($
 $b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a$
 $^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(1105*a^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44$
 $*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[$

$a] * x^{(1/3)}]^2 * x^{(1/6)} * \text{EllipticF}[2 * \text{ArcTan}[(a^{(1/4)} * x^{(1/6)}) / b^{(1/4)}], 1/2]] / ((1105 * a^{(15/4)} * \text{Sqrt}[b * x^{(1/3)} + a * x])$

Rule 2018

$\text{Int}[(x_)^{(m_)} * ((a_) * (x_)^{(j_)} + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a * x^{\text{Simplify}[j/n] + b * x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& \text{NeQ}[n^2, 1]$

Rule 2021

$\text{Int}[(c_) * (x_)^{(m_)} * ((a_) * (x_)^{(j_)} + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c * x)^{(m + 1)} * (a * x^j + b * x^n)^p / (c * (m + n * p + 1)), x] + \text{Dist}[(a * (n - j) * p) / (c^j * (m + n * p + 1)), \text{Int}[(c * x)^{(m + j)} * (a * x^j + b * x^n)^{(p - 1)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[m + n * p + 1, 0]$

Rule 2024

$\text{Int}[(c_) * (x_)^{(m_)} * ((a_) * (x_)^{(j_)} + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n - 1)} * (c * x)^{(m - n + 1)} * (a * x^j + b * x^n)^{(p + 1)}) / (b * (m + n * p + 1)), x] - \text{Dist}[(a * c^{(n - j)} * (m + j * p - n + j + 1)) / (b * (m + n * p + 1)), \text{Int}[(c * x)^{(m - (n - j))} * (a * x^j + b * x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[m + j * p + 1 - n + j, 0] \&\& \text{NeQ}[m + n * p + 1, 0]$

Rule 2032

$\text{Int}[(c_) * (x_)^{(m_)} * ((a_) * (x_)^{(j_)} + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]} * (c * x)^{\text{FracPart}[m]} * (a * x^j + b * x^n)^{\text{FracPart}[p]}) / (x^{(\text{FracPart}[m] + j * \text{FracPart}[p])} * (a + b * x^{(n - j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m + j * p)} * (a + b * x^{(n - j)})^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 329

$\text{Int}[(c_) * (x_)^{(m_)} * ((a_) + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k * (m + 1) - 1)} * (a + (b * x^{(k * n)}) / c^{(n - 1)})^p, x], x, (c * x)^{(1/k)}], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int x (b\sqrt[3]{x} + ax)^{3/2} dx &= 3 \text{Subst} \left(\int x^5 (bx + ax^3)^{3/2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{7} (6b) \text{Subst} \left(\int x^6 \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{119} (12b^2) \text{Subst} \left(\int \frac{x^7}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} - \frac{(132b^3) \text{Subst} \left(\int \frac{x^5}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1547a} \\
&= -\frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} + \frac{(44b^3) \text{Subst} \left(\int \frac{x^5}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1547a} \\
&= \frac{88b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{3315a^3} - \frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} \\
&= \frac{88b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{3315a^3} - \frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} \\
&= \frac{88b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{3315a^3} - \frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} \\
&= \frac{88b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{3315a^3} - \frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2} \\
&= -\frac{88b^5 (b + ax^{2/3}) \sqrt[3]{x}}{1105a^{7/2} (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} + \frac{88b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{3315a^3} - \frac{88b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^2} + \frac{24b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a} + \frac{12}{119} bx^{7/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{7} x^2 (b\sqrt[3]{x} + ax)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.10018, size = 123, normalized size = 0.3

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left((ax^{2/3}+b)^2\sqrt{\frac{ax^{2/3}}{b}+1}\left(221a^2x^{4/3}-143abx^{2/3}+77b^2\right)-77b^4{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{1547a^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]

[Out] $(2*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]*((b + a*x^{(2/3)})^2*\text{Sqrt}[1 + (a*x^{(2/3)})/b] * (77*b^2 - 143*a*b*x^{(2/3)} + 221*a^2*x^{(4/3)}) - 77*b^4*\text{Hypergeometric2F1}[-3/2, 3/4, 7/4, -((a*x^{(2/3)})/b)]))/((1547*a^3*\text{Sqrt}[1 + (a*x^{(2/3)})/b])$

Maple [A] time = 0.018, size = 263, normalized size = 0.6

$$-\frac{2}{23205 a^4} \left(-4665 x^{8/3} a^4 b^2 - 7800 x^{10/3} a^5 b + 40 x^2 a^3 b^3 + 924 b^6 \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticE} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(b*x^{(1/3)}+a*x)^{(3/2)}, x)$

[Out] $-2/23205/a^4*(-4665*x^{(8/3)}*a^4*b^2-7800*x^{(10/3)}*a^5*b+40*x^2*a^3*b^3+924*b^6*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*\text{EllipticE}(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})-462*b^6*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*\text{EllipticF}(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})-3315*x^4*a^6-308*x^{(2/3)}*a*b^5-88*x^{(4/3)}*a^2*b^4)/(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x*(b*x^{(1/3)}+a*x)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((a*x + b*x^{(1/3)})^{(3/2)}*x, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(ax^2 + bx^{\frac{4}{3}} \right) \sqrt{ax + bx^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x (ax + b\sqrt[3]{x})^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**(1/3)+a*x)**(3/2),x)`

[Out] `Integral(x*(a*x + b*x**(1/3))**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax + bx^{\frac{1}{3}})^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2)*x, x)`

3.142 $\int (b\sqrt[3]{x} + ax)^{3/2} dx$

Optimal. Leaf size=208

$$\frac{4b^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2} + \frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a} + \frac{12}{55}bx^{4/3}$$

[Out] $(-8*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^2) + (24*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a) + (12*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/55 + (2*x*(b*x^{(1/3)} + a*x)^{(3/2)})/5 + (4*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})]/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2)*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.270748, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2004, 2018, 2021, 2024, 2011, 329, 220}

$$\frac{4b^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2} + \frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a} + \frac{12}{55}bx^{4/3}\sqrt{ax+b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x^{(1/3)} + a*x)^{(3/2)}, x]$

[Out] $(-8*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^2) + (24*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a) + (12*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/55 + (2*x*(b*x^{(1/3)} + a*x)^{(3/2)})/5 + (4*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})]/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2)*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2004

$\text{Int}[(a_.)*(x_.)^{(j_.)} + (b_.)*(x_.)^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*(n - j)*p)/(n*p + 1), \text{Int}[x^j*(a*x^j + b*x^n)^{(p - 1)}, x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[n*p + 1, 0]$

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x],
x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int (b\sqrt[3]{x} + ax)^{3/2} dx &= \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{5}(2b) \int \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} dx \\
&= \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{5}(6b) \text{Subst} \left(\int x^3 \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{55} (12b^2) \text{Subst} \left(\int \frac{x^4}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{24b^2x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a} + \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} - \frac{(12b^3) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a} \\
&= -\frac{8b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^2} + \frac{24b^2x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a} + \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{(4b^4) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a} \\
&= -\frac{8b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^2} + \frac{24b^2x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a} + \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{(4b^4 \sqrt{b + ax^3}) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a} \\
&= -\frac{8b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^2} + \frac{24b^2x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a} + \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{(8b^4 \sqrt{b + ax^3}) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a} \\
&= -\frac{8b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^2} + \frac{24b^2x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a} + \frac{12}{55}bx^{4/3} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{5}x (b\sqrt[3]{x} + ax)^{3/2} + \frac{4b^{15/4} (\sqrt{b + ax^3}) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a}
\end{aligned}$$

Mathematica [C] time = 0.0809067, size = 106, normalized size = 0.51

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(5b^3 {}_2F_1 \left(-\frac{3}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{ax^{2/3}}{b} \right) - (5b - 11ax^{2/3}) (ax^{2/3} + b)^2 \sqrt{\frac{ax^{2/3}}{b} + 1} \right)}{55a^2 \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2),x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-(5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -(a*x^(2/3))/b])/((55*a^2*Sqrt[1 + (a*x^(2/3))/b])

Maple [A] time = 0.017, size = 164, normalized size = 0.8

$$\frac{2}{385 a^3} \left(10 b^4 \sqrt{-ab} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, \frac{1}{2} \sqrt{2} \right) + 131 x^{5/3} a^3 b^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(1/3)+a*x)^(3/2),x)`

[Out] `2/385*(10*b^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+131*x^(5/3)*a^3*b^2+196*x^(7/3)*a^4*b-8*x*a^2*b^3+77*x^3*a^5-20*x^(1/3)*a*b^4)/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((a*x + b*x^(1/3))^(3/2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax + b\sqrt[3]{x})^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**(1/3)+a*x)**(3/2),x)`

[Out] `Integral((a*x + b*x**(1/3))**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax + bx^{\frac{1}{3}})^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2), x)`

$$3.143 \quad \int \frac{(b \sqrt[3]{x+ax})^{3/2}}{x} dx$$

Optimal. Leaf size=319

$$\frac{4b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{5a^{3/4} \sqrt{ax + b \sqrt[3]{x}}} - \frac{8b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{5a^{3/4} \sqrt{ax + b \sqrt[3]{x}}}$$

[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x] + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3) + a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x] + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.32934, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4} \sqrt{ax + b \sqrt[3]{x}}} - \frac{8b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4} \sqrt{ax + b \sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1/3) + a*x)^(3/2)/x,x]

[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x] + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3) + a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x] + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j
+ b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x} dx &= 3 \operatorname{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} + (2b) \operatorname{Subst} \left(\int \sqrt{bx + ax^3} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4}{5} b\sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} + \frac{1}{5} (4b^2) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{4}{5} b\sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} + \frac{(4b^2 \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{5\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{4}{5} b\sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} + \frac{(8b^2 \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{5\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{4}{5} b\sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} + \frac{(8b^{5/2} \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{5\sqrt{a}\sqrt{b\sqrt[3]{x} + ax}} - \frac{(8b^5)}{5} \\
&= \frac{8b^2 (b + ax^{2/3}) \sqrt[3]{x}}{5\sqrt{a} (\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} + \frac{4}{5} b\sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax} + \frac{2}{3} (b\sqrt[3]{x} + ax)^{3/2} - \frac{8b^{9/4} (\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}}{5}
\end{aligned}$$

Mathematica [C] time = 0.0457798, size = 60, normalized size = 0.19

$$\frac{2b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}{}_2F_1\left(-\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{ax^{2/3}}{b}\right)}{\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]

[Out] (2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -(a*x^(2/3))/b])/Sqrt[1 + (a*x^(2/3))/b]

Maple [A] time = 0.017, size = 230, normalized size = 0.7

$$\frac{2}{15a} \left(12b^3 \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) - 6b^3 \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(3/2)/x,x)

[Out] 2/15/a*(12*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-6*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+11*x^(2/3)*a*b^2+16*x^(4/3)*a^2*b+5*x^2*a^3)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")

[Out] integral((a*x + b*x^(1/3))^(3/2)/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + b\sqrt[3]{x} \right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(3/2)/x,x)

[Out] Integral((a*x + b*x**(1/3))**(3/2)/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```


$$3.144 \quad \int \frac{(b\sqrt[3]{x+ax})^{3/2}}{x^2} dx$$

Optimal. Leaf size=144

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}} - \frac{2(ax+b\sqrt[3]{x})^{3/2}}{x} + 4a\sqrt{ax+b\sqrt[3]{x}}$$

[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[b*x^(1/3) + a*x]

Rubi [A] time = 0.20206, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}} - \frac{2(ax+b\sqrt[3]{x})^{3/2}}{x} + 4a\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]

[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[b*x^(1/3) + a*x]

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2020

```
Int[((c_.)*(x_)^(m_))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]], Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^2} dx &= 3 \operatorname{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x^4} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2(b\sqrt[3]{x} + ax)^{3/2}}{x} + (6a) \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x} dx, x, \sqrt[3]{x} \right) \\
&= 4a\sqrt{b\sqrt[3]{x} + ax} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{x} + (4ab) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= 4a\sqrt{b\sqrt[3]{x} + ax} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{x} + \frac{(4ab\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{\sqrt{b\sqrt[3]{x} + ax}} \\
&= 4a\sqrt{b\sqrt[3]{x} + ax} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{x} + \frac{(8ab\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{\sqrt{b\sqrt[3]{x} + ax}} \\
&= 4a\sqrt{b\sqrt[3]{x} + ax} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}} \sqrt[6]{x} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0582228, size = 60, normalized size = 0.42

$$\frac{2b\sqrt{ax + b\sqrt[3]{x}} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4}; \frac{1}{4}; -\frac{ax^{2/3}}{b}\right)}{x^{2/3} \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]

[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3))/b)]/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))

Maple [A] time = 0.02, size = 130, normalized size = 0.9

$$2 \frac{1}{\sqrt[3]{x}\sqrt[3]{x}(b + ax^{2/3})} \left(2 \sqrt[3]{x} \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2, \sqrt{2}\right) \sqrt{-abb} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(1/3)+a*x)^(3/2)/x^2,x)`

[Out] $2/x^{1/3}*(2*x^{1/3}*((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-2*(a*x^{1/3})-(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x^{1/3}*a/(-a*b)^{1/2})^{1/2}*EllipticF(((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2},1/2*2^{1/2})*(-a*b)^{1/2}*(1/2)*b+x^{4/3}*a^2-b^2/(x^{1/3}*(b+a*x^{2/3}))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(ax + b\sqrt[3]{x})^{\frac{3}{2}}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")`

[Out] `integral((a*x + b*x^(1/3))^(3/2)/x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + b\sqrt[3]{x})^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)

[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)

$$3.145 \quad \int \frac{(b\sqrt[3]{x+ax})^{3/2}}{x^3} dx$$

Optimal. Leaf size=350

$$\frac{4a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{8a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $(8*a^{(5/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(5*b*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*x) - (8*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b*x^{(1/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(3*x^2) - (8*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.431339, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{8a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]

[Out] $(8*a^{(5/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(5*b*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*x) - (8*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b*x^{(1/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(3*x^2) - (8*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^3} dx &= 3 \operatorname{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x^7} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + (2a) \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^4} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + \frac{1}{5}(4a^2) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + \frac{(4a^3) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{5b} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + \frac{(4a^3\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b + ax^2}} dx \right)}{5b\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + \frac{(8a^3\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{b + ax^4}} dx \right)}{5b\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} + \frac{(8a^{5/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b + ax^4}} dx \right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{8a^{5/2}(b + ax^{2/3})\sqrt[3]{x}}{5b(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{4a\sqrt{b\sqrt[3]{x} + ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{5b\sqrt[3]{x}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{3x^2} - \frac{8a^{9/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})}{5b\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0529985, size = 62, normalized size = 0.18

$$\frac{2b\sqrt{ax + b\sqrt[3]{x}} {}_2F_1\left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{ax^{2/3}}{b}\right)}{3x^{5/3}\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]

[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -(a*x^(2/3))/b])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))

Maple [A] time = 0.022, size = 339, normalized size = 1.

$$-\frac{2}{15bx^3} \left(-12a^2b \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} x^{8/3} \sqrt{\sqrt[3]{x}(b+ax^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(3/2)/x^3,x)

[Out]
$$-2/15 * (-12 * a^2 * b * ((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-2 * (a * x^{1/3} - (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a * b)^{1/2})^{1/2} * x^{8/3} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * \text{EllipticE}(((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2}, 1/2 * 2^{1/2})) + 6 * a^2 * b * ((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-2 * (a * x^{1/3} - (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a * b)^{1/2})^{1/2} * x^{8/3} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * \text{EllipticF}(((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2}, 1/2 * 2^{1/2})) + 12 * x^{10/3} * (b * x^{1/3} + a * x)^{1/2} * a^3 + 12 * x^{8/3} * (b * x^{1/3} + a * x)^{1/2} * a^2 * b + 16 * x^2 * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * a * b^2 + 11 * x^{8/3} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * a^2 * b + 5 * x^{4/3} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * b^3) / b / x^3 / (b + a * x^{2/3})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")`

[Out] `integral((a*x + b*x^(1/3))^(3/2)/x^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + b\sqrt[3]{x})^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)`

[Out] `Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)`

$$3.146 \quad \int \frac{(b\sqrt[3]{x+ax})^{3/2}}{x^4} dx$$

Optimal. Leaf size=213

$$\frac{4a^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}} - \frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{55x^2}$$

[Out] $(-12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*x^2) - (24*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*b*x^{(4/3)}) + (8*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^2*x^{(2/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(5*x^3) + (4*a^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*b^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.308573, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}} + \frac{4a^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{55x^2} - 2$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x^{(1/3)} + a*x)^{(3/2)}/x^4, x]$

[Out] $(-12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*x^2) - (24*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*b*x^{(4/3)}) + (8*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^2*x^{(2/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(5*x^3) + (4*a^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*b^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x^p)}, x], x, x^n], x] /;$ FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m+1)/n]] && NeQ[n^2, 1]

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^4} dx &= 3 \text{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x^{10}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{1}{5}(6a) \text{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^7} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{1}{55}(12a^2) \text{Subst} \left(\int \frac{1}{x^4\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{385bx^{4/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} - \frac{(12a^3) \text{Subst} \left(\int \frac{1}{x^2\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{77b} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{(4a^4) \text{Subst} \left(\int \frac{1}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{77b^2} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{(4a^4\sqrt{b + ax^{2/3}}\sqrt[6]{x})}{77b} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{(8a^4\sqrt{b + ax^{2/3}}\sqrt[6]{x})}{77b^2} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x} + ax}}{77b^2x^{2/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{5x^3} + \frac{4a^{15/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})}{77b^2}
\end{aligned}$$

Mathematica [C] time = 0.0661396, size = 62, normalized size = 0.29

$$\frac{2b\sqrt{ax + b\sqrt[3]{x}} {}_2F_1\left(-\frac{15}{4}, -\frac{3}{2}; -\frac{11}{4}; -\frac{ax^{2/3}}{b}\right)}{5x^{8/3}\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4, x]

[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -(a*x^(2/3))/b])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))

Maple [A] time = 0.023, size = 168, normalized size = 0.8

$$\frac{2}{385 b^2} \left(10 a^3 \sqrt{-ab} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a \sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) x^{14/3} - 131 x^{11/3} a^2 b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(3/2)/x^4,x)

[Out] 2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*x^5*a^4-77*x^(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```


$$3.147 \quad \int \frac{(b\sqrt[3]{x+ax})^{3/2}}{x^5} dx$$

Optimal. Leaf size=438

$$\frac{44a^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{88a^{11/2}\sqrt[3]{x}(ax^{2/3}+b)}{1105b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{88a^3\sqrt{a}}{4641b^2x^{5/3}}$$

[Out] $(-88*a^{(11/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*x^3) - (24*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*b*x^{(7/3)}) + (88*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*b^2*x^{(5/3)}) - (88*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*b^3*x) + (88*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1105*b^4*x^{(1/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(7*x^4) + (88*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.640416, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{88a^{11/2}\sqrt[3]{x}(ax^{2/3}+b)}{1105b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{88a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^2x^{5/3}} - \frac{44a^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]

[Out] $(-88*a^{(11/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(1105*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (12*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*x^3) - (24*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*b*x^{(7/3)}) + (88*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*b^2*x^{(5/3)}) - (88*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3315*b^3*x) + (88*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1105*b^4*x^{(1/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(7*x^4) + (88*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (44*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1105*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

$$\left[\text{Sqrt}[a] * x^{(1/3)} \right]^2 * x^{(1/6)} * \text{EllipticE}\left[2 * \text{ArcTan}\left[\frac{a^{(1/4)} * x^{(1/6)}}{b^{(1/4)}}\right], \frac{1}{2}\right] / \left(1105 * b^{(15/4)} * \text{Sqrt}[b * x^{(1/3)} + a * x]\right) - \left(44 * a^{(21/4)} * \left(\text{Sqrt}[b] + \text{Sqrt}[a] * x^{(1/3)}\right) * \text{Sqrt}\left[\frac{b + a * x^{(2/3)}}{\left(\text{Sqrt}[b] + \text{Sqrt}[a] * x^{(1/3)}\right)^2}\right] * x^{(1/6)} * \text{EllipticF}\left[2 * \text{ArcTan}\left[\frac{a^{(1/4)} * x^{(1/6)}}{b^{(1/4)}}\right], \frac{1}{2}\right] / \left(1105 * b^{(15/4)} * \text{Sqrt}[b * x^{(1/3)} + a * x]\right)\right]$$

Rule 2018

$$\text{Int}\left[\left(x\right)^{\left(m\right)} * \left(\left(a\right) * \left(x\right)^{\left(j\right)} + \left(b\right) * \left(x\right)^{\left(n\right)}\right)^{\left(p\right)}, x_Symbol\right] \rightarrow \text{Dist}\left[\frac{1}{n}, \text{Subst}\left[\text{Int}\left[x^{\left(\text{Simplify}\left[\frac{m+1}{n}\right] - 1\right)} * \left(a * x^{\text{Simplify}\left[\frac{j}{n}\right]} + b * x\right)^p, x\right], x, x^n\right], x\right] /; \text{FreeQ}\left[\{a, b, j, m, n, p\}, x\right] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}\left[\text{Simplify}\left[\frac{j}{n}\right]\right] \&\& \text{IntegerQ}\left[\text{Simplify}\left[\frac{m+1}{n}\right]\right] \&\& \text{NeQ}[n^2, 1]$$

Rule 2020

$$\text{Int}\left[\left(\left(c\right) * \left(x\right)\right)^{\left(m\right)} * \left(\left(a\right) * \left(x\right)^{\left(j\right)} + \left(b\right) * \left(x\right)^{\left(n\right)}\right)^{\left(p\right)}, x_Symbol\right] \rightarrow \text{Simp}\left[\frac{\left(c * x\right)^{\left(m+1\right)} * \left(a * x^j + b * x^n\right)^p}{c * \left(m + j * p + 1\right)}, x\right] - \text{Dist}\left[\frac{b * p * \left(n - j\right)}{c^n * \left(m + j * p + 1\right)}, \text{Int}\left[\left(c * x\right)^{\left(m+n\right)} * \left(a * x^j + b * x^n\right)^{\left(p-1\right)}, x\right], x\right] /; \text{FreeQ}\left[\{a, b, c\}, x\right] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& \left(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]\right) \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m + j * p + 1, 0]$$

Rule 2025

$$\text{Int}\left[\left(\left(c\right) * \left(x\right)\right)^{\left(m\right)} * \left(\left(a\right) * \left(x\right)^{\left(j\right)} + \left(b\right) * \left(x\right)^{\left(n\right)}\right)^{\left(p\right)}, x_Symbol\right] \rightarrow \text{Simp}\left[\frac{c^{(j-1)} * \left(c * x\right)^{\left(m-j+1\right)} * \left(a * x^j + b * x^n\right)^{\left(p+1\right)}}{a * \left(m + j * p + 1\right)}, x\right] - \text{Dist}\left[\frac{b * \left(m + n * p + n - j + 1\right)}{a * c^{(n-j)} * \left(m + j * p + 1\right)}, \text{Int}\left[\left(c * x\right)^{\left(m+n-j\right)} * \left(a * x^j + b * x^n\right)^p, x\right], x\right] /; \text{FreeQ}\left[\{a, b, c, m, p\}, x\right] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& \left(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]\right) \&\& \text{LtQ}[m + j * p + 1, 0]$$

Rule 2032

$$\text{Int}\left[\left(\left(c\right) * \left(x\right)\right)^{\left(m\right)} * \left(\left(a\right) * \left(x\right)^{\left(j\right)} + \left(b\right) * \left(x\right)^{\left(n\right)}\right)^{\left(p\right)}, x_Symbol\right] \rightarrow \text{Dist}\left[\frac{c^{\text{IntPart}[m]} * \left(c * x\right)^{\text{FracPart}[m]} * \left(a * x^j + b * x^n\right)^{\text{FracPart}[p]}}{x^{\left(\text{FracPart}[m] + j * \text{FracPart}[p]\right)} * \left(a + b * x^{(n-j)}\right)^{\text{FracPart}[p]}}\right], \text{Int}\left[x^{\left(m + j * p\right)} * \left(a + b * x^{(n-j)}\right)^p, x\right], x\right] /; \text{FreeQ}\left[\{a, b, c, j, m, n, p\}, x\right] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$$

Rule 329

$$\text{Int}\left[\left(\left(c\right) * \left(x\right)\right)^{\left(m\right)} * \left(\left(a\right) + \left(b\right) * \left(x\right)^{\left(n\right)}\right)^{\left(p\right)}, x_Symbol\right] \rightarrow \text{With}\left[\{k = \text{Denominator}[m]\}, \text{Dist}\left[\frac{k}{c}, \text{Subst}\left[\text{Int}\left[x^{\left(k * \left(m+1\right) - 1\right)} * \left(a + \left(b * x^{(k * n)}\right)\right) / c^n\right]^p, x\right], x, \left(c * x\right)^{\left(1/k\right)}, x\right] /; \text{FreeQ}\left[\{a, b, c, p\}, x\right] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^5} dx &= 3 \operatorname{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x^{13}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2(b\sqrt[3]{x} + ax)^{3/2}}{7x^4} + \frac{1}{7}(6a) \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^{10}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{7x^4} + \frac{1}{119}(12a^2) \operatorname{Subst} \left(\int \frac{1}{x^7\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{7x^4} - \frac{(132a^3) \operatorname{Subst} \left(\int \frac{1}{x^5\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1547b} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{7x^4} + \frac{(44a^4) \operatorname{Subst} \left(\int \frac{1}{x^3\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{663} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{7x^4} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}} \\
&= -\frac{88a^{11/2}(b + ax^{2/3})\sqrt[3]{x}}{1105b^4(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{12a\sqrt{b\sqrt[3]{x} + ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x} + ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x} + ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x} + ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x} + ax}}{1105b^4\sqrt[3]{x}}
\end{aligned}$$

Mathematica [C] time = 0.0548633, size = 62, normalized size = 0.14

$$-\frac{2b\sqrt{ax + b\sqrt[3]{x}} {}_2F_1\left(-\frac{21}{4}, -\frac{3}{2}; -\frac{17}{4}; -\frac{ax^{2/3}}{b}\right)}{7x^{11/3}\sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]

[Out] $(-2*b*\text{Sqrt}[b*x^{1/3} + a*x]*\text{Hypergeometric2F1}[-21/4, -3/2, -17/4, -((a*x^{2/3})/b)])/(7*\text{Sqrt}[1 + (a*x^{2/3})/b]*x^{11/3})$

Maple [A] time = 0.026, size = 411, normalized size = 0.9

$$\frac{2}{23205 b^4 x^7} \left(-924 a^5 b \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a \sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} x^{20/3} \sqrt{\sqrt[3]{x} (b + a x^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(3/2)/x^5,x)

[Out] $2/23205*(-924*a^5*b*((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-2*(a*x^{1/3})-(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x^{1/3}*a/(-a*b)^{1/2})^{1/2}*x^{20/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*\text{EllipticE}(((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2})+462*a^5*b*((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-2*(a*x^{1/3})-(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x^{1/3}*a/(-a*b)^{1/2})^{1/2}*x^{20/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*\text{EllipticF}(((a*x^{1/3})+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2})+924*x^{22/3}*(b*x^{1/3})+a*x^{1/2}*a^6+924*x^{20/3}*(b*x^{1/3})+a*x^{1/2}*a^5*b-88*x^6*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*a^4*b^2-308*x^{20/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*a^5*b-4665*x^{14/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*a^2*b^4+40*x^{16/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*a^3*b^3-7800*x^4*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*a*b^5-3315*x^{10/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*b^6)/b^4/x^7/(b+a*x^{2/3})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{1/3})^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")

[Out] integral((a*x + b*x^(1/3))^(3/2)/x^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)

$$3.148 \quad \int \frac{(b\sqrt[3]{x+ax})^{3/2}}{x^6} dx$$

Optimal. Leaf size=301

$$\frac{884a^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{100947b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}} + \frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}$$

[Out] $(-4*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(69*x^4) - (8*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1311*b*x^{(10/3)}) + (136*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(19665*b^2*x^{(8/3)}) - (1768*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(216315*b^3*x^2) + (1768*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(168245*b^4*x^{(4/3)}) - (1768*a^6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(100947*b^5*x^{(2/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(9*x^5) - (884*a^{(27/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(100947*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.478874, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}} + \frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}} - \frac{1768a^4\sqrt{ax+b\sqrt[3]{x}}}{216315b^3x^2} + \frac{136a^3\sqrt{ax+b\sqrt[3]{x}}}{19665b^2x^{8/3}} - \frac{884a^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}}{100947b^{21/4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x^{(1/3)} + a*x)^{(3/2)}/x^6, x]$

[Out] $(-4*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(69*x^4) - (8*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1311*b*x^{(10/3)}) + (136*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(19665*b^2*x^{(8/3)}) - (1768*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(216315*b^3*x^2) + (1768*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(168245*b^4*x^{(4/3)}) - (1768*a^6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(100947*b^5*x^{(2/3)}) - (2*(b*x^{(1/3)} + a*x)^{(3/2)})/(9*x^5) - (884*a^{(27/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(100947*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```


Rubi steps

$$\begin{aligned}
\int \frac{(b\sqrt[3]{x} + ax)^{3/2}}{x^6} dx &= 3 \operatorname{Subst} \left(\int \frac{(bx + ax^3)^{3/2}}{x^{16}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2(b\sqrt[3]{x} + ax)^{3/2}}{9x^5} + \frac{1}{3}(2a) \operatorname{Subst} \left(\int \frac{\sqrt{bx + ax^3}}{x^{13}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{9x^5} + \frac{1}{69}(4a^2) \operatorname{Subst} \left(\int \frac{1}{x^{10}\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{9x^5} - \frac{(68a^3) \operatorname{Subst} \left(\int \frac{1}{x^8\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311b} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{9x^5} + \frac{(884a^4) \operatorname{Subst} \left(\int \frac{1}{x^7\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{19665b^2} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x} + ax}}{216315b^3x^2} - \frac{2(b\sqrt[3]{x} + ax)^{3/2}}{9x^5} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x} + ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x} + ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x} + ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}} \\
&= -\frac{4a\sqrt{b\sqrt[3]{x} + ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x} + ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x} + ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x} + ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x} + ax}}{168245b^4x^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.06984, size = 62, normalized size = 0.21

$$-\frac{2b\sqrt{ax + b\sqrt[3]{x}} {}_2F_1 \left(-\frac{27}{4}, -\frac{3}{2}; -\frac{23}{4}; -\frac{ax^{2/3}}{b} \right)}{9x^{14/3} \sqrt{\frac{ax^{2/3}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]

[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(2/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))

Maple [A] time = 0.026, size = 201, normalized size = 0.7

$$-\frac{2}{1514205 b^5} \left(6630 a^6 \sqrt{-ab} \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticF} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) x^{\frac{26}{3}} - 1768 \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(3/2)/x^6,x)

[Out] -2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*x^7*a^4*b^3+216755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*x^5*a*b^6+13260*x^9*a^7+168245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")

[Out] integral((a*x + b*x^(1/3))^(3/2)/x^6, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)

$$3.149 \quad \int \frac{x^4}{\sqrt{b\sqrt[3]{x+ax}}} dx$$

Optimal. Leaf size=304

$$\frac{5525b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5}$$

[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6))*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.50605, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 2024, 2011, 329, 220}

$$-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5} - \frac{1190b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^3} - \frac{5525b^{27/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[b*x^(1/3) + a*x], x]

[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6))*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{b\sqrt[3]{x} + ax}} dx &= 3 \text{Subst} \left(\int \frac{x^{14}}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} - \frac{(25b) \text{Subst} \left(\int \frac{x^{12}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{9a} \\
&= -\frac{50bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a^2} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} + \frac{(175b^2) \text{Subst} \left(\int \frac{x^{10}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{69a^2} \\
&= \frac{350b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^3} - \frac{50bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a^2} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} - \frac{(2975b^3) \text{Subst} \left(\int \frac{x^8}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^3} \\
&= -\frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{350b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^3} - \frac{50bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a^2} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} + \frac{(7735b^4) \text{Subst} \left(\int \frac{x^6}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1311a^3} \\
&= \frac{15470b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{43263a^5} - \frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{350b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^3} - \frac{50bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}}{207a^2} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} \\
&= -\frac{2210b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{4807a^6} + \frac{15470b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{43263a^5} - \frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{350b^2 x^{8/3} \sqrt{b\sqrt[3]{x} + ax}}{1311a^3} \\
&= \frac{11050b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^7} - \frac{2210b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{4807a^6} + \frac{15470b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{43263a^5} - \frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} \\
&= \frac{11050b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^7} - \frac{2210b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{4807a^6} + \frac{15470b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{43263a^5} - \frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a} \\
&= \frac{11050b^6 \sqrt{b\sqrt[3]{x} + ax}}{14421a^7} - \frac{2210b^5 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{4807a^6} + \frac{15470b^4 x^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{43263a^5} - \frac{1190b^3 x^2 \sqrt{b\sqrt[3]{x} + ax}}{3933a^4} + \frac{2x^4 \sqrt{b\sqrt[3]{x} + ax}}{9a}
\end{aligned}$$

Mathematica [C] time = 0.0922186, size = 161, normalized size = 0.53

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(550a^5 b^2 x^{10/3} - 770a^4 b^3 x^{8/3} + 1190a^3 b^4 x^2 - 2210a^2 b^5 x^{4/3} - 418a^6 b x^4 + 4807a^7 x^{14/3} - 16575b^7 \sqrt{\frac{ax^{2/3}}{b}} + 12 \right)}{43263a^7 (ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^(4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 418*a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))

Maple [A] time = 0.032, size = 196, normalized size = 0.6

$$-\frac{1}{43263 a^8} \left(-1100 x^{11/3} a^6 b^2 + 836 x^{13/3} a^7 b + 1540 x^3 a^5 b^3 + 4420 x^{5/3} a^3 b^5 - 2380 x^{7/3} a^4 b^4 - 9614 x^5 a^8 + 16575 b^7 \sqrt{-ab} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^(1/3)+a*x)^(1/2),x)

[Out] -1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*x^3*a^5*b^3+4420*x^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*x^5*a^8+16575*b^7*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*x*a^2*b^6-33150*x^(1/3)*a*b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^5 - a b x^{\frac{13}{3}} + b^2 x^{\frac{11}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^2 + b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^2 + b^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a x + b \sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)

[Out] Integral(x**4/sqrt(a*x + b*x**(1/3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a x + b x^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)

$$3.150 \quad \int \frac{x^3}{\sqrt{b} \sqrt[3]{x+ax}} dx$$

Optimal. Leaf size=414

$$\frac{209b^{21/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{221a^{23/4} \sqrt{ax + b} \sqrt[3]{x}} - \frac{418b^5 \sqrt[3]{x} (ax^{2/3} + b)}{221a^{11/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b} \sqrt[3]{x}} + \frac{570b^2}{221a^{11/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b} \sqrt[3]{x}}$$

[Out] $(-418*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(11/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (418*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^5) - (2090*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^4) + (570*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^3) - (38*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a^2) + (2*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*a) + (418*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (221*a^{(23/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (209*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (221*a^{(23/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.566179, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$\frac{418b^5 \sqrt[3]{x} (ax^{2/3} + b)}{221a^{11/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b} \sqrt[3]{x}} + \frac{570b^2 x^{5/3} \sqrt{ax + b} \sqrt[3]{x}}{1547a^3} - \frac{209b^{21/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{221a^{23/4} \sqrt{ax + b} \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[b*x^(1/3) + a*x], x]

[Out] $(-418*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(11/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (418*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^5) - (2090*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^4) + (570*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^3) - (38*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a^2) + (2*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*a) + (418*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (221*a^{(23/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (209*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (221*a^{(23/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

+ a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2024

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2032

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{b\sqrt[3]{x} + ax}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^{11}}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} - \frac{(19b) \operatorname{Subst} \left(\int \frac{x^9}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{7a} \\
&= -\frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} + \frac{(285b^2) \operatorname{Subst} \left(\int \frac{x^7}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{119a^2} \\
&= \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} - \frac{(3135b^3) \operatorname{Subst} \left(\int \frac{x^5}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{1547a^3} \\
&= -\frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} + \frac{(1045b^4)}{663a^5} \\
&= \frac{418b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^5} - \frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} \\
&= \frac{418b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^5} - \frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} \\
&= \frac{418b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^5} - \frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} \\
&= \frac{418b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^5} - \frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3} - \frac{38bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}}{119a^2} + \frac{2x^3 \sqrt{b\sqrt[3]{x} + ax}}{7a} \\
&= -\frac{418b^5 (b + ax^{2/3}) \sqrt[3]{x}}{221a^{11/2} (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} + \frac{418b^4 \sqrt[3]{x} \sqrt{b\sqrt[3]{x} + ax}}{663a^5} - \frac{2090b^3 x \sqrt{b\sqrt[3]{x} + ax}}{4641a^4} + \frac{570b^2 x^{5/3} \sqrt{b\sqrt[3]{x} + ax}}{1547a^3}
\end{aligned}$$

Mathematica [C] time = 0.0707897, size = 143, normalized size = 0.35

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(114a^3 b^2 x^{7/3} - 190a^2 b^3 x^{5/3} - 78a^4 b x^3 + 663a^5 x^{11/3} - 1463b^5 \sqrt[3]{x} \sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{ax^{2/3}}{b} \right) + 418ab^4 x + \right)}{4641a^5 (ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(5/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^5*(b + a*x^(2/3)))

Maple [A] time = 0.024, size = 261, normalized size = 0.6

$$-\frac{1}{4641 a^6} \left(-228 x^{8/3} a^4 b^2 + 156 x^{10/3} a^5 b + 380 x^2 a^3 b^3 + 8778 b^6 \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a \sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticE} \left(\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^(1/3)+a*x)^(1/2), x)

[Out] -1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*x^2*a^3*b^3+8778*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))-4389*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))-1326*x^4*a^6-2926*x^(2/3)*a*b^5-836*x^(4/3)*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^4 - a b x^{\frac{10}{3}} + b^2 x^{\frac{8}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^2 + b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^2 + b^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a x + b \sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)

[Out] Integral(x**3/sqrt(a*x + b*x**(1/3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a x + b x^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)

$$3.151 \quad \int \frac{x^2}{\sqrt{b} \sqrt[3]{x+ax}} dx$$

Optimal. Leaf size=216

$$\frac{39b^{15/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{77a^{17/4} \sqrt{ax + b} \sqrt[3]{x}} + \frac{234b^2 x^{2/3} \sqrt{ax + b} \sqrt[3]{x}}{385a^3} - \frac{78b^3 \sqrt{ax + b} \sqrt[3]{x}}{77a^4} - \frac{26bx^{4/3} \sqrt{ax + b} \sqrt[3]{x}}{55a^4}$$

[Out] $(-78*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^4) + (234*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a^3) - (26*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*a^2) + (2*x^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*a) + (39*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(17/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.318692, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 2024, 2011, 329, 220}

$$\frac{234b^2 x^{2/3} \sqrt{ax + b} \sqrt[3]{x}}{385a^3} + \frac{39b^{15/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{77a^{17/4} \sqrt{ax + b} \sqrt[3]{x}} - \frac{78b^3 \sqrt{ax + b} \sqrt[3]{x}}{77a^4} - \frac{26bx^{4/3} \sqrt{ax + b} \sqrt[3]{x}}{55a^4}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[b*x^(1/3) + a*x], x]

[Out] $(-78*b^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*a^4) + (234*b^2*x^{(2/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*a^3) - (26*b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*a^2) + (2*x^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*a) + (39*b^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(77*a^{(17/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]

&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt[3]{x} + ax}} dx &= 3 \text{Subst} \left(\int \frac{x^8}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} - \frac{(13b) \text{Subst} \left(\int \frac{x^6}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{5a} \\
&= -\frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} + \frac{(117b^2) \text{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{55a^2} \\
&= \frac{234b^2 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a^3} - \frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} - \frac{(117b^3) \text{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a^3} \\
&= -\frac{78b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^4} + \frac{234b^2 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a^3} - \frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} + \frac{(39b^4) \text{Subst} \left(\int \frac{1}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{77a^3} \\
&= -\frac{78b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^4} + \frac{234b^2 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a^3} - \frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} + \frac{(39b^4 \sqrt{b + ax^{2/3}})}{77a^3} \\
&= -\frac{78b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^4} + \frac{234b^2 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a^3} - \frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} + \frac{(78b^4 \sqrt{b + ax^{2/3}})}{77a^3} \\
&= -\frac{78b^3 \sqrt{b\sqrt[3]{x} + ax}}{77a^4} + \frac{234b^2 x^{2/3} \sqrt{b\sqrt[3]{x} + ax}}{385a^3} - \frac{26bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}}{55a^2} + \frac{2x^2 \sqrt{b\sqrt[3]{x} + ax}}{5a} + \frac{39b^{15/4} (\sqrt{b + ax^{2/3}})}{77a^3}
\end{aligned}$$

Mathematica [C] time = 0.066839, size = 124, normalized size = 0.57

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(26a^2 b^2 x^{4/3} - 14a^3 b x^2 + 77a^4 x^{8/3} + 195b^4 \sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{ax^{2/3}}{b} \right) - 78ab^3 x^{2/3} - 195b^4 \right)}{385a^4 (ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3) - 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))

Maple [A] time = 0.007, size = 164, normalized size = 0.8

$$\frac{1}{385 a^5} \left(195 b^4 \sqrt{-ab} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, \frac{1}{2} \sqrt{2} \right) + 52 x^{5/3} a^3 b^2 - 28 x^{7/3} a^4 b - 156 x a^2 b^3 + 154 x^3 a^5 - 390 x^{1/3} a b^4 \right) / (x^{1/3} (b + a x^{2/3}))^{1/2} / a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^(1/3)+a*x)^(1/2),x)`

[Out] `1/385*(195*b^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+52*x^(5/3)*a^3*b^2-28*x^(7/3)*a^4*b-156*x*a^2*b^3+154*x^3*a^5-390*x^(1/3)*a*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/sqrt(a*x + b*x^(1/3)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\left(a^2 x^3 - a b x^{\frac{7}{3}} + b^2 x^{\frac{5}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^2 + b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^2 + b^3), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + b\sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**(1/3)+a*x)**(1/2), x)`

[Out] `Integral(x**2/sqrt(a*x + b*x**(1/3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/3)+a*x)^(1/2), x, algorithm="giac")`

[Out] `integrate(x^2/sqrt(a*x + b*x^(1/3)), x)`

$$3.152 \quad \int \frac{x}{\sqrt{b} \sqrt[3]{x+ax}} dx$$

Optimal. Leaf size=326

$$\frac{7b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{5a^{11/4} \sqrt{ax + b} \sqrt[3]{x}} + \frac{14b^2 \sqrt[3]{x} (ax^{2/3} + b)}{5a^{5/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b} \sqrt[3]{x}} - \frac{14b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b})}{5a^{11/4} \sqrt{ax + b} \sqrt[3]{x}}$$

[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2 + (2*x*Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.349393, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$\frac{14b^2 \sqrt[3]{x} (ax^{2/3} + b)}{5a^{5/2} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b} \sqrt[3]{x}} + \frac{7b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{5a^{11/4} \sqrt{ax + b} \sqrt[3]{x}} - \frac{14b^{9/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b})}{5a^{11/4} \sqrt{ax + b} \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[b*x^(1/3) + a*x], x]

[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2 + (2*x*Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{b\sqrt[3]{x} + ax}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^5}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} - \frac{(7b) \operatorname{Subst} \left(\int \frac{x^3}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{3a} \\
&= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} + \frac{(7b^2) \operatorname{Subst} \left(\int \frac{x}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{5a^2} \\
&= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} + \frac{(7b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b + ax^2}} dx, x, \sqrt[3]{x} \right)}{5a^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} + \frac{(14b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{5a^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} + \frac{(14b^{5/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{5a^{5/2}\sqrt{b\sqrt[3]{x} + ax}} - \frac{(14b^{5/2})}{5a^{11/2}} \\
&= \frac{14b^2(b + ax^{2/3})\sqrt[3]{x}}{5a^{5/2}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x} + ax}}{3a} - \frac{14b^{9/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{\frac{1}{b\sqrt[3]{x} + ax}}}{5a^{11/2}}
\end{aligned}$$

Mathematica [C] time = 0.0566559, size = 106, normalized size = 0.33

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(5a^2x^{5/3} + 7b^2\sqrt[3]{x}\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{ax^{2/3}}{b} \right) - 2abx - 7b^2\sqrt[3]{x} \right)}{15a^2(ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[b*x^(1/3) + a*x],x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(15*a^2*(b + a*x^(2/3)))

Maple [A] time = 0.008, size = 230, normalized size = 0.7

$$\frac{1}{15 a^3} \left(42 b^3 \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticE} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) - 21 b^3 \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^(1/3)+a*x)^(1/2),x)

[Out] 1/15/a^3*(42*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^2^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2),1/2*2^(1/2))-21*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^2^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^2^(1/2),1/2*2^(1/2))-14*x^(2/3)*a*b^2-4*x^(4/3)*a^2*b+10*x^2*a^3)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^2 - a b x^{\frac{4}{3}} + b^2 x^{\frac{2}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^2 + b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^2 + b^3), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a x + b \sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**(1/3)+a*x)**(1/2),x)`

[Out] `Integral(x/sqrt(a*x + b*x**(1/3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a x + b x^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")`

[Out] `integrate(x/sqrt(a*x + b*x^(1/3)), x)`

$$3.153 \quad \int \frac{1}{\sqrt{b} \sqrt[3]{x+ax}} dx$$

Optimal. Leaf size=126

$$\frac{2\sqrt{ax+b}\sqrt[3]{x}}{a} - \frac{b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b}\sqrt[3]{x}}$$

[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.11811, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2010, 2013, 2011, 329, 220}

$$\frac{2\sqrt{ax+b}\sqrt[3]{x}}{a} - \frac{b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2010

Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n - 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] && LtQ[2*(n - 1), j, n]

Rule 2013

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{b\sqrt[3]{x} + ax}} dx &= \frac{2\sqrt{b\sqrt[3]{x} + ax}}{a} - \frac{b \int \frac{1}{x^{2/3}\sqrt{b\sqrt[3]{x} + ax}} dx}{3a} \\
&= \frac{2\sqrt{b\sqrt[3]{x} + ax}}{a} - \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x}\right)}{a} \\
&= \frac{2\sqrt{b\sqrt[3]{x} + ax}}{a} - \frac{(b\sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x}\right)}{a\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{2\sqrt{b\sqrt[3]{x} + ax}}{a} - \frac{(2b\sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x}\right)}{a\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{2\sqrt{b\sqrt[3]{x} + ax}}{a} - \frac{b^{3/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}} \sqrt[6]{x} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0363058, size = 80, normalized size = 0.63

$$\frac{2\sqrt{ax + b\sqrt[3]{x}} \left(-b\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{ax^{2/3}}{b}\right) + ax^{2/3} + b \right)}{a(ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x^(1/3) + a*x], x]

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b])*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)])/(a*(b + a*x^(2/3)))

Maple [A] time = 0.005, size = 128, normalized size = 1.

$$-\frac{1}{a^2} \left(b\sqrt{-ab} \sqrt{\left(a\sqrt[3]{x} + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{2} \sqrt{\left(-a\sqrt[3]{x} + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{-a\sqrt[3]{x} \frac{1}{\sqrt{-ab}}} \operatorname{EllipticF}\left(\sqrt{\left(a\sqrt[3]{x} + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^(1/3)+a*x)^(1/2),x)`

[Out] $-(b*(-a*b)^{(1/2)}*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*EllipticF(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)},1/2*2^{(1/2)})-2*x^{(1/3)}*a*b-2*a^2*x)/(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}/a^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(a*x + b*x^(1/3)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^3x^3 + b^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^3 + b^3*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

$$3.154 \quad \int \frac{1}{x\sqrt{b}\sqrt[3]{x+ax}} dx$$

Optimal. Leaf size=294

$$\frac{3\sqrt[4]{a}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{b^{3/4}\sqrt{ax + b\sqrt[3]{x}}} - \frac{6\sqrt[4]{a}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{b^{3/4}\sqrt{ax + b\sqrt[3]{x}}}$$

[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.293531, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt[4]{a}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax + b\sqrt[3]{x}}} - \frac{6\sqrt[4]{a}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[b*x^(1/3) + a*x]), x]

[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

$1 + q^2 x^2) \sqrt{(a + c x^4) / (a (1 + q^2 x^2)^2)} \text{EllipticE}[2 \text{ArcTan}[q x], 1/2] / (q \sqrt{a + c x^4}), x] / ; \text{EqQ}[e + d q^2, 0] / ; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x \sqrt{b \sqrt[3]{x} + ax}} dx &= 3 \text{Subst} \left(\int \frac{1}{x \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
 &= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{b \sqrt[3]{x}} + \frac{(3a) \text{Subst} \left(\int \frac{x}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{b} \\
 &= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{b \sqrt[3]{x}} + \frac{(3a \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \text{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b + ax^2}} dx, x, \sqrt[3]{x} \right)}{b \sqrt{b \sqrt[3]{x} + ax}} \\
 &= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{b \sqrt[3]{x}} + \frac{(6a \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \text{Subst} \left(\int \frac{x^2}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{b \sqrt{b \sqrt[3]{x} + ax}} \\
 &= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{b \sqrt[3]{x}} + \frac{(6 \sqrt{a} \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \text{Subst} \left(\int \frac{1}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{\sqrt{b} \sqrt{b \sqrt[3]{x} + ax}} - \frac{(6 \sqrt{a} \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \text{Subst} \left(\int \frac{1}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{\sqrt{b} \sqrt{b \sqrt[3]{x} + ax}} \\
 &= \frac{6 \sqrt{a} (b + ax^{2/3}) \sqrt[3]{x}}{b (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b \sqrt[3]{x} + ax}} - \frac{6 \sqrt{b \sqrt[3]{x} + ax}}{b \sqrt[3]{x}} - \frac{6^4 \sqrt{a} (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{\frac{b + ax^{2/3}}{(\sqrt{b} + \sqrt{a} \sqrt[3]{x})^2}} \sqrt[6]{x} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{a}}{\sqrt[4]{b}} \right) \right)}{b^{3/4} \sqrt{b \sqrt[3]{x} + ax}}
 \end{aligned}$$

Mathematica [C] time = 0.0494866, size = 54, normalized size = 0.18

$$\frac{6 \sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1 \left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{ax^{2/3}}{b} \right)}{\sqrt{ax + b \sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]

[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3))/b)]/Sqrt[b*x^(1/3) + a*x]

Maple [A] time = 0.018, size = 253, normalized size = 0.9

$$-3 \frac{1}{\sqrt[3]{x}(b+ax^{2/3})b} \left(-2 \sqrt{\sqrt[3]{x}(b+ax^{2/3})} \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^(1/3)+a*x)^(1/2),x)

[Out] $-3 * (-2 * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * ((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-2 * (a * x^{1/3} - (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a * b)^{1/2})^{1/2} * \text{EllipticE}(((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2}, 1/2 * 2^{1/2}) * b + (x^{1/3} * (b + a * x^{2/3}))^{1/2} * ((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-2 * (a * x^{1/3} - (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a * b)^{1/2})^{1/2} * \text{EllipticF}(((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2}, 1/2 * 2^{1/2}) * b + 2 * x^{2/3} * (b * x^{1/3} + a * x)^{1/2} * a + 2 * (b * x^{1/3} + a * x)^{1/2} * b / x^{1/3} / (b + a * x^{2/3}) / b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^2 - a b x^{\frac{4}{3}} + b^2 x^{\frac{2}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^4 + b^3 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^4 + b^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax + b\sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

$$3.155 \quad \int \frac{1}{x^2 \sqrt{b \sqrt[3]{x+ax}}} dx$$

Optimal. Leaf size=163

$$\frac{5a^{7/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{7b^{9/4} \sqrt{ax + b \sqrt[3]{x}}} + \frac{10a \sqrt{ax + b \sqrt[3]{x}}}{7b^2 x^{2/3}} - \frac{6 \sqrt{ax + b \sqrt[3]{x}}}{7bx^{4/3}}$$

[Out] $(-6 \sqrt{bx^{1/3} + ax}) / (7bx^{4/3}) + (10a \sqrt{bx^{1/3} + ax}) / (7b^2 x^{2/3}) + (5a^{7/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{(b + ax^{2/3}) / (\sqrt{b} + \sqrt{a} x^{1/3})^2}) x^{1/6} \text{EllipticF}[2 \text{ArcTan}[(a^{1/4} x^{1/6}) / b^{1/4}], 1/2]) / (7b^{9/4} \sqrt{bx^{1/3} + ax})$

Rubi [A] time = 0.200723, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 2025, 2011, 329, 220}

$$\frac{5a^{7/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{7b^{9/4} \sqrt{ax + b \sqrt[3]{x}}} + \frac{10a \sqrt{ax + b \sqrt[3]{x}}}{7b^2 x^{2/3}} - \frac{6 \sqrt{ax + b \sqrt[3]{x}}}{7bx^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]

[Out] $(-6 \sqrt{bx^{1/3} + ax}) / (7bx^{4/3}) + (10a \sqrt{bx^{1/3} + ax}) / (7b^2 x^{2/3}) + (5a^{7/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{(b + ax^{2/3}) / (\sqrt{b} + \sqrt{a} x^{1/3})^2}) x^{1/6} \text{EllipticF}[2 \text{ArcTan}[(a^{1/4} x^{1/6}) / b^{1/4}], 1/2]) / (7b^{9/4} \sqrt{bx^{1/3} + ax})$

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{b \sqrt[3]{x} + ax}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x^4 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6\sqrt{b \sqrt[3]{x} + ax}}{7bx^{4/3}} - \frac{(15a) \operatorname{Subst} \left(\int \frac{1}{x^2 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{7b} \\
&= -\frac{6\sqrt{b \sqrt[3]{x} + ax}}{7bx^{4/3}} + \frac{10a\sqrt{b \sqrt[3]{x} + ax}}{7b^2 x^{2/3}} + \frac{(5a^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{7b^2} \\
&= -\frac{6\sqrt{b \sqrt[3]{x} + ax}}{7bx^{4/3}} + \frac{10a\sqrt{b \sqrt[3]{x} + ax}}{7b^2 x^{2/3}} + \frac{(5a^2 \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x} \sqrt{b + ax^2}} dx, x, \sqrt[3]{x} \right)}{7b^2 \sqrt{b \sqrt[3]{x} + ax}} \\
&= -\frac{6\sqrt{b \sqrt[3]{x} + ax}}{7bx^{4/3}} + \frac{10a\sqrt{b \sqrt[3]{x} + ax}}{7b^2 x^{2/3}} + \frac{(10a^2 \sqrt{b + ax^{2/3}} \sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b + ax^4}} dx, x, \sqrt[6]{x} \right)}{7b^2 \sqrt{b \sqrt[3]{x} + ax}} \\
&= -\frac{6\sqrt{b \sqrt[3]{x} + ax}}{7bx^{4/3}} + \frac{10a\sqrt{b \sqrt[3]{x} + ax}}{7b^2 x^{2/3}} + \frac{5a^{7/4} (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{\frac{b + ax^{2/3}}{(\sqrt{b} + \sqrt{a} \sqrt[3]{x})^2}} \sqrt[6]{x} F \left(2 \tan^{-1} \left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}} \right) \middle| \frac{1}{2} \right)}{7b^{9/4} \sqrt{b \sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0499106, size = 59, normalized size = 0.36

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1 \left(-\frac{7}{4}, \frac{1}{2}; -\frac{3}{4}; -\frac{ax^{2/3}}{b} \right)}{7x \sqrt{ax + b \sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*sqrt[b*x^(1/3) + a*x]),x]

[Out] (-6*sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3))/b)])/(7*x*sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.022, size = 142, normalized size = 0.9

$$\frac{1}{7b^2} \left(5a\sqrt{-ab} \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) x^{4/3} + 4abx + 10x^{5/3}a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x)`

[Out] $\frac{1}{7} * (5 * a * (-a * b)^{(1/2)} * ((a * x^{(1/3)} + (-a * b)^{(1/2)}) / (-a * b)^{(1/2)})^{(1/2)} * (-2 * (a * x^{(1/3)} - (-a * b)^{(1/2)}) / (-a * b)^{(1/2)})^{(1/2)} * (-x^{(1/3)} * a / (-a * b)^{(1/2)})^{(1/2)} * \text{EllipticF}(((a * x^{(1/3)} + (-a * b)^{(1/2)}) / (-a * b)^{(1/2)})^{(1/2)}, 1/2 * 2^{(1/2)}) * x^{(4/3)} + 4 * a * b * x + 10 * x^{(5/3)} * a^2 - 6 * x^{(1/3)} * b^2) / b^2 / (x^{(1/3)} * (b + a * x^{(2/3)}))^{(1/2)} / x^{(4/3)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^2 - a b x^{\frac{4}{3}} + b^2 x^{\frac{2}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^5 + b^3 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^5 + b^3*x^3), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{ax + b\sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2), x)

[Out] Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)

$$3.156 \quad \int \frac{1}{x^3 \sqrt{b \sqrt[3]{x+ax}}} dx$$

Optimal. Leaf size=388

$$\frac{77a^{13/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{65b^{15/4} \sqrt{ax + b \sqrt[3]{x}}} - \frac{154a^{7/2} \sqrt[3]{x} (ax^{2/3} + b)}{65b^4 (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b \sqrt[3]{x}}} + \frac{154a^{13/4} \sqrt[6]{x}}{65b^4 (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b \sqrt[3]{x}}}$$

[Out] $(-154*a^{(7/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(65*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(13*b*x^{(7/3)}) + (22*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(39*b^2*x^{(5/3)}) - (154*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(195*b^3*x) + (154*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(65*b^4*x^{(1/3)}) + (154*a^{(13/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(65*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (77*a^{(13/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(65*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.475911, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$\frac{154a^{7/2} \sqrt[3]{x} (ax^{2/3} + b)}{65b^4 (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b \sqrt[3]{x}}} - \frac{77a^{13/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{65b^{15/4} \sqrt{ax + b \sqrt[3]{x}}} + \frac{154a^{13/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b})}{65b^4 (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{ax + b \sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]

[Out] $(-154*a^{(7/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(65*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(13*b*x^{(7/3)}) + (22*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(39*b^2*x^{(5/3)}) - (154*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(195*b^3*x) + (154*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(65*b^4*x^{(1/3)}) + (154*a^{(13/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(65*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (77*a^{(13/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(65*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

$$[(a^{1/4}x^{1/6})/b^{1/4}], 1/2]/(65b^{15/4}\sqrt{bx^{1/3} + ax})$$

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{b \sqrt[3]{x} + ax}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x^7 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} - \frac{(33a) \operatorname{Subst} \left(\int \frac{1}{x^5 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{13b} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} + \frac{(77a^2) \operatorname{Subst} \left(\int \frac{1}{x^3 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{39b^2} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} - \frac{(77a^3) \operatorname{Subst} \left(\int \frac{1}{x \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{65b^3} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} + \frac{154a^3 \sqrt{b \sqrt[3]{x} + ax}}{65b^4 \sqrt[3]{x}} - \frac{(77a^4) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b + ax^2/3}} dx, x, \sqrt[3]{x} \right)}{65b^4} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} + \frac{154a^3 \sqrt{b \sqrt[3]{x} + ax}}{65b^4 \sqrt[3]{x}} - \frac{(77a^4 \sqrt{b + ax^{2/3}}) \sqrt[6]{x}}{65b^4} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} + \frac{154a^3 \sqrt{b \sqrt[3]{x} + ax}}{65b^4 \sqrt[3]{x}} - \frac{(154a^4 \sqrt{b + ax^{2/3}}) \sqrt[9]{x}}{65b^4} \\
&= -\frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} + \frac{154a^3 \sqrt{b \sqrt[3]{x} + ax}}{65b^4 \sqrt[3]{x}} - \frac{(154a^{7/2} \sqrt{b + ax^{2/3}}) \sqrt[12]{x}}{65b^4} \\
&= -\frac{154a^{7/2} (b + ax^{2/3}) \sqrt[3]{x}}{65b^4 (\sqrt{b + \sqrt{a} \sqrt[3]{x}}) \sqrt{b \sqrt[3]{x} + ax}} - \frac{6 \sqrt{b \sqrt[3]{x} + ax}}{13bx^{7/3}} + \frac{22a \sqrt{b \sqrt[3]{x} + ax}}{39b^2 x^{5/3}} - \frac{154a^2 \sqrt{b \sqrt[3]{x} + ax}}{195b^3 x} + \frac{154a^3}{65b^4}
\end{aligned}$$

Mathematica [C] time = 0.049712, size = 59, normalized size = 0.15

$$\frac{6\sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1\left(-\frac{13}{4}, \frac{1}{2}; -\frac{9}{4}; -\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{ax + b}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]

[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/3))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.025, size = 363, normalized size = 0.9

$$-\frac{1}{195b^4} \left(462a^3b \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a\sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} x^{10/3} \sqrt{\sqrt[3]{x}(b + ax^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x)

[Out] -1/195*(462*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-231*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-462*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)*a^3*b+44*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^2+154*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b-20*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^3-462*x^4*(b*x^(1/3)+a*x)^(1/2)*a^4+90*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^4)/x^(11/3)/(b+a*x^(2/3))/b^4

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^2 x^2 - a b x^{\frac{4}{3}} + b^2 x^{\frac{2}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^6 + b^3 x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^6 + b^3*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{a x + b \sqrt[3]{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a x + b x^{\frac{1}{3}} x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

$$3.157 \quad \int \frac{1}{x^4 \sqrt{b \sqrt[3]{x+ax}}} dx$$

Optimal. Leaf size=251

$$\frac{663a^{19/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1463b^{21/4} \sqrt{ax + b \sqrt[3]{x}}} - \frac{1326a^4 \sqrt{ax + b \sqrt[3]{x}}}{1463b^5 x^{2/3}} + \frac{3978a^3 \sqrt{ax + b \sqrt[3]{x}}}{7315b^4 x^{4/3}} - \frac{442a^2 \sqrt{ax + b \sqrt[3]{x}}}{1045b^3 x^2}$$

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(19*b*x^{(10/3)}) + (34*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(95*b^2*x^{(8/3)}) - (442*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1045*b^3*x^2) + (3978*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7315*b^4*x^{(4/3)}) - (1326*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1463*b^5*x^{(2/3)}) - (663*a^{(19/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1463*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.345415, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2018, 2025, 2011, 329, 220}

$$-\frac{1326a^4 \sqrt{ax + b \sqrt[3]{x}}}{1463b^5 x^{2/3}} + \frac{3978a^3 \sqrt{ax + b \sqrt[3]{x}}}{7315b^4 x^{4/3}} - \frac{442a^2 \sqrt{ax + b \sqrt[3]{x}}}{1045b^3 x^2} - \frac{663a^{19/4} \sqrt[6]{x} (\sqrt{a} \sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a} \sqrt[3]{x} + \sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a} \sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{1463b^{21/4} \sqrt{ax + b \sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]

[Out] $(-6*\text{Sqrt}[b*x^{(1/3)} + a*x])/(19*b*x^{(10/3)}) + (34*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(95*b^2*x^{(8/3)}) - (442*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1045*b^3*x^2) + (3978*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7315*b^4*x^{(4/3)}) - (1326*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1463*b^5*x^{(2/3)}) - (663*a^{(19/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(1463*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]

, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2025

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{b\sqrt[3]{x} + ax}} dx &= 3 \text{Subst} \left(\int \frac{1}{x^{10} \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} - \frac{(51a) \text{Subst} \left(\int \frac{1}{x^8 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{19b} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} + \frac{(221a^2) \text{Subst} \left(\int \frac{1}{x^6 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{95b^2} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x} + ax}}{1045b^3x^2} - \frac{(1989a^3) \text{Subst} \left(\int \frac{1}{x^4 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1045b^3} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x} + ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x} + ax}}{7315b^4x^{4/3}} + \frac{(1989a^4) \text{Subst} \left(\int \frac{1}{x^2 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{1463b^5x^{2/3}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x} + ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x} + ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x} + ax}}{1463b^5x^{2/3}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x} + ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x} + ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x} + ax}}{1463b^5x^{2/3}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x} + ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x} + ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x} + ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x} + ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x} + ax}}{1463b^5x^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.056562, size = 59, normalized size = 0.24

$$\frac{6\sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1\left(-\frac{19}{4}, \frac{1}{2}; -\frac{15}{4}; -\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]

[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -(a*x^(2/3))/b])/(19*x^3*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.026, size = 179, normalized size = 0.7

$$-\frac{1}{7315 b^5} \left(3315 a^4 \sqrt{-ab} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-2 \frac{a \sqrt[3]{x} - \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) x^{16/3} + 2652 x^{16/3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^(1/3)+a*x)^(1/2), x)`

[Out] `-1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*x^(16/3)+2652*x^5*a^4*b+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^(13/3)*a^3*b^2-308*x^3*a*b^4+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(16/3)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\left(a^2 x^2 - a b x^{\frac{4}{3}} + b^2 x^{\frac{2}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^3 x^7 + b^3 x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^7 + b^3*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

$$3.158 \quad \int \frac{x^4}{(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=437

$$\frac{4807b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{4807b^5\sqrt[3]{x}(ax^{2/3}+b)}{221a^{13/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4}$$

[Out] $(-4807*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(13/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (3*x^4)/(a*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4807*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^6) - (24035*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^5) + (6555*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^4) - (437*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a^3) + (23*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*a^2) + (4807*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(442*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.673587, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4807b^5\sqrt[3]{x}(ax^{2/3}+b)}{221a^{13/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4} - \frac{4807b^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(b*x^(1/3) + a*x)^(3/2), x]

[Out] $(-4807*b^5*(b + a*x^{(2/3)})*x^{(1/3)})/(221*a^{(13/2)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (3*x^4)/(a*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (4807*b^4*x^{(1/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*a^6) - (24035*b^3*x*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*a^5) + (6555*b^2*x^{(5/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*a^4) - (437*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*a^3) + (23*x^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*a^2) + (4807*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(442*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

$$\begin{aligned} & (\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*b^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(442*a^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) \end{aligned}$$

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 305

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^{14}}{(bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{69 \operatorname{Subst} \left(\int \frac{x^{11}}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2a} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{(437b) \operatorname{Subst} \left(\int \frac{x^9}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{14a^2} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} + \frac{(6555b^2) \operatorname{Subst} \left(\int \frac{x^7}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{(72105b^3)}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{72105b^3}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{72105b^3}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{72105b^3}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{72105b^3}{238a^3} \\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x} + ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x} + ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x} + ax}}{7a^2} - \frac{72105b^3}{238a^3} \\
&= -\frac{4807b^5(b + ax^{2/3})\sqrt[3]{x}}{221a^{13/2}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{3x^4}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x} + ax}}{4641a^5}
\end{aligned}$$

Mathematica [C] time = 0.0933211, size = 131, normalized size = 0.3

$$\frac{2x^{2/3} \left(1311a^3b^2x^2 - 2185a^2b^3x^{4/3} - 897a^4bx^{8/3} + 663a^5x^{10/3} + 33649b^5\sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{ax^{2/3}}{b}\right) + 4807ab^4x^{2/3} - 3 \right)}{4641a^6\sqrt{ax + b}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(b*x^(1/3) + a*x)^(3/2), x]

[Out] (2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*a^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^6*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.026, size = 384, normalized size = 0.9

$$-\frac{1}{9282a^7} \left(-5244x^{8/3}\sqrt[3]{x(b+ax^{2/3})}a^4b^2 + 3588x^{10/3}\sqrt[3]{x(b+ax^{2/3})}a^5b + 8740x^2\sqrt[3]{x(b+ax^{2/3})}a^3b^3 + 201894b^6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^(1/3)+a*x)^(3/2), x)

[Out] -1/9282/a^7*(-5244*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b^2+3588*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b+8740*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b^3+201894*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2)*(-x^(1/3)*a/(-a*b)^(1/2)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2), 1/2*2^(1/2))-100947*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2)*(-x^(1/3)*a/(-a*b)^(1/2)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^((1/2), 1/2*2^(1/2))-2652*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^4*a^6-39452*x^(2/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-27846*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)*a*b^5-19228*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4)/x^(1/3)/(b+a*x^(2/3))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^6 + 3 a^2 b^2 x^{\frac{14}{3}} - 2 a b^3 x^4 - (2 a^3 b x^5 - b^4 x^3) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^4 + 2 a^3 b^3 x^2 + b^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*x^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

$$3.159 \quad \int \frac{x^3}{(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=239

$$\frac{663b^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4} - \frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5} - \frac{221bx^{4/3}}{55a^3}$$

[Out] $(-3*x^3)/(a*\text{Sqrt}[b*x^{1/3} + a*x]) - (663*b^3*\text{Sqrt}[b*x^{1/3} + a*x])/(77*a^5) + (1989*b^2*x^{2/3}*\text{Sqrt}[b*x^{1/3} + a*x])/(385*a^4) - (221*b*x^{4/3}*\text{Sqrt}[b*x^{1/3} + a*x])/(55*a^3) + (17*x^2*\text{Sqrt}[b*x^{1/3} + a*x])/(5*a^2) + (663*b^{15/4}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})*\text{Sqrt}[(b + a*x^{2/3})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})^2]*x^{1/6}*\text{EllipticF}[2*\text{ArcTan}[(a^{1/4})*x^{1/6}]/b^{1/4}], 1/2)]/(154*a^{21/4}*\text{Sqrt}[b*x^{1/3} + a*x])$

Rubi [A] time = 0.377479, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4} + \frac{663b^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5} - \frac{221bx^{4/3}}{55a^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b*x^(1/3) + a*x)^(3/2), x]

[Out] $(-3*x^3)/(a*\text{Sqrt}[b*x^{1/3} + a*x]) - (663*b^3*\text{Sqrt}[b*x^{1/3} + a*x])/(77*a^5) + (1989*b^2*x^{2/3}*\text{Sqrt}[b*x^{1/3} + a*x])/(385*a^4) - (221*b*x^{4/3}*\text{Sqrt}[b*x^{1/3} + a*x])/(55*a^3) + (17*x^2*\text{Sqrt}[b*x^{1/3} + a*x])/(5*a^2) + (663*b^{15/4}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})*\text{Sqrt}[(b + a*x^{2/3})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{1/3})^2]*x^{1/6}*\text{EllipticF}[2*\text{ArcTan}[(a^{1/4})*x^{1/6}]/b^{1/4}], 1/2)]/(154*a^{21/4}*\text{Sqrt}[b*x^{1/3} + a*x])$

Rule 2018

Int[(x_)^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]

, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2022

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^{11}}{(bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{51 \operatorname{Subst} \left(\int \frac{x^8}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2a} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} - \frac{(221b) \operatorname{Subst} \left(\int \frac{x^6}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{10a^2} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} + \frac{(1989b^2) \operatorname{Subst} \left(\int \frac{x^4}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{110a^3} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} - \frac{(1989b^3) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{110a^3} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x} + ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x} + ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x} + ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x} + ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x} + ax}}{5a^2}
\end{aligned}$$

Mathematica [C] time = 0.0826685, size = 124, normalized size = 0.52

$$\frac{\sqrt{ax + b\sqrt[3]{x}} \left(442a^2b^2x^{4/3} - 238a^3bx^2 + 154a^4x^{8/3} + 3315b^4\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{ax^{2/3}}{b} \right) - 1326ab^3x^{2/3} - 3315b^4 \right)}{385a^5(ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]

[Out] (Sqrt[b*x^(1/3) + a*x]*(-3315*b^4 - 1326*a*b^3*x^(2/3) + 442*a^2*b^2*x^(4/3) - 238*a^3*b*x^2 + 154*a^4*x^(8/3) + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^5*(b + a*x^(2/3)))

Maple [A] time = 0.024, size = 261, normalized size = 1.1

$$\frac{1}{770 a^6} \left(3315 \sqrt{-ab} \sqrt{\sqrt[3]{x} (b + ax^{2/3})} \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^(1/3)+a*x)^(3/2),x)

[Out] 1/770*(3315*(-a*b)^(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))^(-1/2))/(-a*b)^(1/2))^2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(-1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(-1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(-1/2), 1/2*2^(1/2))*b^4+884*x^(5/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b^2-476*x^(7/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b-2652*x*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^3+308*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^3*a^5-4320*x^(1/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^4-2310*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)*a*b^4)/x^(1/3)/(b+a*x^(2/3))/a^6

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^5 + 3 a^2 b^2 x^{\frac{11}{3}} - 2 a b^3 x^3 - (2 a^3 b x^4 - b^4 x^2) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^4 + 2 a^3 b^3 x^2 + b^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*x^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax + b\sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)

$$3.160 \quad \int \frac{x^2}{(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=349

$$\frac{77b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{77b^2\sqrt[3]{x}(ax^{2/3}+b)}{5a^{7/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} - \frac{77b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x] - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]/(5*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.426236, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$\frac{77b^2\sqrt[3]{x}(ax^{2/3}+b)}{5a^{7/2}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{77b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{77b^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(b*x^(1/3) + a*x)^(3/2), x]

[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x] - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]/(5*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))]^2)*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol
] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol
] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```


$b*x^4], x], x]] /; FreeQ[\{a, b\}, x] \&\& PosQ[b/a]$

Rule 220

$Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[\{q = Rt[b/a, 4]\}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[\{a, b\}, x] \&\& PosQ[b/a]$

Rule 1196

$Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[\{q = Rt[c/a, 4]\}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[\{a, c, d, e\}, x] \&\& PosQ[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \text{Subst} \left(\int \frac{x^8}{(bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{33 \text{Subst} \left(\int \frac{x^5}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2a} \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} - \frac{(77b) \text{Subst} \left(\int \frac{x^3}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{6a^2} \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} + \frac{(77b^2) \text{Subst} \left(\int \frac{x}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{10a^3} \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} + \frac{(77b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst} \left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}} dx \right)}{10a^3\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} + \frac{(77b^2\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst} \left(\int \frac{x^2}{\sqrt{b+ax^4}} dx \right)}{5a^3\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} + \frac{(77b^{5/2}\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \text{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx \right)}{5a^{7/2}\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{77b^2(b + ax^{2/3})\sqrt[3]{x}}{5a^{7/2}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} - \frac{3x^2}{a\sqrt{b\sqrt[3]{x} + ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x} + ax}}{3a^2} - \frac{77b^{9/4}}{5a^{7/2}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0674355, size = 94, normalized size = 0.27

$$\frac{2x^{2/3} \left(5a^2x^{4/3} - 77b^2\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1 \left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{ax^{2/3}}{b} \right) - 11abx^{2/3} + 77b^2 \right)}{15a^3\sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(b*x^(1/3) + a*x)^(3/2), x]

[Out] $(2x^{2/3})(77b^2 - 11abx^{2/3} + 5a^2x^{4/3} - 77b^2\sqrt{1 + (ax^{2/3})/b})\text{Hypergeometric2F1}[3/4, 3/2, 7/4, -((ax^{2/3})/b)]/(15a^3\sqrt{bx^{1/3} + ax})$

Maple [A] time = 0.011, size = 314, normalized size = 0.9

$$-\frac{1}{30a^4} \left(-462b^3 \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \sqrt{\sqrt[3]{x}(b + ax^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^(1/3)+a*x)^(3/2), x)`

[Out] $-1/30/a^4 * (-462*b^3 * ((a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2} * 2^{1/2} * ((-a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a*b)^{1/2})^{1/2} * (x^{1/3} * (b + a*x^{2/3}))^{1/2} * \text{EllipticE}(((a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2}, 1/2 * 2^{1/2}) + 231 * b^3 * ((a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2} * 2^{1/2} * ((-a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a*b)^{1/2})^{1/2} * (x^{1/3} * (b + a*x^{2/3}))^{1/2} * \text{EllipticF}(((a*x^{1/3}) + (-a*b)^{1/2}) / (-a*b)^{1/2})^{1/2}, 1/2 * 2^{1/2}) + 90 * (b*x^{1/3} + a*x)^{1/2} * x^{2/3} * a*b^2 + 64*x^{2/3} * (x^{1/3} * (b + a*x^{2/3}))^{1/2} * a*b^2 + 44*x^{4/3} * (x^{1/3} * (b + a*x^{2/3}))^{1/2} * a^2 * b - 20 * (x^{1/3} * (b + a*x^{2/3}))^{1/2} * x^2 * a^3 / x^{1/3} / (b + a*x^{2/3})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax + bx^{1/3})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(1/3)+a*x)^(3/2), x, algorithm="maxima")`

[Out] `integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^4 + 3 a^2 b^2 x^{\frac{8}{3}} - 2 a b^3 x^2 - (2 a^3 b x^3 - b^4 x) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^4 + 2 a^3 b^3 x^2 + b^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax + b\sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + bx^{\frac{1}{3}} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)

$$3.161 \quad \int \frac{x}{(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=149

$$\frac{5b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2} - \frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $(-3*x)/(a*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (5*\text{Sqrt}[b*x^{(1/3)} + a*x])/a^2 - (5*b^{(3/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(2*a^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.208698, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$\frac{5b^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2} - \frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(b*x^{(1/3)} + a*x)^{(3/2)}, x]$

[Out] $(-3*x)/(a*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (5*\text{Sqrt}[b*x^{(1/3)} + a*x])/a^2 - (5*b^{(3/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(2*a^{(9/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

$\text{Int}[(x_)^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a*x^{\text{Simplify}[j/n]} + b*x)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, j, m, n, p\}, x \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{IntegerQ}[\text{Simplify}[j/n]] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& \text{NeQ}[n^2, 1]$

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*(p + 1)), x]
- Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] && GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x]
- Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x]
/; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]
/; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^5}{(bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{15 \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2a} \\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{5\sqrt{b\sqrt[3]{x} + ax}}{a^2} - \frac{(5b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2a^2} \\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{5\sqrt{b\sqrt[3]{x} + ax}}{a^2} - \frac{(5b\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{2a^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{5\sqrt{b\sqrt[3]{x} + ax}}{a^2} - \frac{(5b\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{a^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x} + ax}} + \frac{5\sqrt{b\sqrt[3]{x} + ax}}{a^2} - \frac{5b^{3/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}\sqrt[3]{x})^2}}\sqrt[6]{x}F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0632735, size = 82, normalized size = 0.55

$$\frac{\sqrt{ax + b\sqrt[3]{x}} \left(-5b\sqrt{\frac{ax^{2/3}}{b}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{ax^{2/3}}{b}\right) + 2ax^{2/3} + 5b \right)}{a^2(ax^{2/3} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b*x^(1/3) + a*x)^(3/2), x]

[Out] (Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b])*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)])/(a^2*(b + a*x^(2/3)))

Maple [A] time = 0.01, size = 185, normalized size = 1.2

$$\frac{1}{2a^3} \left(-5\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2}\sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2, \sqrt{2}\right) \sqrt{-ab}\sqrt{\sqrt[3]{x}(b + ax^{2/3})} + 6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*x^(1/3)+a*x)^(3/2),x)`

[Out] $\frac{1}{2} * (-5 * ((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * 2^{1/2} * ((-a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2} * (-x^{1/3} * a / (-a * b)^{1/2})^{1/2} * \text{EllipticF}(((a * x^{1/3} + (-a * b)^{1/2}) / (-a * b)^{1/2})^{1/2}, 1/2 * 2^{1/2})) * (-a * b)^{1/2} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * b + 6 * (b * x^{1/3} + a * x)^{1/2} * x^{1/3} * a * b + 4 * x^{1/3} * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * a * b + 4 * (x^{1/3} * (b + a * x^{2/3}))^{1/2} * x * a^2) / x^{1/3} / (b + a * x^{2/3}) / a^3$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x/(a*x + b*x^(1/3))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^3 + 3 a^2 b^2 x^{\frac{5}{3}} - 2 a b^3 x - (2 a^3 b x^2 - b^4) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^4 + 2 a^3 b^3 x^2 + b^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax + b\sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(x/(a*x + b*x**(1/3))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)

$$3.162 \quad \int \frac{1}{(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=296

$$\frac{3\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{3\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right)}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $(-3*(b + a*x^{(2/3)})*x^{(1/3)})/(\text{Sqrt}[a]*b*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (3*x^{(2/3)})/(b*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (3*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (a^{(3/4)}*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (3*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2))/(2*a^{(3/4)}*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.25996, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2006, 2018, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{3\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{3\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b}) \sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1/3) + a*x)^(-3/2), x]

[Out] $(-3*(b + a*x^{(2/3)})*x^{(1/3)})/(\text{Sqrt}[a]*b*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (3*x^{(2/3)})/(b*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (3*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/ (a^{(3/4)}*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (3*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2))/(2*a^{(3/4)}*b^{(3/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(a*x^j +
  b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2032

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
```

x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(b\sqrt[3]{x} + ax)^{3/2}} dx &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} - \frac{\int \frac{1}{\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} dx}{2b} \\
 &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} - \frac{3 \operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x}\right)}{2b} \\
 &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}} dx, x, \sqrt[3]{x}\right)}{2b\sqrt{b\sqrt[3]{x} + ax}} \\
 &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right) \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x} + ax}} \\
 &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}} + \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right) \operatorname{Subst}\left(\int \frac{1-\sqrt{ax}}{\sqrt{b+ax}} dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x} + ax}} \\
 &= -\frac{3(b+ax^{2/3})\sqrt[3]{x}}{\sqrt{ab}(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{b\sqrt[3]{x} + ax}} + \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x} + ax}} + \frac{3(\sqrt{b} + \sqrt{a}\sqrt[3]{x})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}}\sqrt[6]{x}E\left(2 \tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{b+ax}}\right)\right)}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x} + ax}}
 \end{aligned}$$

Mathematica [C] time = 0.0291438, size = 62, normalized size = 0.21

$$\frac{2x^{2/3}\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{ax^{2/3}}{b}\right)}{b\sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1/3) + a*x)^(-3/2), x]

[Out] (2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.005, size = 245, normalized size = 0.8

$$-\frac{3}{2ab} \left(2\sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2} \right) \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\sqrt[3]{x}(b+ax^{2/3})} b - \sqrt{2} \sqrt{\left(\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^(1/3)+a*x)^(3/2),x)`

[Out]
$$-3/2/a*(2*2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*\operatorname{EllipticE}(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}))*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*b-2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*\operatorname{EllipticF}(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}))*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*b-2*x^{(2/3)}*(b*x^{(1/3)}+a*x)^{(1/2)}*a/x^{(1/3)}/(b+a*x^{(2/3)})/b$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(1/3))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\left(a^4 x^3 + 3 a^2 b^2 x^{\frac{5}{3}} - 2 a b^3 x - \left(2 a^3 b x^2 - b^4 \right) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^5 + 2 a^3 b^3 x^3 + b^6 x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + b\sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral((a*x + b*x**(1/3))**(-3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate((a*x + b*x^(1/3))^(3/2), x)

$$3.163 \quad \int \frac{1}{x(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=158

$$\frac{5a^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}} + \frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])

Rubi [A] time = 0.205232, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$\frac{5a^{3/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}} + \frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]

[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2023

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x(bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} + \frac{15 \operatorname{Subst} \left(\int \frac{1}{x^2\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2b} \\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} - \frac{5\sqrt{b\sqrt[3]{x} + ax}}{b^2x^{2/3}} - \frac{(5a) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2b^2} \\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} - \frac{5\sqrt{b\sqrt[3]{x} + ax}}{b^2x^{2/3}} - \frac{(5a\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}} dx, x, \sqrt[3]{x} \right)}{2b^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} - \frac{5\sqrt{b\sqrt[3]{x} + ax}}{b^2x^{2/3}} - \frac{(5a\sqrt{b + ax^{2/3}}\sqrt[6]{x}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^4}} dx, x, \sqrt[6]{x} \right)}{b^2\sqrt{b\sqrt[3]{x} + ax}} \\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x} + ax}} - \frac{5\sqrt{b\sqrt[3]{x} + ax}}{b^2x^{2/3}} - \frac{5a^{3/4}(\sqrt{b} + \sqrt{a}\sqrt[3]{x}) \sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}\sqrt[3]{x})^2}} \sqrt[6]{x} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right) \middle| \frac{1}{2}\right)}{2b^{9/4}\sqrt{b\sqrt[3]{x} + ax}}
\end{aligned}$$

Mathematica [C] time = 0.0597793, size = 62, normalized size = 0.39

$$\frac{2\sqrt{\frac{ax^{2/3}}{b}} + {}_1F_1\left(-\frac{3}{4}, \frac{3}{2}; \frac{1}{4}; -\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)), x]

[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -(a*x^(2/3))/b])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.012, size = 181, normalized size = 1.2

$$-\frac{1}{2b^2x} \left(5 \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} \operatorname{EllipticF} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2, \sqrt{2} \right) \sqrt{-abx^{2/3}} \sqrt{\sqrt[3]{x}(b + ax^{2/3})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^(1/3)+a*x)^(3/2),x)`

[Out]
$$-1/2*5*((a*x^{1/3}+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*2^{1/2}*((-a*x^{1/3}+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x^{1/3}*a/(-a*b)^{1/2})^{1/2}*EllipticF(((a*x^{1/3}+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2},1/2*2^{1/2})*(-a*b)^{1/2}*x^{2/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}+6*(b*x^{1/3}+a*x)^{1/2}*x*a+4*x^{1/3}*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*b+4*(x^{1/3}*(b+a*x^{2/3}))^{1/2}*x*a)/b^2/x/(b+a*x^{2/3})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^3 + 3a^2b^2x^{\frac{5}{3}} - 2ab^3x - (2a^3bx^2 - b^4)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^6 + 2a^3b^3x^4 + b^6x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x (ax + b\sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + bx^{\frac{1}{3}})^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)

$$3.164 \quad \int \frac{1}{x^2(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=383

$$\frac{77a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{77a^{5/2}\sqrt[3]{x}(ax^{2/3}+b)}{5b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} - \frac{77a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{5b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $3/(b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (77*a^{(5/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(5*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (11*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3*b^2*x^{(5/3)}) + (77*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(15*b^3*x) - (77*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b^4*x^{(1/3)}) - (77*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)}))*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2]/(5*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (77*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2]/(10*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.478506, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{77a^{5/2}\sqrt[3]{x}(ax^{2/3}+b)}{5b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{77a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{77a^{9/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})}{5b^4(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]

[Out] $3/(b*x^{(4/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (77*a^{(5/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(5*b^4*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (11*\text{Sqrt}[b*x^{(1/3)} + a*x])/(3*b^2*x^{(5/3)}) + (77*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(15*b^3*x) - (77*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b^4*x^{(1/3)}) - (77*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)}))*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2]/(5*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) + (77*a^{(9/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2]/(10*b^{(15/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

$(1/4)], 1/2)]/(10*b^{(15/4)*Sqrt[b*x^{(1/3)} + a*x])$

Rule 2018

`Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]`

Rule 2023

`Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]`

Rule 2025

`Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]`

Rule 2032

`Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]`

Rule 329

`Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]`

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x^4 (bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} + \frac{33 \operatorname{Subst} \left(\int \frac{1}{x^5 \sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{2b} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} - \frac{(77a) \operatorname{Subst} \left(\int \frac{1}{x^3 \sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{6b^2} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} + \frac{(77a^2) \operatorname{Subst} \left(\int \frac{1}{x \sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{10b^3} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} - \frac{77a^2 \sqrt{b\sqrt[3]{x} + ax}}{5b^4 \sqrt[3]{x}} + \frac{(77a^3) \operatorname{Subst} \left(\int \frac{1}{\sqrt{bx+ax^3}} dx, x, \sqrt[3]{x} \right)}{10b^3} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} - \frac{77a^2 \sqrt{b\sqrt[3]{x} + ax}}{5b^4 \sqrt[3]{x}} + \frac{(77a^3 \sqrt{b + ax^{2/3}} \sqrt[3]{x})}{10b^3} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} - \frac{77a^2 \sqrt{b\sqrt[3]{x} + ax}}{5b^4 \sqrt[3]{x}} + \frac{(77a^3 \sqrt{b + ax^{2/3}} \sqrt[3]{x})}{5b^3} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} - \frac{77a^2 \sqrt{b\sqrt[3]{x} + ax}}{5b^4 \sqrt[3]{x}} + \frac{(77a^{5/2} \sqrt{b + ax^{2/3}} \sqrt[3]{x})}{5b^3} \\
&= \frac{3}{bx^{4/3} \sqrt{b\sqrt[3]{x} + ax}} + \frac{77a^{5/2} (b + ax^{2/3}) \sqrt[3]{x}}{5b^4 (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} - \frac{11\sqrt{b\sqrt[3]{x} + ax}}{3b^2 x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x} + ax}}{15b^3 x} - \frac{77a^2 \sqrt{b\sqrt[3]{x} + ax}}{5b^4 \sqrt[3]{x}}
\end{aligned}$$

Mathematica [C] time = 0.05572777, size = 64, normalized size = 0.17

$$\frac{2\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1 \left(-\frac{9}{4}, \frac{3}{2}; -\frac{5}{4}; -\frac{ax^{2/3}}{b} \right)}{3bx^{4/3} \sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]

[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3))/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.011, size = 341, normalized size = 0.9

$$-\frac{1}{30x^3b^4} \left(-462a^2b \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-a\sqrt[3]{x}}{\sqrt{-ab}}} x^{8/3} \sqrt{\sqrt[3]{x}(b + ax^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x)

[Out] -1/30*(-462*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+231*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)*a^3+372*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)*a^2*b-44*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^2-64*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b+20*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^3)/x^3/(b+a*x^(2/3))/b^4

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^3 + 3 a^2 b^2 x^{\frac{5}{3}} - 2 a b^3 x - (2 a^3 b x^2 - b^4) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^7 + 2 a^3 b^3 x^5 + b^6 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a x + b \sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a x + b x^{\frac{1}{3}})^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)

$$3.165 \quad \int \frac{1}{x^3(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=246

$$\frac{663a^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}} - \frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}} + \frac{221a\sqrt{ax+b\sqrt[3]{x}}}{55b^3x^2}$$

[Out] $3/(b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (17*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b^2*x^{(8/3)}) + (221*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*b^3*x^2) - (1989*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*b^4*x^{(4/3)}) + (663*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^5*x^{(2/3)}) + (663*a^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})]/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2)*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(154*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.364747, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}} - \frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}} + \frac{663a^{15/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{F}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{221a\sqrt{ax+b\sqrt[3]{x}}}{55b^3x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)), x]

[Out] $3/(b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (17*\text{Sqrt}[b*x^{(1/3)} + a*x])/(5*b^2*x^{(8/3)}) + (221*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(55*b^3*x^2) - (1989*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(385*b^4*x^{(4/3)}) + (663*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(77*b^5*x^{(2/3)}) + (663*a^{(15/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})]/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2)*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2)]/(154*b^{(21/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rule 2018

Int[(x_)^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]

, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2023

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]

Rule 2025

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x^7 (bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} + \frac{51 \operatorname{Subst} \left(\int \frac{1}{x^8 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{2b} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} - \frac{(221a) \operatorname{Subst} \left(\int \frac{1}{x^6 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{10b^2} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} + \frac{(1989a^2) \operatorname{Subst} \left(\int \frac{1}{x^4 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{110b^3} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} - \frac{1989a^2 \sqrt{b\sqrt[3]{x} + ax}}{385b^4 x^{4/3}} - \frac{(1989a^3) \operatorname{Subst} \left(\int \frac{1}{x^2 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{110b^3} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} - \frac{1989a^2 \sqrt{b\sqrt[3]{x} + ax}}{385b^4 x^{4/3}} + \frac{663a^3 \sqrt{b\sqrt[3]{x} + ax}}{77b^5 x^{2/3}} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} - \frac{1989a^2 \sqrt{b\sqrt[3]{x} + ax}}{385b^4 x^{4/3}} + \frac{663a^3 \sqrt{b\sqrt[3]{x} + ax}}{77b^5 x^{2/3}} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} - \frac{1989a^2 \sqrt{b\sqrt[3]{x} + ax}}{385b^4 x^{4/3}} + \frac{663a^3 \sqrt{b\sqrt[3]{x} + ax}}{77b^5 x^{2/3}} \\
&= \frac{3}{bx^{7/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{17 \sqrt{b\sqrt[3]{x} + ax}}{5b^2 x^{8/3}} + \frac{221a \sqrt{b\sqrt[3]{x} + ax}}{55b^3 x^2} - \frac{1989a^2 \sqrt{b\sqrt[3]{x} + ax}}{385b^4 x^{4/3}} + \frac{663a^3 \sqrt{b\sqrt[3]{x} + ax}}{77b^5 x^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0561036, size = 64, normalized size = 0.26

$$\frac{2\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1\left(-\frac{15}{4}, \frac{3}{2}; -\frac{11}{4}; -\frac{ax^{2/3}}{b}\right)}{5bx^{7/3} \sqrt{ax + b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]

[Out] $(-2*\text{Sqrt}[1 + (a*x^{(2/3)})/b]*\text{Hypergeometric2F1}[-15/4, 3/2, -11/4, -((a*x^{(2/3)})/b)])/(5*b*x^{(7/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Maple [A] time = 0.014, size = 262, normalized size = 1.1

$$\frac{1}{770 b^5 x^5} \left(3315 \sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a \sqrt[3]{x}}{\sqrt{-ab}}} \text{EllipticF} \left(\sqrt{\frac{a \sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{-ab} x^{14/3} \sqrt{\sqrt[3]{x} (b + a x^{1/3})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x)

[Out] $1/770*(3315*((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*2^{(1/2)}*((-a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x^{(1/3)}*a/(-a*b)^{(1/2)})^{(1/2)}*\text{EllipticF}(((a*x^{(1/3)}+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})*(-a*b)^{(1/2)}*x^{(14/3)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*a^3-884*x^{(11/3)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*a^2*b^2+2652*x^{(13/3)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*a^3*b+476*x^3*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*a*b^3+2310*(b*x^{(1/3)}+a*x)^{(1/2)}*x^5*a^4+4320*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*x^5*a^4-308*x^{(7/3)}*(x^{(1/3)}*(b+a*x^{(2/3)}))^{(1/2)}*b^4)/b^5/x^5/(b+a*x^{(2/3)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\left(a^4 x^3 + 3 a^2 b^2 x^{\frac{5}{3}} - 2 a b^3 x - (2 a^3 b x^2 - b^4) x^{\frac{1}{3}} \right) \sqrt{a x + b x^{\frac{1}{3}}}}{a^6 x^8 + 2 a^3 b^3 x^6 + b^6 x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a x + b \sqrt[3]{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a x + b x^{\frac{1}{3}} \right)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)

$$3.166 \quad \int \frac{1}{x^4(b\sqrt[3]{x+ax})^{3/2}} dx$$

Optimal. Leaf size=471

$$\frac{4807a^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}+b}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right), \frac{1}{2}\right)}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{4807a^{11/2}\sqrt[3]{x}(ax^{2/3}+b)}{221b^7(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}} - \frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}} - \frac{4807a^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

[Out] $3/(b*x^{(10/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*a^{(11/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(221*b^7*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (23*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*b^2*x^{(11/3)}) + (437*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*b^3*x^3) - (6555*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*b^4*x^{(7/3)}) + (24035*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*b^5*x^{(5/3)}) - (4807*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/(663*b^6*x) + (4807*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(221*b^7*x^{(1/3)}) + (4807*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*b^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(442*b^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$

Rubi [A] time = 0.693825, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4807a^{11/2}\sqrt[3]{x}(ax^{2/3}+b)}{221b^7(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{ax+b\sqrt[3]{x}}} + \frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}} - \frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}} - \frac{4807a^{21/4}\sqrt[6]{x}(\sqrt{a}\sqrt[3]{x} + \sqrt{b})\sqrt{\frac{ax^{2/3}}{(\sqrt{a}\sqrt[3]{x}+\sqrt{b})^2}}}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)), x]

[Out] $3/(b*x^{(10/3)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*a^{(11/2)}*(b + a*x^{(2/3)})*x^{(1/3)})/(221*b^7*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (23*\text{Sqrt}[b*x^{(1/3)} + a*x])/(7*b^2*x^{(11/3)}) + (437*a*\text{Sqrt}[b*x^{(1/3)} + a*x])/(119*b^3*x^3) - (6555*a^2*\text{Sqrt}[b*x^{(1/3)} + a*x])/(1547*b^4*x^{(7/3)}) + (24035*a^3*\text{Sqrt}[b*x^{(1/3)} + a*x])/(4641*b^5*x^{(5/3)}) - (4807*a^4*\text{Sqrt}[b*x^{(1/3)} + a*x])/($

$$(663*b^6*x) + (4807*a^5*\text{Sqrt}[b*x^{(1/3)} + a*x])/(221*b^7*x^{(1/3)}) + (4807*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticE}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(221*b^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x]) - (4807*a^{(21/4)}*(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})*\text{Sqrt}[(b + a*x^{(2/3)})/(\text{Sqrt}[b] + \text{Sqrt}[a]*x^{(1/3)})^2]*x^{(1/6)}*\text{EllipticF}[2*\text{ArcTan}[(a^{(1/4)}*x^{(1/6)})/b^{(1/4)}], 1/2])/(442*b^{(27/4)}*\text{Sqrt}[b*x^{(1/3)} + a*x])$$

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
```

Rule 2025

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)
*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:= Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m]
+ j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)
*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
```


$n)^p, x], x, (c*x)^{(1/k)], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{FractionQ}[m] \ \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 305

$\text{Int}[(x_)^2/\text{Sqrt}[(a_)+(b_)*(x_)^4], x_Symbol] :> \text{With}[\{q = \text{Rt}[b/a, 2]\}, \text{Dist}[1/q, \text{Int}[1/\text{Sqrt}[a + b*x^4], x], x] - \text{Dist}[1/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + b*x^4], x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \text{PosQ}[b/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_)+(b_)*(x_)^4], x_Symbol] :> \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[((d_)+(e_)*(x_)^2)/\text{Sqrt}[(a_)+(c_)*(x_)^4], x_Symbol] :> \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2]]/(q*\text{Sqrt}[a + c*x^4]), x]] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (b\sqrt[3]{x} + ax)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{1}{x^{10} (bx + ax^3)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} + \frac{69 \operatorname{Subst} \left(\int \frac{1}{x^{11} \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{2b} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} - \frac{(437a) \operatorname{Subst} \left(\int \frac{1}{x^9 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{14b^2} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} + \frac{(6555a^2) \operatorname{Subst} \left(\int \frac{1}{x^7 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{238b^3} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} - \frac{(72105a^3) \operatorname{Subst} \left(\int \frac{1}{x^5 \sqrt{bx + ax^3}} dx, x, \sqrt[3]{x} \right)}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}} \\
&= \frac{3}{bx^{10/3} \sqrt{b\sqrt[3]{x} + ax}} - \frac{4807a^{11/2} (b + ax^{2/3}) \sqrt[3]{x}}{221b^7 (\sqrt{b} + \sqrt{a} \sqrt[3]{x}) \sqrt{b\sqrt[3]{x} + ax}} - \frac{23 \sqrt{b\sqrt[3]{x} + ax}}{7b^2 x^{11/3}} + \frac{437a \sqrt{b\sqrt[3]{x} + ax}}{119b^3 x^3} - \frac{6555a^2 \sqrt{b\sqrt[3]{x} + ax}}{1547b^4 x^{7/3}} + \frac{24035a^3 \sqrt{b\sqrt[3]{x} + ax}}{4641b^5 x^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.0547918, size = 64, normalized size = 0.14

$$\frac{2\sqrt{\frac{ax^{2/3}}{b}} + 1 {}_2F_1\left(-\frac{21}{4}, \frac{3}{2}; -\frac{17}{4}; -\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{ax + b}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)), x]

[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2/3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])

Maple [A] time = 0.013, size = 413, normalized size = 0.9

$$\frac{1}{9282 x^7 b^7} \left(-201894 a^5 b \sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{2} \sqrt{\frac{-a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{a\sqrt[3]{x}}{\sqrt{-ab}}} x^{\frac{20}{3}} \sqrt{\sqrt[3]{x} (b + ax^{2/3})} \text{EllipticE} \left(\sqrt{\frac{a\sqrt[3]{x} + \sqrt{-ab}}{\sqrt{-ab}}}, \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^(1/3)+a*x)^(3/2), x)

[Out] 1/9282*(-201894*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))+100947*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))+201894*x^(22/3)*(b*x^(1/3)+a*x)^(1/2)*a^6-19228*x^6*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b^2-39452*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b+174048*x^(20/3)*(b*x^(1/3)+a*x)^(1/2)*a^5*b+3588*x^4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-5244*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4+8740*x^(16/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b^3-2652*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^6/x^7/(b+a*x^(2/3))/b^7

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^3 + 3a^2b^2x^{\frac{5}{3}} - 2ab^3x - (2a^3bx^2 - b^4)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^9 + 2a^3b^3x^7 + b^6x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

3.167 $\int x^3 \sqrt{bx^{2/3} + ax} dx$

Optimal. Leaf size=371

$$\frac{8388608b^{12}(ax + bx^{2/3})^{3/2}}{152108775a^{13}x} - \frac{4194304b^{11}(ax + bx^{2/3})^{3/2}}{50702925a^{12}x^{2/3}} + \frac{1048576b^{10}(ax + bx^{2/3})^{3/2}}{10140585a^{11}\sqrt[3]{x}} - \frac{524288b^9(ax + bx^{2/3})^{3/2}}{4345965a^{10}} + \frac{65536b^8(ax + bx^{2/3})^{3/2}}{10140585a^{11}\sqrt[3]{x}}$$

[Out] $(-524288*b^9*(b*x^{(2/3)} + a*x)^{(3/2)})/(4345965*a^{10}) + (8388608*b^{12}*(b*x^{(2/3)} + a*x)^{(3/2)})/(152108775*a^{13}*x) - (4194304*b^{11}*(b*x^{(2/3)} + a*x)^{(3/2)})/(50702925*a^{12}*x^{(2/3)}) + (1048576*b^{10}*(b*x^{(2/3)} + a*x)^{(3/2)})/(10140585*a^{11}*x^{(1/3)}) + (65536*b^8*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(482885*a^9) - (360448*b^7*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(2414425*a^8) + (90112*b^6*x*(b*x^{(2/3)} + a*x)^{(3/2)})/(557175*a^7) - (45056*b^5*x^{(4/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(260015*a^6) + (2816*b^4*x^{(5/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(15295*a^5) - (1408*b^3*x^2*(b*x^{(2/3)} + a*x)^{(3/2)})/(7245*a^4) + (352*b^2*x^{(7/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(1725*a^3) - (16*b*x^{(8/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(75*a^2) + (2*x^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(9*a)$

Rubi [A] time = 0.627282, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$\frac{8388608b^{12}(ax + bx^{2/3})^{3/2}}{152108775a^{13}x} - \frac{4194304b^{11}(ax + bx^{2/3})^{3/2}}{50702925a^{12}x^{2/3}} + \frac{1048576b^{10}(ax + bx^{2/3})^{3/2}}{10140585a^{11}\sqrt[3]{x}} - \frac{524288b^9(ax + bx^{2/3})^{3/2}}{4345965a^{10}} + \frac{65536b^8(ax + bx^{2/3})^{3/2}}{10140585a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[b*x^(2/3) + a*x], x]

[Out] $(-524288*b^9*(b*x^{(2/3)} + a*x)^{(3/2)})/(4345965*a^{10}) + (8388608*b^{12}*(b*x^{(2/3)} + a*x)^{(3/2)})/(152108775*a^{13}*x) - (4194304*b^{11}*(b*x^{(2/3)} + a*x)^{(3/2)})/(50702925*a^{12}*x^{(2/3)}) + (1048576*b^{10}*(b*x^{(2/3)} + a*x)^{(3/2)})/(10140585*a^{11}*x^{(1/3)}) + (65536*b^8*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(482885*a^9) - (360448*b^7*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(2414425*a^8) + (90112*b^6*x*(b*x^{(2/3)} + a*x)^{(3/2)})/(557175*a^7) - (45056*b^5*x^{(4/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(260015*a^6) + (2816*b^4*x^{(5/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(15295*a^5) - (1408*b^3*x^2*(b*x^{(2/3)} + a*x)^{(3/2)})/(7245*a^4) + (352*b^2*x^{(7/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(1725*a^3) - (16*b*x^{(8/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(75*a^2) + (2*x^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(9*a)$

Rule 2016

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

```

Rule 2002

```

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]

```

Rule 2014

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{bx^{2/3} + ax} dx &= \frac{2x^3 (bx^{2/3} + ax)^{3/2}}{9a} - \frac{(8b) \int x^{8/3} \sqrt{bx^{2/3} + ax} dx}{9a} \\
&= -\frac{16bx^{8/3} (bx^{2/3} + ax)^{3/2}}{75a^2} + \frac{2x^3 (bx^{2/3} + ax)^{3/2}}{9a} + \frac{(176b^2) \int x^{7/3} \sqrt{bx^{2/3} + ax} dx}{225a^2} \\
&= \frac{352b^2 x^{7/3} (bx^{2/3} + ax)^{3/2}}{1725a^3} - \frac{16bx^{8/3} (bx^{2/3} + ax)^{3/2}}{75a^2} + \frac{2x^3 (bx^{2/3} + ax)^{3/2}}{9a} - \frac{(704b^3) \int x^2 \sqrt{bx^{2/3} + ax} dx}{1035a^3} \\
&= -\frac{1408b^3 x^2 (bx^{2/3} + ax)^{3/2}}{7245a^4} + \frac{352b^2 x^{7/3} (bx^{2/3} + ax)^{3/2}}{1725a^3} - \frac{16bx^{8/3} (bx^{2/3} + ax)^{3/2}}{75a^2} + \frac{2x^3 (bx^{2/3} + ax)^{3/2}}{9a} \\
&= \frac{2816b^4 x^{5/3} (bx^{2/3} + ax)^{3/2}}{15295a^5} - \frac{1408b^3 x^2 (bx^{2/3} + ax)^{3/2}}{7245a^4} + \frac{352b^2 x^{7/3} (bx^{2/3} + ax)^{3/2}}{1725a^3} - \frac{16bx^{8/3} (bx^{2/3} + ax)^{3/2}}{75a^2} \\
&= -\frac{45056b^5 x^{4/3} (bx^{2/3} + ax)^{3/2}}{260015a^6} + \frac{2816b^4 x^{5/3} (bx^{2/3} + ax)^{3/2}}{15295a^5} - \frac{1408b^3 x^2 (bx^{2/3} + ax)^{3/2}}{7245a^4} + \frac{352b^2 x^{7/3} (bx^{2/3} + ax)^{3/2}}{1725a^3} \\
&= \frac{90112b^6 x (bx^{2/3} + ax)^{3/2}}{557175a^7} - \frac{45056b^5 x^{4/3} (bx^{2/3} + ax)^{3/2}}{260015a^6} + \frac{2816b^4 x^{5/3} (bx^{2/3} + ax)^{3/2}}{15295a^5} - \frac{1408b^3 x^2 (bx^{2/3} + ax)^{3/2}}{7245a^4} \\
&= -\frac{360448b^7 x^{2/3} (bx^{2/3} + ax)^{3/2}}{2414425a^8} + \frac{90112b^6 x (bx^{2/3} + ax)^{3/2}}{557175a^7} - \frac{45056b^5 x^{4/3} (bx^{2/3} + ax)^{3/2}}{260015a^6} + \frac{2816b^4 x^{5/3} (bx^{2/3} + ax)^{3/2}}{15295a^5} \\
&= \frac{65536b^8 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{482885a^9} - \frac{360448b^7 x^{2/3} (bx^{2/3} + ax)^{3/2}}{2414425a^8} + \frac{90112b^6 x (bx^{2/3} + ax)^{3/2}}{557175a^7} - \frac{45056b^5 x^{4/3} (bx^{2/3} + ax)^{3/2}}{260015a^6} \\
&= -\frac{524288b^9 (bx^{2/3} + ax)^{3/2}}{4345965a^{10}} + \frac{65536b^8 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{482885a^9} - \frac{360448b^7 x^{2/3} (bx^{2/3} + ax)^{3/2}}{2414425a^8} + \frac{90112b^6 x (bx^{2/3} + ax)^{3/2}}{15295a^5} \\
&= -\frac{524288b^9 (bx^{2/3} + ax)^{3/2}}{4345965a^{10}} + \frac{1048576b^{10} (bx^{2/3} + ax)^{3/2}}{10140585a^{11} \sqrt[3]{x}} + \frac{65536b^8 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{482885a^9} - \frac{360448b^7 x^{2/3} (bx^{2/3} + ax)^{3/2}}{2414425a^8} \\
&= -\frac{524288b^9 (bx^{2/3} + ax)^{3/2}}{4345965a^{10}} - \frac{4194304b^{11} (bx^{2/3} + ax)^{3/2}}{50702925a^{12} x^{2/3}} + \frac{1048576b^{10} (bx^{2/3} + ax)^{3/2}}{10140585a^{11} \sqrt[3]{x}} + \frac{65536b^8 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{482885a^9} \\
&= -\frac{524288b^9 (bx^{2/3} + ax)^{3/2}}{4345965a^{10}} + \frac{8388608b^{12} (bx^{2/3} + ax)^{3/2}}{152108775a^{13} x} - \frac{4194304b^{11} (bx^{2/3} + ax)^{3/2}}{50702925a^{12} x^{2/3}} + \frac{1048576b^{10} (bx^{2/3} + ax)^{3/2}}{10140585a^{11} \sqrt[3]{x}}
\end{aligned}$$

Mathematica [A] time = 0.135226, size = 181, normalized size = 0.49

$$\frac{2(a\sqrt[3]{x} + b)\sqrt{ax + bx^{2/3}}(15519504a^{10}b^2x^{10/3} - 14780480a^9b^3x^3 + 14002560a^8b^4x^{8/3} - 13178880a^7b^5x^{7/3} + 12300288a^6b^6x^{2/3})}{152108775a^{13}x}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]

[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(4194304*b^12 - 6291456*a*b^11*x^(1/3) + 7864320*a^2*b^10*x^(2/3) - 9175040*a^3*b^9*x + 10321920*a^4*b^8*x^(4/3) - 11354112*a^5*b^7*x^(5/3) + 12300288*a^6*b^6*x^2 - 13178880*a^7*b^5*x^(7/3) + 14002560*a^8*b^4*x^(8/3) - 14780480*a^9*b^3*x^3 + 15519504*a^10*b^2*x^(10/3) - 16224936*a^11*b*x^(11/3) + 16900975*a^12*x^4)/(152108775*a^13*x^(1/3))

Maple [A] time = 0.01, size = 156, normalized size = 0.4

$$-\frac{2}{152108775 a^{13}} \sqrt{b x^{\frac{2}{3}} + a x} (b + a \sqrt[3]{x}) (16224936 x^{11/3} a^{11} b - 15519504 x^{10/3} a^{10} b^2 - 14002560 x^{8/3} a^8 b^4 + 13178880 x^{7/3} a^7 b^5 + 11354112 x^{5/3} a^5 b^7 - 10321920 x^{4/3} a^4 b^8 - 16900975 x^4 a^{12} + 14780480 x^3 a^9 b^3 - 7864320 x^{2/3} a^2 b^{10} - 12300288 x^2 a^6 b^6 + 6291456 x^{1/3} a^3 b^9 - 4194304 b^{12}) / x^{1/3} / a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x)

[Out] -2/152108775*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(16224936*x^(11/3)*a^11*b-15519504*x^(10/3)*a^10*b^2-14002560*x^(8/3)*a^8*b^4+13178880*x^(7/3)*a^7*b^5+11354112*x^(5/3)*a^5*b^7-10321920*x^(4/3)*a^4*b^8-16900975*x^4*a^12+14780480*x^3*a^9*b^3-7864320*x^(2/3)*a^2*b^10-12300288*x^2*a^6*b^6+6291456*x^(1/3)*a^3*b^9+9175040*x*a^3*b^9-4194304*b^12)/x^(1/3)/a^13

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{ax + bx^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)`

[Out] `Integral(x**3*sqrt(a*x + b*x**(2/3)), x)`

Giac [A] time = 1.13299, size = 259, normalized size = 0.7

$$-\frac{8388608 b^{\frac{27}{2}}}{152108775 a^{13}} + \frac{2 \left(16900975 \left(ax^{\frac{1}{3}} + b \right)^{\frac{27}{2}} - 219036636 \left(ax^{\frac{1}{3}} + b \right)^{\frac{25}{2}} b + 1309458150 \left(ax^{\frac{1}{3}} + b \right)^{\frac{23}{2}} b^2 - 4780561500 \left(ax^{\frac{1}{3}} + b \right)^{\frac{21}{2}} b^3 + 11888501625 \left(ax^{\frac{1}{3}} + b \right)^{\frac{19}{2}} b^4 - 21259438200 \left(ax^{\frac{1}{3}} + b \right)^{\frac{17}{2}} b^5 + 28109701620 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} b^6 - 27800803800 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} b^7 + 20534684625 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} b^8 - 11154643500 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} b^9 + 4302505350 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b^{10} - 1095183180 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^{11} + 152108775 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^{12} \right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")`

[Out] `-8388608/152108775*b^(27/2)/a^13 + 2/152108775*(16900975*(a*x^(1/3) + b)^(27/2) - 219036636*(a*x^(1/3) + b)^(25/2)*b + 1309458150*(a*x^(1/3) + b)^(23/2)*b^2 - 4780561500*(a*x^(1/3) + b)^(21/2)*b^3 + 11888501625*(a*x^(1/3) + b)^(19/2)*b^4 - 21259438200*(a*x^(1/3) + b)^(17/2)*b^5 + 28109701620*(a*x^(1/3) + b)^(15/2)*b^6 - 27800803800*(a*x^(1/3) + b)^(13/2)*b^7 + 20534684625*(a*x^(1/3) + b)^(11/2)*b^8 - 11154643500*(a*x^(1/3) + b)^(9/2)*b^9 + 4302505350*(a*x^(1/3) + b)^(7/2)*b^10 - 1095183180*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2)*b^12)/a^13`

3.168 $\int x^2 \sqrt{bx^{2/3} + ax} dx$

Optimal. Leaf size=283

$$\frac{131072b^9 (ax + bx^{2/3})^{3/2}}{1616615a^{10}x} + \frac{196608b^8 (ax + bx^{2/3})^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7 (ax + bx^{2/3})^{3/2}}{323323a^8\sqrt[3]{x}} + \frac{8192b^6 (ax + bx^{2/3})^{3/2}}{46189a^7} - \frac{9216b^5\sqrt[3]{x}}{46}$$

[Out] $(8192*b^6*(b*x^{2/3} + a*x)^{(3/2)})/(46189*a^7) - (131072*b^9*(b*x^{2/3} + a*x)^{(3/2)})/(1616615*a^{10}*x) + (196608*b^8*(b*x^{2/3} + a*x)^{(3/2)})/(1616615*a^9*x^{2/3}) - (49152*b^7*(b*x^{2/3} + a*x)^{(3/2)})/(323323*a^8*x^{1/3}) - (9216*b^5*x^{1/3}*(b*x^{2/3} + a*x)^{(3/2)})/(46189*a^6) + (4608*b^4*x^{2/3}*(b*x^{2/3} + a*x)^{(3/2)})/(20995*a^5) - (384*b^3*x*(b*x^{2/3} + a*x)^{(3/2)})/(1615*a^4) + (576*b^2*x^{4/3}*(b*x^{2/3} + a*x)^{(3/2)})/(2261*a^3) - (36*b*x^{5/3}*(b*x^{2/3} + a*x)^{(3/2)})/(133*a^2) + (2*x^2*(b*x^{2/3} + a*x)^{(3/2)})/(7*a)$

Rubi [A] time = 0.441085, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$\frac{131072b^9 (ax + bx^{2/3})^{3/2}}{1616615a^{10}x} + \frac{196608b^8 (ax + bx^{2/3})^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7 (ax + bx^{2/3})^{3/2}}{323323a^8\sqrt[3]{x}} + \frac{8192b^6 (ax + bx^{2/3})^{3/2}}{46189a^7} - \frac{9216b^5\sqrt[3]{x}}{46}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[b*x^(2/3) + a*x], x]

[Out] $(8192*b^6*(b*x^{2/3} + a*x)^{(3/2)})/(46189*a^7) - (131072*b^9*(b*x^{2/3} + a*x)^{(3/2)})/(1616615*a^{10}*x) + (196608*b^8*(b*x^{2/3} + a*x)^{(3/2)})/(1616615*a^9*x^{2/3}) - (49152*b^7*(b*x^{2/3} + a*x)^{(3/2)})/(323323*a^8*x^{1/3}) - (9216*b^5*x^{1/3}*(b*x^{2/3} + a*x)^{(3/2)})/(46189*a^6) + (4608*b^4*x^{2/3}*(b*x^{2/3} + a*x)^{(3/2)})/(20995*a^5) - (384*b^3*x*(b*x^{2/3} + a*x)^{(3/2)})/(1615*a^4) + (576*b^2*x^{4/3}*(b*x^{2/3} + a*x)^{(3/2)})/(2261*a^3) - (36*b*x^{5/3}*(b*x^{2/3} + a*x)^{(3/2)})/(133*a^2) + (2*x^2*(b*x^{2/3} + a*x)^{(3/2)})/(7*a)$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{bx^{2/3} + ax} dx &= \frac{2x^2 (bx^{2/3} + ax)^{3/2}}{7a} - \frac{(6b) \int x^{5/3} \sqrt{bx^{2/3} + ax} dx}{7a} \\
&= -\frac{36bx^{5/3} (bx^{2/3} + ax)^{3/2}}{133a^2} + \frac{2x^2 (bx^{2/3} + ax)^{3/2}}{7a} + \frac{(96b^2) \int x^{4/3} \sqrt{bx^{2/3} + ax} dx}{133a^2} \\
&= \frac{576b^2 x^{4/3} (bx^{2/3} + ax)^{3/2}}{2261a^3} - \frac{36bx^{5/3} (bx^{2/3} + ax)^{3/2}}{133a^2} + \frac{2x^2 (bx^{2/3} + ax)^{3/2}}{7a} - \frac{(192b^3) \int x \sqrt{bx^{2/3} + ax} dx}{323a^3} \\
&= -\frac{384b^3 x (bx^{2/3} + ax)^{3/2}}{1615a^4} + \frac{576b^2 x^{4/3} (bx^{2/3} + ax)^{3/2}}{2261a^3} - \frac{36bx^{5/3} (bx^{2/3} + ax)^{3/2}}{133a^2} + \frac{2x^2 (bx^{2/3} + ax)^{3/2}}{7a} \\
&= \frac{4608b^4 x^{2/3} (bx^{2/3} + ax)^{3/2}}{20995a^5} - \frac{384b^3 x (bx^{2/3} + ax)^{3/2}}{1615a^4} + \frac{576b^2 x^{4/3} (bx^{2/3} + ax)^{3/2}}{2261a^3} - \frac{36bx^{5/3} (bx^{2/3} + ax)^{3/2}}{133a^2} \\
&= -\frac{9216b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{46189a^6} + \frac{4608b^4 x^{2/3} (bx^{2/3} + ax)^{3/2}}{20995a^5} - \frac{384b^3 x (bx^{2/3} + ax)^{3/2}}{1615a^4} + \frac{576b^2 x^{4/3} (bx^{2/3} + ax)^{3/2}}{2261a^3} \\
&= \frac{8192b^6 (bx^{2/3} + ax)^{3/2}}{46189a^7} - \frac{9216b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{46189a^6} + \frac{4608b^4 x^{2/3} (bx^{2/3} + ax)^{3/2}}{20995a^5} - \frac{384b^3 x (bx^{2/3} + ax)^{3/2}}{1615a^4} \\
&= \frac{8192b^6 (bx^{2/3} + ax)^{3/2}}{46189a^7} - \frac{49152b^7 (bx^{2/3} + ax)^{3/2}}{323323a^8 \sqrt[3]{x}} - \frac{9216b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{46189a^6} + \frac{4608b^4 x^{2/3} (bx^{2/3} + ax)^{3/2}}{20995a^5} \\
&= \frac{8192b^6 (bx^{2/3} + ax)^{3/2}}{46189a^7} + \frac{196608b^8 (bx^{2/3} + ax)^{3/2}}{1616615a^9 x^{2/3}} - \frac{49152b^7 (bx^{2/3} + ax)^{3/2}}{323323a^8 \sqrt[3]{x}} - \frac{9216b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{3/2}}{46189a^6} \\
&= \frac{8192b^6 (bx^{2/3} + ax)^{3/2}}{46189a^7} - \frac{131072b^9 (bx^{2/3} + ax)^{3/2}}{1616615a^{10} x} + \frac{196608b^8 (bx^{2/3} + ax)^{3/2}}{1616615a^9 x^{2/3}} - \frac{49152b^7 (bx^{2/3} + ax)^{3/2}}{323323a^8 \sqrt[3]{x}}
\end{aligned}$$

Mathematica [A] time = 0.0951391, size = 144, normalized size = 0.51

$$\frac{2(a\sqrt[3]{x} + b)\sqrt{ax + bx^{2/3}}(205920a^7b^2x^{7/3} - 192192a^6b^3x^2 + 177408a^5b^4x^{5/3} - 161280a^4b^5x^{4/3} - 122880a^2b^7x^{2/3} + 143360ab^8x^{1/3} - 161280a^4b^5x^{4/3} - 122880a^2b^7x^{2/3} + 143360ab^8x^{1/3} - 161280a^4b^5x^{4/3} - 122880a^2b^7x^{2/3} + 143360ab^8x^{1/3})}{1616615a^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x], x]

[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3) + 230945*a^9*x^3))/(1616615*a^10*x^(1/3))

Maple [A] time = 0.004, size = 123, normalized size = 0.4

$$-\frac{2}{1616615 a^{10}} \sqrt{bx^{\frac{2}{3}} + ax} (b + a\sqrt[3]{x}) (218790 x^{8/3} a^8 b - 205920 x^{7/3} a^7 b^2 - 177408 x^{5/3} a^5 b^4 + 161280 x^{4/3} a^4 b^5 - 230945 x^3 a^3 b^6 + 122880 x^2 a^2 b^7 + 192192 x a b^8 - 98304 b^9) / x^{1/3} / a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^(2/3)+a*x)^(1/2), x)

[Out] -2/1616615*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(218790*x^(8/3)*a^8*b-205920*x^(7/3)*a^7*b^2-177408*x^(5/3)*a^5*b^4+161280*x^(4/3)*a^4*b^5-230945*x^3*a^3*b^6+122880*x^2*a^2*b^7+192192*x*a*b^8-98304*b^9)/x^(1/3)/a^10

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{ax + bx^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2), x)

[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.12632, size = 203, normalized size = 0.72

$$\frac{131072 b^{\frac{21}{2}}}{1616615 a^{10}} + \frac{2 \left(230945 \left(ax^{\frac{1}{3}} + b \right)^{\frac{21}{2}} - 2297295 \left(ax^{\frac{1}{3}} + b \right)^{\frac{19}{2}} b + 10270260 \left(ax^{\frac{1}{3}} + b \right)^{\frac{17}{2}} b^2 - 27159132 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} b^3 + \dots \right)}{1616615 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2), x, algorithm="giac")

[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(230945*(a*x^(1/3) + b)^(21/2) - 2297295*(a*x^(1/3) + b)^(19/2)*b + 10270260*(a*x^(1/3) + b)^(17/2)*b^2 - 27159132*(a*x^(1/3) + b)^(15/2)*b^3 + 47006190*(a*x^(1/3) + b)^(13/2)*b^4 - 55552770*(a*x^(1/3) + b)^(11/2)*b^5 + 45265220*(a*x^(1/3) + b)^(9/2)*b^6 - 24942060*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 1616615*(a*x^(1/3) + b)^(3/2)*b^9)/a^10

3.169 $\int x\sqrt{bx^{2/3} + ax} dx$

Optimal. Leaf size=195

$$\frac{2048b^6 (ax + bx^{2/3})^{3/2}}{15015a^7x} - \frac{1024b^5 (ax + bx^{2/3})^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4 (ax + bx^{2/3})^{3/2}}{1001a^5\sqrt[3]{x}} - \frac{128b^3 (ax + bx^{2/3})^{3/2}}{429a^4} + \frac{48b^2\sqrt[3]{x} (ax + bx^{2/3})^{3/2}}{143a^3}$$

[Out] $(-128*b^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(429*a^4) + (2048*b^6*(b*x^{(2/3)} + a*x)^{(3/2)})/(15015*a^7*x) - (1024*b^5*(b*x^{(2/3)} + a*x)^{(3/2)})/(5005*a^6*x^{(2/3)}) + (256*b^4*(b*x^{(2/3)} + a*x)^{(3/2)})/(1001*a^5*x^{(1/3)}) + (48*b^2*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(143*a^3) - (24*b*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(65*a^2) + (2*x*(b*x^{(2/3)} + a*x)^{(3/2)})/(5*a)$

Rubi [A] time = 0.273272, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2016, 2002, 2014}

$$\frac{2048b^6 (ax + bx^{2/3})^{3/2}}{15015a^7x} - \frac{1024b^5 (ax + bx^{2/3})^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4 (ax + bx^{2/3})^{3/2}}{1001a^5\sqrt[3]{x}} - \frac{128b^3 (ax + bx^{2/3})^{3/2}}{429a^4} + \frac{48b^2\sqrt[3]{x} (ax + bx^{2/3})^{3/2}}{143a^3}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[b*x^(2/3) + a*x], x]

[Out] $(-128*b^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(429*a^4) + (2048*b^6*(b*x^{(2/3)} + a*x)^{(3/2)})/(15015*a^7*x) - (1024*b^5*(b*x^{(2/3)} + a*x)^{(3/2)})/(5005*a^6*x^{(2/3)}) + (256*b^4*(b*x^{(2/3)} + a*x)^{(3/2)})/(1001*a^5*x^{(1/3)}) + (48*b^2*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(143*a^3) - (24*b*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(3/2)})/(65*a^2) + (2*x*(b*x^{(2/3)} + a*x)^{(3/2)})/(5*a)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002


```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j
)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{bx^{2/3} + ax} dx &= \frac{2x(bx^{2/3} + ax)^{3/2}}{5a} - \frac{(4b) \int x^{2/3}\sqrt{bx^{2/3} + ax} dx}{5a} \\
&= -\frac{24bx^{2/3}(bx^{2/3} + ax)^{3/2}}{65a^2} + \frac{2x(bx^{2/3} + ax)^{3/2}}{5a} + \frac{(8b^2) \int \sqrt[3]{x}\sqrt{bx^{2/3} + ax} dx}{13a^2} \\
&= \frac{48b^2\sqrt[3]{x}(bx^{2/3} + ax)^{3/2}}{143a^3} - \frac{24bx^{2/3}(bx^{2/3} + ax)^{3/2}}{65a^2} + \frac{2x(bx^{2/3} + ax)^{3/2}}{5a} - \frac{(64b^3) \int \sqrt{bx^{2/3} + ax} dx}{143a^3} \\
&= -\frac{128b^3(bx^{2/3} + ax)^{3/2}}{429a^4} + \frac{48b^2\sqrt[3]{x}(bx^{2/3} + ax)^{3/2}}{143a^3} - \frac{24bx^{2/3}(bx^{2/3} + ax)^{3/2}}{65a^2} + \frac{2x(bx^{2/3} + ax)^{3/2}}{5a} + \\
&= -\frac{128b^3(bx^{2/3} + ax)^{3/2}}{429a^4} + \frac{256b^4(bx^{2/3} + ax)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}(bx^{2/3} + ax)^{3/2}}{143a^3} - \frac{24bx^{2/3}(bx^{2/3} + ax)^{3/2}}{65a^2} \\
&= -\frac{128b^3(bx^{2/3} + ax)^{3/2}}{429a^4} - \frac{1024b^5(bx^{2/3} + ax)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4(bx^{2/3} + ax)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}(bx^{2/3} + ax)^{3/2}}{143a^3} \\
&= -\frac{128b^3(bx^{2/3} + ax)^{3/2}}{429a^4} + \frac{2048b^6(bx^{2/3} + ax)^{3/2}}{15015a^7x} - \frac{1024b^5(bx^{2/3} + ax)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4(bx^{2/3} + ax)^{3/2}}{1001a^5\sqrt[3]{x}}
\end{aligned}$$

Mathematica [A] time = 0.0687215, size = 107, normalized size = 0.55

$$\frac{2(a\sqrt[3]{x} + b)\sqrt{ax + bx^{2/3}}(2520a^4b^2x^{4/3} + 1920a^2b^4x^{2/3} - 2240a^3b^3x - 2772a^5bx^{5/3} + 3003a^6x^2 - 1536ab^5\sqrt[3]{x} + 1024b^6)}{15015a^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]

[Out] $(2*(b + a*x^{(1/3)})*\text{Sqrt}[b*x^{(2/3)} + a*x]*(1024*b^6 - 1536*a*b^5*x^{(1/3)} + 1920*a^2*b^4*x^{(2/3)} - 2240*a^3*b^3*x + 2520*a^4*b^2*x^{(4/3)} - 2772*a^5*b*x^{(5/3)} + 3003*a^6*x^2))/(15015*a^7*x^{(1/3)})$

Maple [A] time = 0.002, size = 90, normalized size = 0.5

$$-\frac{2}{15015 a^7} \sqrt{b x^{\frac{2}{3}} + a x} (b + a \sqrt[3]{x}) (2772 x^{5/3} a^5 b - 2520 x^{4/3} a^4 b^2 - 1920 x^{2/3} a^2 b^4 - 3003 x^2 a^6 + 1536 \sqrt[3]{x} a b^5 + 2240 x a^3 b^3 - 1024 b^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^(2/3)+a*x)^(1/2),x)

[Out] $-2/15015*(b*x^{(2/3)}+a*x)^{(1/2)}*(b+a*x^{(1/3)})*(2772*x^{(5/3)}*a^5*b-2520*x^{(4/3)}*a^4*b^2-1920*x^{(2/3)}*a^2*b^4-3003*x^2*a^6+1536*x^{(1/3)}*a*b^5+2240*x*a^3*b^3-1024*b^6)/x^{(1/3)}/a^7$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{ax + bx^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.13058, size = 146, normalized size = 0.75

$$-\frac{2048 b^{\frac{15}{2}}}{15015 a^7} + \frac{2 \left(3003 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} - 20790 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} b + 61425 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} b^2 - 100100 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} b^3 + 96525 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b^4 - 54054 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^5 + 15015 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^6 \right)}{15015 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(3003*(a*x^(1/3) + b)^(15/2) - 20790*(a*x^(1/3) + b)^(13/2)*b + 61425*(a*x^(1/3) + b)^(11/2)*b^2 - 100100*(a*x^(1/3) + b)^(9/2)*b^3 + 96525*(a*x^(1/3) + b)^(7/2)*b^4 - 54054*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6)/a^7

3.170 $\int \sqrt{bx^{2/3} + ax} dx$

Optimal. Leaf size=109

$$-\frac{32b^3(ax + bx^{2/3})^{3/2}}{105a^4x} + \frac{16b^2(ax + bx^{2/3})^{3/2}}{35a^3x^{2/3}} - \frac{4b(ax + bx^{2/3})^{3/2}}{7a^2\sqrt[3]{x}} + \frac{2(ax + bx^{2/3})^{3/2}}{3a}$$

[Out] $(2*(b*x^{(2/3)} + a*x)^{(3/2)})/(3*a) - (32*b^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(105*a^4*x) + (16*b^2*(b*x^{(2/3)} + a*x)^{(3/2)})/(35*a^3*x^{(2/3)}) - (4*b*(b*x^{(2/3)} + a*x)^{(3/2)})/(7*a^2*x^{(1/3)})$

Rubi [A] time = 0.136737, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2002, 2016, 2014}

$$-\frac{32b^3(ax + bx^{2/3})^{3/2}}{105a^4x} + \frac{16b^2(ax + bx^{2/3})^{3/2}}{35a^3x^{2/3}} - \frac{4b(ax + bx^{2/3})^{3/2}}{7a^2\sqrt[3]{x}} + \frac{2(ax + bx^{2/3})^{3/2}}{3a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x], x]

[Out] $(2*(b*x^{(2/3)} + a*x)^{(3/2)})/(3*a) - (32*b^3*(b*x^{(2/3)} + a*x)^{(3/2)})/(105*a^4*x) + (16*b^2*(b*x^{(2/3)} + a*x)^{(3/2)})/(35*a^3*x^{(2/3)}) - (4*b*(b*x^{(2/3)} + a*x)^{(3/2)})/(7*a^2*x^{(1/3)})$

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2016

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/

$(n - j)], 0] \&\& \text{NeQ}[m + j*p + 1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2014

$\text{Int}[\left((c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^{p+1}\right), x_{\text{Symbol}}] \rightarrow -\text{Simp}\left[\frac{c^{j-1} \cdot (c \cdot x)^{m-j+1} \cdot (a \cdot x^j + b \cdot x^n)^{p+1}}{a \cdot (n-j) \cdot (p+1)}, x\right] /;$ $\text{FreeQ}\{a, b, c, j, m, n, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[m + n \cdot p + n - j + 1, 0] \&\& (\text{IntegerQ}[j] \parallel \text{GtQ}[c, 0])$

Rubi steps

$$\begin{aligned} \int \sqrt{bx^{2/3} + ax} dx &= \frac{2(bx^{2/3} + ax)^{3/2}}{3a} - \frac{(2b) \int \frac{\sqrt{bx^{2/3} + ax}}{\sqrt[3]{x}} dx}{3a} \\ &= \frac{2(bx^{2/3} + ax)^{3/2}}{3a} - \frac{4b(bx^{2/3} + ax)^{3/2}}{7a^2 \sqrt[3]{x}} + \frac{(8b^2) \int \frac{\sqrt{bx^{2/3} + ax}}{x^{2/3}} dx}{21a^2} \\ &= \frac{2(bx^{2/3} + ax)^{3/2}}{3a} + \frac{16b^2(bx^{2/3} + ax)^{3/2}}{35a^3 x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{3/2}}{7a^2 \sqrt[3]{x}} - \frac{(16b^3) \int \frac{\sqrt{bx^{2/3} + ax}}{x} dx}{105a^3} \\ &= \frac{2(bx^{2/3} + ax)^{3/2}}{3a} - \frac{32b^3(bx^{2/3} + ax)^{3/2}}{105a^4 x} + \frac{16b^2(bx^{2/3} + ax)^{3/2}}{35a^3 x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{3/2}}{7a^2 \sqrt[3]{x}} \end{aligned}$$

Mathematica [A] time = 0.0395579, size = 70, normalized size = 0.64

$$\frac{2(a\sqrt[3]{x} + b)\sqrt{ax + bx^{2/3}}(-30a^2bx^{2/3} + 35a^3x + 24ab^2\sqrt[3]{x} - 16b^3)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x], x]

[Out] $(2(b + a \cdot x^{1/3}) \cdot \text{Sqrt}[b \cdot x^{2/3} + a \cdot x] \cdot (-16 \cdot b^3 + 24 \cdot a \cdot b^2 \cdot x^{1/3} - 30 \cdot a^2 \cdot b \cdot x^{2/3} + 35 \cdot a^3 \cdot x)) / (105 \cdot a^4 \cdot x^{1/3})$

Maple [A] time = 0.003, size = 57, normalized size = 0.5

$$-\frac{2}{105a^4} \sqrt{bx^{2/3} + ax} (b + a\sqrt[3]{x}) (30x^{2/3}a^2b - 24\sqrt[3]{x}ab^2 - 35xa^3 + 16b^3) \frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(2/3)+a*x)^(1/2),x)`

[Out] $-2/105*(b*x^{(2/3)+a*x}^{(1/2)}*(b+a*x^{(1/3)})*(30*x^{(2/3)*a^2*b-24*x^{(1/3)*a*b}^{2-35*x*a^3+16*b^3})/x^{(1/3)}/a^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^(2/3)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**(2/3)+a*x)**(1/2),x)`

[Out] Integral(sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.11866, size = 89, normalized size = 0.82

$$\frac{32 b^{\frac{9}{2}}}{105 a^4} + \frac{2 \left(35 \left(a x^{\frac{1}{3}} + b \right)^{\frac{9}{2}} - 135 \left(a x^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b + 189 \left(a x^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^2 - 105 \left(a x^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^3 \right)}{105 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 32/105*b^(9/2)/a^4 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 135*(a*x^(1/3) + b)^(7/2)*b + 189*(a*x^(1/3) + b)^(5/2)*b^2 - 105*(a*x^(1/3) + b)^(3/2)*b^3)/a^4

$$3.171 \quad \int \frac{\sqrt{bx^{2/3}+ax}}{x} dx$$

Optimal. Leaf size=23

$$\frac{2(ax + bx^{2/3})^{3/2}}{ax}$$

[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)

Rubi [A] time = 0.0404968, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$\frac{2(ax + bx^{2/3})^{3/2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]

[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3} + ax}}{x} dx = \frac{2(bx^{2/3} + ax)^{3/2}}{ax}$$

Mathematica [A] time = 0.0108147, size = 23, normalized size = 1.

$$\frac{2(ax + bx^{2/3})^{3/2}}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]

[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)

Maple [A] time = 0.003, size = 27, normalized size = 1.2

$$2 \frac{\sqrt{bx^{2/3} + ax} (b + a\sqrt[3]{x})}{a\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(1/2)/x,x)

[Out] 2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(b+a*x^(1/3))/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)

[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)

Giac [A] time = 1.126, size = 31, normalized size = 1.35

$$\frac{2 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}}}{a} - \frac{2b^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")

[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a

$$3.172 \quad \int \frac{\sqrt{bx^{2/3}+ax}}{x^2} dx$$

Optimal. Leaf size=90

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2*x) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4*b*x^{(2/3)}) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(4*b^{(3/2)})$

Rubi [A] time = 0.138582, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[b*x^{(2/3)} + a*x]/x^2, x]$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2*x) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4*b*x^{(2/3)}) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(4*b^{(3/2)})$

Rule 2020

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:\> \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*x^p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x$ && $!\text{IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$ && $\text{GtQ}[p, 0]$ && $\text{LtQ}[m+j*p+1, 0]$

Rule 2025

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:\> \text{Simp}[c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)}/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, m, p\}, x$ && $!\text{IntegerQ}[p]$ && $\text{LtQ}[0, j, n]$ && $(\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$ && $\text{LtQ}[m$

+ j*p + 1, 0]

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{bx^{2/3} + ax}}{x^2} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{2x} + \frac{1}{4}a \int \frac{1}{x\sqrt{bx^{2/3} + ax}} dx \\ &= -\frac{3\sqrt{bx^{2/3} + ax}}{2x} - \frac{3a\sqrt{bx^{2/3} + ax}}{4bx^{2/3}} - \frac{a^2 \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{8b} \\ &= -\frac{3\sqrt{bx^{2/3} + ax}}{2x} - \frac{3a\sqrt{bx^{2/3} + ax}}{4bx^{2/3}} + \frac{(3a^2) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{4b} \\ &= -\frac{3\sqrt{bx^{2/3} + ax}}{2x} - \frac{3a\sqrt{bx^{2/3} + ax}}{4bx^{2/3}} + \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{4b^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.0415882, size = 57, normalized size = 0.63

$$\frac{2a^2 (a\sqrt[3]{x} + b) \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^3 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2, x]
```

```
[Out] (-2*a^2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 3, 5/2,
1 + (a*x^(1/3))/b])/(b^3*x^(1/3))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.9

$$\frac{3}{4x} \sqrt{bx^{\frac{2}{3}} + ax} \left(\operatorname{Arctanh} \left(\sqrt{b + a\sqrt[3]{x}} \frac{1}{\sqrt{b}} \right) ba^2 x^{\frac{2}{3}} - b^{\frac{3}{2}} (b + a\sqrt[3]{x})^{\frac{3}{2}} - b^{\frac{5}{2}} \sqrt{b + a\sqrt[3]{x}} \right) b^{-\frac{5}{2}} \frac{1}{\sqrt{b + a\sqrt[3]{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(2/3)+a*x)^(1/2)/x^2,x)`

[Out] $\frac{3}{4} * (b*x^{(2/3)}+a*x)^{(1/2)} * (\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)}) * b*a^2*x^{(2/3)} - b^{(3/2)} * (b+a*x^{(1/3)})^{(3/2)} - b^{(5/2)} * (b+a*x^{(1/3)})^{(1/2)}) / x / (b+a*x^{(1/3)})^{(1/2)} / b^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^(2/3))/x^2, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)

[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)

Giac [A] time = 1.21738, size = 97, normalized size = 1.08

$$\frac{3 \left(\frac{a^3 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^3 + \sqrt{ax^{\frac{1}{3}}+ba^3b}}{a^2 b x^{\frac{2}{3}}} \right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")

[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) + b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a

$$3.173 \quad \int \frac{\sqrt{bx^{2/3}+ax}}{x^3} dx$$

Optimal. Leaf size=178

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}} - \frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x} + \frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}} - \frac{21a^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(5*x^2) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(40*b*x^{(5/3)}) + (7*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(80*b^2*x^{(4/3)}) - (7*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(64*b^3*x) + (21*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(128*b^4*x^{(2/3)}) - (21*a^5*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(128*b^{(9/2)})$

Rubi [A] time = 0.295909, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}} - \frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x} + \frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}} - \frac{21a^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3, x]

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(5*x^2) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(40*b*x^{(5/3)}) + (7*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(80*b^2*x^{(4/3)}) - (7*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(64*b^3*x) + (21*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(128*b^4*x^{(2/3)}) - (21*a^5*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(128*b^{(9/2)})$

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2025

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{bx^{2/3} + ax}}{x^3} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} + \frac{1}{10}a \int \frac{1}{x^2\sqrt{bx^{2/3} + ax}} dx \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} - \frac{(7a^2) \int \frac{1}{x^{5/3}\sqrt{bx^{2/3} + ax}} dx}{80b} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3} + ax}}{80b^2x^{4/3}} + \frac{(7a^3) \int \frac{1}{x^{4/3}\sqrt{bx^{2/3} + ax}} dx}{96b^2} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3} + ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3} + ax}}{64b^3x} - \frac{(7a^4) \int \frac{1}{x\sqrt{bx^{2/3} + ax}} dx}{128b^3} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3} + ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3} + ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3} + ax}}{128b^4x^{2/3}} + \frac{(7a^5) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{128b^4} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3} + ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3} + ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3} + ax}}{128b^4x^{2/3}} - \frac{(21a^5) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{128b^4} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3} + ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3} + ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3} + ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3} + ax}}{128b^4x^{2/3}} - \frac{21a^5 \operatorname{arctanh}\left(\frac{\sqrt{bx^{2/3} + ax}}{\sqrt{ax + bx^{2/3}}}\right)}{128b^4}
 \end{aligned}$$

Mathematica [C] time = 0.0408602, size = 57, normalized size = 0.32

$$\frac{2a^5 (a\sqrt[3]{x} + b) \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{3}{2}, 6; \frac{5}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^6 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]

[Out] (2*a^5*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 6, 5/2, 1 + (a*x^(1/3))/b])/(b^6*x^(1/3))

Maple [A] time = 0.011, size = 125, normalized size = 0.7

$$-\frac{1}{640x^2}\sqrt{bx^{\frac{2}{3}}+ax}\left(105b^{17/2}\sqrt{b+a\sqrt[3]{x}}+790b^{15/2}(b+a\sqrt[3]{x})^{3/2}-896b^{13/2}(b+a\sqrt[3]{x})^{5/2}+490b^{11/2}(b+a\sqrt[3]{x})^{7/2}-105b^{9/2}(b+a\sqrt[3]{x})^{9/2}+105\operatorname{arctanh}\left(\frac{(b+a\sqrt[3]{x})^{1/2}}{b^{1/2}}\right)*b^4a^5x^{5/3}\right)/x^2/(b+a\sqrt[3]{x})^{1/2}/b^{17/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(1/2)/x^3,x)

[Out] -1/640*(b*x^(2/3)+a*x)^(1/2)*(105*b^(17/2)*(b+a*x^(1/3))^(1/2)+790*b^(15/2)*(b+a*x^(1/3))^(3/2)-896*b^(13/2)*(b+a*x^(1/3))^(5/2)+490*b^(11/2)*(b+a*x^(1/3))^(7/2)-105*b^(9/2)*(b+a*x^(1/3))^(9/2)+105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^5*x^(5/3))/x^2/(b+a*x^(1/3))^(1/2)/b^(17/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)

[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)

Giac [A] time = 1.20498, size = 170, normalized size = 0.96

$$\frac{105 a^6 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^6 - 490 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^6 b + 896 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^6 b^2 - 790 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^6 b^3 - 105 \sqrt{ax^{\frac{1}{3}}+b} a^6 b^4}{640 a^5 b^4 x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")

[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3) + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3) + b)*a^6*b^4)/(a^5*b^4*x^(5/3))/a

$$3.174 \quad \int \frac{\sqrt{bx^{2/3}+ax}}{x^4} dx$$

Optimal. Leaf size=266

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}} + \frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x} - \frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}} + \frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}} - \frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2} + \frac{13a^2\sqrt{ax}}{448b^2}$$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8*x^3) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(112*b*x^{(8/3)}) + (13*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(448*b^2*x^{(7/3)}) - (143*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4480*b^3*x^2) + (1287*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(35840*b^4*x^{(5/3)}) - (429*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(10240*b^5*x^{(4/3)}) + (429*a^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8192*b^6*x) - (1287*a^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16384*b^7*x^{(2/3)}) + (1287*a^8*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(16384*b^{(15/2)})$

Rubi [A] time = 0.475003, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}} + \frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x} - \frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}} + \frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}} - \frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2} + \frac{13a^2\sqrt{ax}}{448b^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4, x]

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8*x^3) - (3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(112*b*x^{(8/3)}) + (13*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(448*b^2*x^{(7/3)}) - (143*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4480*b^3*x^2) + (1287*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(35840*b^4*x^{(5/3)}) - (429*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(10240*b^5*x^{(4/3)}) + (429*a^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8192*b^6*x) - (1287*a^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16384*b^7*x^{(2/3)}) + (1287*a^8*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(16384*b^{(15/2)})$

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers

$Q[j, n] \parallel \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m + j*p + 1, 0]$

Rule 2025

$\text{Int}[(c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{j-1} \cdot (c \cdot x)^{m-j+1} \cdot (a \cdot x^j + b \cdot x^n)^{p+1}) / (a \cdot (m + j \cdot p + 1)), x] - \text{Dist}[(b \cdot (m + n \cdot p + n - j + 1)) / (a \cdot c^{n-j} \cdot (m + j \cdot p + 1)), \text{Int}[(c \cdot x)^{m+n-j} \cdot (a \cdot x^j + b \cdot x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, m, p\}, x]$
 $\&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \ \&\& \ \text{LtQ}[m + j \cdot p + 1, 0]$

Rule 2029

$\text{Int}[x^m / \text{Sqrt}[a \cdot x^j + b \cdot x^n], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a \cdot x^2), x], x, x^{(j/2)} / \text{Sqrt}[a \cdot x^j + b \cdot x^n]], x] /;$ $\text{FreeQ}\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^{2/3} + ax}}{x^4} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} + \frac{1}{16}a \int \frac{1}{x^3\sqrt{bx^{2/3} + ax}} dx \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} - \frac{(13a^2) \int \frac{1}{x^{8/3}\sqrt{bx^{2/3} + ax}} dx}{224b} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} + \frac{(143a^3) \int \frac{1}{x^{7/3}\sqrt{bx^{2/3} + ax}} dx}{2688b^2} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} - \frac{(429a^4) \int \frac{1}{x^2\sqrt{bx^{2/3} + ax}} dx}{8960b^3} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} + \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3} + ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3} + ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3} + ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3} + ax}}{35840b^4x^{5/3}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.0456889, size = 57, normalized size = 0.21

$$\frac{2a^8 (a\sqrt[3]{x} + b) \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{3}{2}, 9; \frac{5}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^9 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]

[Out] (-2*a^8*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 9, 5/2, 1 + (a*x^(1/3))/b])/(b^9*x^(1/3))

Maple [A] time = 0.013, size = 167, normalized size = 0.6

$$-\frac{1}{573440x^3}\sqrt{bx^{\frac{2}{3}}+ax}\left(45045b^{15/2}(b+a\sqrt[3]{x})^{15/2}-345345b^{17/2}(b+a\sqrt[3]{x})^{13/2}+1150149b^{19/2}(b+a\sqrt[3]{x})^{11/2}-2167737b^{21/2}(b+a\sqrt[3]{x})^{9/2}+2518087b^{23/2}(b+a\sqrt[3]{x})^{7/2}-1831739b^{25/2}(b+a\sqrt[3]{x})^{5/2}+801535b^{27/2}(b+a\sqrt[3]{x})^{3/2}-45045\operatorname{arc}\tanh\left(\frac{(b+a\sqrt[3]{x})^{1/2}}{b^{1/2}}\right)*b^7a^8x^{8/3}+45045b^{29/2}(b+a\sqrt[3]{x})^{1/2}\right)/x^3/(b+a\sqrt[3]{x})^{1/2}/b^{29/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(1/2)/x^4,x)

[Out] -1/573440*(b*x^(2/3)+a*x)^(1/2)*(45045*b^(15/2)*(b+a*x^(1/3))^(15/2)-345345*b^(17/2)*(b+a*x^(1/3))^(13/2)+1150149*b^(19/2)*(b+a*x^(1/3))^(11/2)-2167737*b^(21/2)*(b+a*x^(1/3))^(9/2)+2518087*b^(23/2)*(b+a*x^(1/3))^(7/2)-1831739*b^(25/2)*(b+a*x^(1/3))^(5/2)+801535*b^(27/2)*(b+a*x^(1/3))^(3/2)-45045*arc tanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^8*x^(8/3)+45045*b^(29/2)*(b+a*x^(1/3))^(1/2))/x^3/(b+a*x^(1/3))^(1/2)/b^(29/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)

[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)

Giac [A] time = 1.2688, size = 239, normalized size = 0.9

$$\frac{45045 a^9 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb^7}} + \frac{45045 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^9 - 345345 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^9 b + 1150149 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^9 b^2 - 2167737 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^9 b^3 + 2518087 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^9 b^4 - 1831739 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^9 b^5 + 801535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^9 b^6 + 45045 \sqrt{ax^{\frac{1}{3}}+b} a^9 b^7}{573440 a a^8 b^7 x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")

[Out]
$$\frac{-1/573440*(45045*a^9*\arctan(\sqrt{a*x^{1/3}+b}/\sqrt{-b}))/(\sqrt{-b}*b^7) + (45045*(a*x^{1/3}+b)^{(15/2)}*a^9 - 345345*(a*x^{1/3}+b)^{(13/2)}*a^9*b + 1150149*(a*x^{1/3}+b)^{(11/2)}*a^9*b^2 - 2167737*(a*x^{1/3}+b)^{(9/2)}*a^9*b^3 + 2518087*(a*x^{1/3}+b)^{(7/2)}*a^9*b^4 - 1831739*(a*x^{1/3}+b)^{(5/2)}*a^9*b^5 + 801535*(a*x^{1/3}+b)^{(3/2)}*a^9*b^6 + 45045*\sqrt{a*x^{1/3}+b}*a^9*b^7)/(a^8*b^7*x^{(8/3)})}{a}$$

$$3.175 \quad \int \frac{\sqrt{bx^{2/3}+ax}}{x^5} dx$$

Optimal. Leaf size=354

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}} - \frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x} + \frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}} - \frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}} + \frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2} - \frac{4199a^5\sqrt{ax+bx^{2/3}}}{236160b^5x^{8/3}}$$

[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqrt[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(19712*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3) + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b*x^(1/3)]/Sqrt[b*x^(2/3) + a*x])]/(262144*b^(21/2)))

Rubi [A] time = 0.660036, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}} - \frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x} + \frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}} - \frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}} + \frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2} - \frac{4199a^5\sqrt{ax+bx^{2/3}}}{236160b^5x^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5, x]

[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqrt[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(19712*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3) + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b*x^(1/3)]/Sqrt[b*x^(2/3) + a*x])]/(262144*b^(21/2)))

Rule 2020


```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^{2/3} + ax}}{x^5} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} + \frac{1}{22}a \int \frac{1}{x^4\sqrt{bx^{2/3} + ax}} dx \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} - \frac{(19a^2) \int \frac{1}{x^{11/3}\sqrt{bx^{2/3} + ax}} dx}{440b} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} + \frac{(323a^3) \int \frac{1}{x^{10/3}\sqrt{bx^{2/3} + ax}} dx}{7920b^2} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} - \frac{(323a^4) \int \frac{1}{x^3\sqrt{bx^{2/3} + ax}} dx}{8448b^3} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} + \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3} + ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3} + ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3} + ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3} + ax}}{19712b^4x^{8/3}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.0490546, size = 57, normalized size = 0.16

$$\frac{2a^{11} (a\sqrt[3]{x} + b) \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{3}{2}, 12; \frac{5}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^{12}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]

[Out] (2*a^11*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 12, 5/2, 1 + (a*x^(1/3))/b])/(b^12*x^(1/3))

Maple [A] time = 0.015, size = 209, normalized size = 0.6

$$-\frac{1}{302776320x^4}\sqrt{bx^{\frac{2}{3}}+ax}\left(-14549535(b+a\sqrt[3]{x})^{21/2}b^{21/2}+155195040(b+a\sqrt[3]{x})^{19/2}b^{23/2}-749786037(b+a\sqrt[3]{x})^{17/2}b^{25/2}+2163862272(b+a\sqrt[3]{x})^{15/2}b^{27/2}-4139920070(b+a\sqrt[3]{x})^{13/2}b^{29/2}+5503713280(b+a\sqrt[3]{x})^{11/2}b^{31/2}-5174056250(b+a\sqrt[3]{x})^9b^{33/2}+3424523520(b+a\sqrt[3]{x})^7b^{35/2}-1551313995(b+a\sqrt[3]{x})^5b^{37/2}+14549535\operatorname{arctanh}\left(\frac{(b+a\sqrt[3]{x})^{1/2}}{b^{1/2}}\right)b^{10}a^{11}x^{11/3}+450357600(b+a\sqrt[3]{x})^3b^{39/2}+14549535(b+a\sqrt[3]{x})^{1/2}b^{41/2}\right)/x^4/(b+a\sqrt[3]{x})^{1/2}/b^{41/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(1/2)/x^5,x)

[Out] -1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*(b+a*x^(1/3))^(21/2)*b^(21/2)+155195040*(b+a*x^(1/3))^(19/2)*b^(23/2)-749786037*(b+a*x^(1/3))^(17/2)*b^(25/2)+2163862272*(b+a*x^(1/3))^(15/2)*b^(27/2)-4139920070*(b+a*x^(1/3))^(13/2)*b^(29/2)+5503713280*(b+a*x^(1/3))^(11/2)*b^(31/2)-5174056250*(b+a*x^(1/3))^9*b^(33/2)+3424523520*(b+a*x^(1/3))^(7/2)*b^(35/2)-1551313995*(b+a*x^(1/3))^5*b^(37/2)+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^11*x^(11/3)+450357600*(b+a*x^(1/3))^(3/2)*b^(39/2)+14549535*(b+a*x^(1/3))^(1/2)*b^(41/2))/x^4/(b+a*x^(1/3))^(1/2)/b^(41/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)

[Out] Timed out

Giac [A] time = 1.39868, size = 308, normalized size = 0.87

$$\frac{14549535 a^{12} \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} a^{12} - 155195040 \left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} a^{12} b + 749786037 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{12} b^2 - 2163862272 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{12} b^3 + 4139920070 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{12} b^4 - 5503713280 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{12} b^5 + 5174056250 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{12} b^6 - 3424523520 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{12} b^7 + 1551313995 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{12} b^8 - 450357600 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^{12} b^9 - 14549535 \sqrt{ax^{\frac{1}{3}}+b} a^{12} b^{10}}{a^{11} b^{10} x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")

[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(19/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 5503713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 14549535*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a

$$3.176 \quad \int x^2 (bx^{2/3} + ax)^{3/2} dx$$

Optimal. Leaf size=343

$$-\frac{1048576b^{11}(ax + bx^{2/3})^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}(ax + bx^{2/3})^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9(ax + bx^{2/3})^{5/2}}{4345965a^{10}x} + \frac{65536b^8(ax + bx^{2/3})^{5/2}}{1448655a^9x^{2/3}} - \frac{90112b^7(ax + bx^{2/3})^{5/2}}{111435a^8}$$

[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3) + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(4345965*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90112*b^7*(b*x^(2/3) + a*x)^(5/2))/(111435*a^8) - (11264*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (176*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)

Rubi [A] time = 0.616286, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$-\frac{1048576b^{11}(ax + bx^{2/3})^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}(ax + bx^{2/3})^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9(ax + bx^{2/3})^{5/2}}{4345965a^{10}x} + \frac{65536b^8(ax + bx^{2/3})^{5/2}}{1448655a^9x^{2/3}} - \frac{90112b^7(ax + bx^{2/3})^{5/2}}{111435a^8}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2), x]

[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3) + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(4345965*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90112*b^7*(b*x^(2/3) + a*x)^(5/2))/(111435*a^8) - (11264*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (176*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)

Rule 2016

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

```

Rule 2002

```

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]

```

Rule 2014

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\int x^2 (bx^{2/3} + ax)^{3/2} dx &= \frac{2x^2 (bx^{2/3} + ax)^{5/2}}{9a} - \frac{(22b) \int x^{5/3} (bx^{2/3} + ax)^{3/2} dx}{27a} \\
&= -\frac{44bx^{5/3} (bx^{2/3} + ax)^{5/2}}{225a^2} + \frac{2x^2 (bx^{2/3} + ax)^{5/2}}{9a} + \frac{(88b^2) \int x^{4/3} (bx^{2/3} + ax)^{3/2} dx}{135a^2} \\
&= \frac{176b^2 x^{4/3} (bx^{2/3} + ax)^{5/2}}{1035a^3} - \frac{44bx^{5/3} (bx^{2/3} + ax)^{5/2}}{225a^2} + \frac{2x^2 (bx^{2/3} + ax)^{5/2}}{9a} - \frac{(176b^3) \int x (bx^{2/3} + ax)^{3/2} dx}{345a^3} \\
&= -\frac{352b^3 x (bx^{2/3} + ax)^{5/2}}{2415a^4} + \frac{176b^2 x^{4/3} (bx^{2/3} + ax)^{5/2}}{1035a^3} - \frac{44bx^{5/3} (bx^{2/3} + ax)^{5/2}}{225a^2} + \frac{2x^2 (bx^{2/3} + ax)^{5/2}}{9a} \\
&= \frac{5632b^4 x^{2/3} (bx^{2/3} + ax)^{5/2}}{45885a^5} - \frac{352b^3 x (bx^{2/3} + ax)^{5/2}}{2415a^4} + \frac{176b^2 x^{4/3} (bx^{2/3} + ax)^{5/2}}{1035a^3} - \frac{44bx^{5/3} (bx^{2/3} + ax)^{5/2}}{225a^2} \\
&= -\frac{11264b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{111435a^6} + \frac{5632b^4 x^{2/3} (bx^{2/3} + ax)^{5/2}}{45885a^5} - \frac{352b^3 x (bx^{2/3} + ax)^{5/2}}{2415a^4} + \frac{176b^2 x^{4/3} (bx^{2/3} + ax)^{5/2}}{1035a^3} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} - \frac{11264b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{111435a^6} + \frac{5632b^4 x^{2/3} (bx^{2/3} + ax)^{5/2}}{45885a^5} - \frac{352b^3 x (bx^{2/3} + ax)^{5/2}}{2415a^4} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} - \frac{90112b^7 (bx^{2/3} + ax)^{5/2}}{1448655a^8 \sqrt[3]{x}} - \frac{11264b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{111435a^6} + \frac{5632b^4 x^{2/3} (bx^{2/3} + ax)^{5/2}}{45885a^5} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} + \frac{65536b^8 (bx^{2/3} + ax)^{5/2}}{1448655a^9 x^{2/3}} - \frac{90112b^7 (bx^{2/3} + ax)^{5/2}}{1448655a^8 \sqrt[3]{x}} - \frac{11264b^5 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{111435a^6} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} - \frac{131072b^9 (bx^{2/3} + ax)^{5/2}}{4345965a^{10} x} + \frac{65536b^8 (bx^{2/3} + ax)^{5/2}}{1448655a^9 x^{2/3}} - \frac{90112b^7 (bx^{2/3} + ax)^{5/2}}{1448655a^8 \sqrt[3]{x}} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} + \frac{524288b^{10} (bx^{2/3} + ax)^{5/2}}{30421755a^{11} x^{4/3}} - \frac{131072b^9 (bx^{2/3} + ax)^{5/2}}{4345965a^{10} x} + \frac{65536b^8 (bx^{2/3} + ax)^{5/2}}{1448655a^9 x^{2/3}} \\
&= \frac{45056b^6 (bx^{2/3} + ax)^{5/2}}{557175a^7} - \frac{1048576b^{11} (bx^{2/3} + ax)^{5/2}}{152108775a^{12} x^{5/3}} + \frac{524288b^{10} (bx^{2/3} + ax)^{5/2}}{30421755a^{11} x^{4/3}} - \frac{131072b^9 (bx^{2/3} + ax)^{5/2}}{4345965a^{10} x}
\end{aligned}$$

Mathematica [A] time = 0.131135, size = 172, normalized size = 0.5

$$\frac{2(a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} (12932920a^9 b^2 x^3 - 11085360a^8 b^3 x^{8/3} + 9335040a^7 b^4 x^{7/3} - 7687680a^6 b^5 x^2 + 6150144a^5 b^6 x^{5/3} - 2488320a^4 b^7 x^{4/3} + 524288a^3 b^8 x^{2/3} - 90112a^2 b^9 x^{1/3} + 11264ab^{10} - 11264b^{11})}{152108775a^{12} x^{5/3}}$$

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Antiderivative was successfully verified.

[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]

[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-524288*b^11 + 1310720*a*b^10*x^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(10/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x^(1/3))

Maple [A] time = 0.004, size = 145, normalized size = 0.4

$$\frac{2}{152108775 x a^{12}} \left(b x^{\frac{2}{3}} + a x \right)^{\frac{3}{2}} \left(b + a \sqrt[3]{x} \right) \left(16900975 x^{11/3} a^{11} - 14872858 x^{10/3} a^{10} b + 12932920 x^3 a^9 b^2 - 11085360 x^{8/3} a^8 b^3 + 9335040 x^{7/3} a^7 b^4 - 7687680 x^2 a^6 b^5 + 6150144 x^{5/3} a^5 b^6 - 4730880 x^{4/3} a^4 b^7 + 3440640 x a^3 b^8 - 2293760 x^{2/3} a^2 b^9 + 1310720 x^{1/3} a b^{10} - 524288 b^{11} \right) / x / a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^(2/3)+a*x)^(3/2),x)

[Out] 2/152108775*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(16900975*x^(11/3)*a^11-14872858*x^(10/3)*a^10*b+12932920*x^3*a^9*b^2-11085360*x^(8/3)*a^8*b^3+9335040*x^(7/3)*a^7*b^4-7687680*x^2*a^6*b^5+6150144*x^(5/3)*a^5*b^6-4730880*x^(4/3)*a^4*b^7+3440640*x*a^3*b^8-2293760*x^(2/3)*a^2*b^9+1310720*x^(1/3)*a*b^10-524288*b^11)/x/a^12

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a x + b x^{\frac{2}{3}} \right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18264, size = 508, normalized size = 1.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (2028117*(a*x^(1/3) + b)^(25/2) - 2424
9225*(a*x^(1/3) + b)^(23/2)*b + 132793375*(a*x^(1/3) + b)^(21/2)*b^2 - 4403
14875*(a*x^(1/3) + b)^(19/2)*b^3 + 984233250*(a*x^(1/3) + b)^(17/2)*b^4 - 1
561650090*(a*x^(1/3) + b)^(15/2)*b^5 + 1801903950*(a*x^(1/3) + b)^(13/2)*b^
6 - 1521087750*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)
*b^8 - 398380125*(a*x^(1/3) + b)^(7/2)*b^9 + 111546435*(a*x^(1/3) + b)^(5/2)
)*b^10 - 16900975*(a*x^(1/3) + b)^(3/2)*b^11)/a^12) - 2/152108775*a*(419430
4*b^(27/2)/a^13 - (16900975*(a*x^(1/3) + b)^(27/2) - 219036636*(a*x^(1/3) +
b)^(25/2)*b + 1309458150*(a*x^(1/3) + b)^(23/2)*b^2 - 4780561500*(a*x^(1/3)
+ b)^(21/2)*b^3 + 11888501625*(a*x^(1/3) + b)^(19/2)*b^4 - 21259438200*(a
*x^(1/3) + b)^(17/2)*b^5 + 28109701620*(a*x^(1/3) + b)^(15/2)*b^6 - 2780080
3800*(a*x^(1/3) + b)^(13/2)*b^7 + 20534684625*(a*x^(1/3) + b)^(11/2)*b^8 -
11154643500*(a*x^(1/3) + b)^(9/2)*b^9 + 4302505350*(a*x^(1/3) + b)^(7/2)*b^
10 - 1095183180*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2)
)*b^12)/a^13)
```

3.177 $\int x (bx^{2/3} + ax)^{3/2} dx$

Optimal. Leaf size=255

$$\frac{65536b^8 (ax + bx^{2/3})^{5/2}}{4849845a^9x^{5/3}} - \frac{32768b^7 (ax + bx^{2/3})^{5/2}}{969969a^8x^{4/3}} + \frac{8192b^6 (ax + bx^{2/3})^{5/2}}{138567a^7x} - \frac{4096b^5 (ax + bx^{2/3})^{5/2}}{46189a^6x^{2/3}} + \frac{512b^4 (ax + bx^{2/3})^{5/2}}{4199a^5\sqrt[3]{x}}$$

[Out] $(-256*b^3*(b*x^{(2/3)} + a*x)^{(5/2)})/(1615*a^4) + (65536*b^8*(b*x^{(2/3)} + a*x)^{(5/2)})/(4849845*a^9*x^{(5/3)}) - (32768*b^7*(b*x^{(2/3)} + a*x)^{(5/2)})/(969969*a^8*x^{(4/3)}) + (8192*b^6*(b*x^{(2/3)} + a*x)^{(5/2)})/(138567*a^7*x) - (4096*b^5*(b*x^{(2/3)} + a*x)^{(5/2)})/(46189*a^6*x^{(2/3)}) + (512*b^4*(b*x^{(2/3)} + a*x)^{(5/2)})/(4199*a^5*x^{(1/3)}) + (64*b^2*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(5/2)})/(323*a^3) - (32*b*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(5/2)})/(133*a^2) + (2*x*(b*x^{(2/3)} + a*x)^{(5/2)})/(7*a)$

Rubi [A] time = 0.42205, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2016, 2002, 2014}

$$\frac{65536b^8 (ax + bx^{2/3})^{5/2}}{4849845a^9x^{5/3}} - \frac{32768b^7 (ax + bx^{2/3})^{5/2}}{969969a^8x^{4/3}} + \frac{8192b^6 (ax + bx^{2/3})^{5/2}}{138567a^7x} - \frac{4096b^5 (ax + bx^{2/3})^{5/2}}{46189a^6x^{2/3}} + \frac{512b^4 (ax + bx^{2/3})^{5/2}}{4199a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[x*(b*x^(2/3) + a*x)^(3/2), x]

[Out] $(-256*b^3*(b*x^{(2/3)} + a*x)^{(5/2)})/(1615*a^4) + (65536*b^8*(b*x^{(2/3)} + a*x)^{(5/2)})/(4849845*a^9*x^{(5/3)}) - (32768*b^7*(b*x^{(2/3)} + a*x)^{(5/2)})/(969969*a^8*x^{(4/3)}) + (8192*b^6*(b*x^{(2/3)} + a*x)^{(5/2)})/(138567*a^7*x) - (4096*b^5*(b*x^{(2/3)} + a*x)^{(5/2)})/(46189*a^6*x^{(2/3)}) + (512*b^4*(b*x^{(2/3)} + a*x)^{(5/2)})/(4199*a^5*x^{(1/3)}) + (64*b^2*x^{(1/3)}*(b*x^{(2/3)} + a*x)^{(5/2)})/(323*a^3) - (32*b*x^{(2/3)}*(b*x^{(2/3)} + a*x)^{(5/2)})/(133*a^2) + (2*x*(b*x^{(2/3)} + a*x)^{(5/2)})/(7*a)$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}

, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_)^m)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegersQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int x (bx^{2/3} + ax)^{3/2} dx &= \frac{2x (bx^{2/3} + ax)^{5/2}}{7a} - \frac{(16b) \int x^{2/3} (bx^{2/3} + ax)^{3/2} dx}{21a} \\
&= -\frac{32bx^{2/3} (bx^{2/3} + ax)^{5/2}}{133a^2} + \frac{2x (bx^{2/3} + ax)^{5/2}}{7a} + \frac{(32b^2) \int \sqrt[3]{x} (bx^{2/3} + ax)^{3/2} dx}{57a^2} \\
&= \frac{64b^2 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{323a^3} - \frac{32bx^{2/3} (bx^{2/3} + ax)^{5/2}}{133a^2} + \frac{2x (bx^{2/3} + ax)^{5/2}}{7a} - \frac{(128b^3) \int (bx^{2/3} + ax)^3}{323a^3} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} + \frac{64b^2 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{323a^3} - \frac{32bx^{2/3} (bx^{2/3} + ax)^{5/2}}{133a^2} + \frac{2x (bx^{2/3} + ax)^{5/2}}{7a} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} + \frac{512b^4 (bx^{2/3} + ax)^{5/2}}{4199a^5 \sqrt[3]{x}} + \frac{64b^2 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{323a^3} - \frac{32bx^{2/3} (bx^{2/3} + ax)^{5/2}}{133a^2} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} - \frac{4096b^5 (bx^{2/3} + ax)^{5/2}}{46189a^6 x^{2/3}} + \frac{512b^4 (bx^{2/3} + ax)^{5/2}}{4199a^5 \sqrt[3]{x}} + \frac{64b^2 \sqrt[3]{x} (bx^{2/3} + ax)^{5/2}}{323a^3} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} + \frac{8192b^6 (bx^{2/3} + ax)^{5/2}}{138567a^7 x} - \frac{4096b^5 (bx^{2/3} + ax)^{5/2}}{46189a^6 x^{2/3}} + \frac{512b^4 (bx^{2/3} + ax)^{5/2}}{4199a^5 \sqrt[3]{x}} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} - \frac{32768b^7 (bx^{2/3} + ax)^{5/2}}{969969a^8 x^{4/3}} + \frac{8192b^6 (bx^{2/3} + ax)^{5/2}}{138567a^7 x} - \frac{4096b^5 (bx^{2/3} + ax)^{5/2}}{46189a^6 x^{2/3}} \\
&= -\frac{256b^3 (bx^{2/3} + ax)^{5/2}}{1615a^4} + \frac{65536b^8 (bx^{2/3} + ax)^{5/2}}{4849845a^9 x^{5/3}} - \frac{32768b^7 (bx^{2/3} + ax)^{5/2}}{969969a^8 x^{4/3}} + \frac{8192b^6 (bx^{2/3} + ax)^{5/2}}{138567a^7 x}
\end{aligned}$$

Mathematica [A] time = 0.0855035, size = 135, normalized size = 0.53

$$\frac{2(a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} (480480a^6 b^2 x^2 - 384384a^5 b^3 x^{5/3} + 295680a^4 b^4 x^{4/3} + 143360a^2 b^6 x^{2/3} - 215040a^3 b^5 x - 583440)}{4849845a^9 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2), x]

[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(32768*b^8 - 81920*a*b^7*x^(1/3) + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^8*x^(8/3)))/(4849845*a^9*x^(1/3))

Maple [A] time = 0.004, size = 112, normalized size = 0.4

$$\frac{2}{4849845 x a^9} \left(b x^{\frac{2}{3}} + a x \right)^{\frac{3}{2}} \left(b + a \sqrt[3]{x} \right) \left(692835 x^{\frac{8}{3}} a^8 - 583440 x^{\frac{7}{3}} a^7 b + 480480 x^2 a^6 b^2 - 384384 x^{\frac{5}{3}} a^5 b^3 + 295680 x^{\frac{4}{3}} a^4 b^4 - 215040 x a^3 b^5 + 143360 x^{\frac{2}{3}} a^2 b^6 - 81920 x^{\frac{1}{3}} a b^7 + 32768 b^8 \right) / x / a^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^(2/3)+a*x)^(3/2),x)`

[Out] $2/4849845*(b*x^{(2/3)+a*x})^{(3/2)}*(b+a*x^{(1/3)})*(692835*x^{(8/3)}*a^8-583440*x^{(7/3)}*a^7*b+480480*x^2*a^6*b^2-384384*x^{(5/3)}*a^5*b^3+295680*x^{(4/3)}*a^4*b^4-215040*x*a^3*b^5+143360*x^{(2/3)}*a^2*b^6-81920*x^{(1/3)}*a*b^7+32768*b^8)/x/a^9$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a x + b x^{\frac{2}{3}} \right)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(2/3))^(3/2)*x, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.16902, size = 394, normalized size = 1.55

$$-\frac{2}{692835}b \left(\frac{32768b^{\frac{19}{2}}}{a^9} - \frac{109395 \left(ax^{\frac{1}{3}} + b\right)^{\frac{19}{2}} - 978120 \left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}}b + 3879876 \left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}}b^2 - 8953560 \left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}}b^3}{a^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (109395*(a*x^(1/3) + b)^(19/2) - 978120*(a*x^(1/3) + b)^(17/2)*b + 3879876*(a*x^(1/3) + b)^(15/2)*b^2 - 8953560*(a*x^(1/3) + b)^(13/2)*b^3 + 13226850*(a*x^(1/3) + b)^(11/2)*b^4 - 12932920*(a*x^(1/3) + b)^(9/2)*b^5 + 8314020*(a*x^(1/3) + b)^(7/2)*b^6 - 3325608*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8)/a^9 + 2/1616615*a*(65536*b^(21/2)/a^10 + (230945*(a*x^(1/3) + b)^(21/2) - 2297295*(a*x^(1/3) + b)^(19/2)*b + 10270260*(a*x^(1/3) + b)^(17/2)*b^2 - 27159132*(a*x^(1/3) + b)^(15/2)*b^3 + 47006190*(a*x^(1/3) + b)^(13/2)*b^4 - 55552770*(a*x^(1/3) + b)^(11/2)*b^5 + 45265220*(a*x^(1/3) + b)^(9/2)*b^6 - 24942060*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 1616615*(a*x^(1/3) + b)^(3/2)*b^9)/a^10)

3.178 $\int (bx^{2/3} + ax)^{3/2} dx$

Optimal. Leaf size=169

$$\frac{512b^5(ax + bx^{2/3})^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4(ax + bx^{2/3})^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(ax + bx^{2/3})^{5/2}}{429a^4x} + \frac{32b^2(ax + bx^{2/3})^{5/2}}{143a^3x^{2/3}} - \frac{4b(ax + bx^{2/3})^{5/2}}{13a^2\sqrt[3]{x}} + \frac{2}{a}$$

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(15015*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2))/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rubi [A] time = 0.249476, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2002, 2016, 2014}

$$\frac{512b^5(ax + bx^{2/3})^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4(ax + bx^{2/3})^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(ax + bx^{2/3})^{5/2}}{429a^4x} + \frac{32b^2(ax + bx^{2/3})^{5/2}}{143a^3x^{2/3}} - \frac{4b(ax + bx^{2/3})^{5/2}}{13a^2\sqrt[3]{x}} + \frac{2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2), x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(15015*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2))/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int (bx^{2/3} + ax)^{3/2} dx &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} - \frac{(2b) \int \frac{(bx^{2/3} + ax)^{3/2}}{\sqrt[3]{x}} dx}{3a} \\
 &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{(16b^2) \int \frac{(bx^{2/3} + ax)^{3/2}}{x^{2/3}} dx}{39a^2} \\
 &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} + \frac{32b^2(bx^{2/3} + ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2\sqrt[3]{x}} - \frac{(32b^3) \int \frac{(bx^{2/3} + ax)^{3/2}}{x} dx}{143a^3} \\
 &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} - \frac{64b^3(bx^{2/3} + ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3} + ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{(128b^4) \int \frac{(bx^{2/3} + ax)^{3/2}}{x^2} dx}{128a^4} \\
 &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} + \frac{256b^4(bx^{2/3} + ax)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(bx^{2/3} + ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3} + ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2\sqrt[3]{x}} \\
 &= \frac{2(bx^{2/3} + ax)^{5/2}}{5a} - \frac{512b^5(bx^{2/3} + ax)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4(bx^{2/3} + ax)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(bx^{2/3} + ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3} + ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3} + ax)^{5/2}}{13a^2\sqrt[3]{x}}
 \end{aligned}$$

Mathematica [A] time = 0.0578055, size = 98, normalized size = 0.58

$$\frac{2(a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} (-1120a^2b^3x^{2/3} + 1680a^3b^2x - 2310a^4bx^{4/3} + 3003a^5x^{5/3} + 640ab^4\sqrt[3]{x} - 256b^5)}{15015a^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2), x]


```
[Out] (2*(b + a*x^(1/3))^2*sqrt[b*x^(2/3) + a*x]*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x^(1/3))
```

Maple [A] time = 0.003, size = 79, normalized size = 0.5

$$\frac{2}{15015 x a^6} \left(b x^{\frac{2}{3}} + a x \right)^{\frac{3}{2}} \left(b + a \sqrt[3]{x} \right) \left(3003 x^{5/3} a^5 - 2310 x^{4/3} a^4 b + 1680 x a^3 b^2 - 1120 x^{2/3} a^2 b^3 + 640 \sqrt[3]{x} a b^4 - 256 b^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/15015*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(3003*x^(5/3)*a^5-2310*x^(4/3)*
a^4*b+1680*x*a^3*b^2-1120*x^(2/3)*a^2*b^3+640*x^(1/3)*a*b^4-256*b^5)/x/a^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a x + b x^{\frac{2}{3}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral((a*x + b*x**(2/3))**(3/2), x)

Giac [A] time = 1.22769, size = 281, normalized size = 1.66

$$\frac{2}{3003} b \left(\frac{256 b^{\frac{13}{2}}}{a^6} + \frac{693 \left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} - 4095 \left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} b + 10010 \left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} b^2 - 12870 \left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} b^3 + 9009 \left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} b^4}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] 2/3003*b*(256*b^(13/2)/a^6 + (693*(a*x^(1/3) + b)^(13/2) - 4095*(a*x^(1/3) + b)^(11/2)*b + 10010*(a*x^(1/3) + b)^(9/2)*b^2 - 12870*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 3003*(a*x^(1/3) + b)^(3/2)*b^5)/a^6 - 2/15015*a*(1024*b^(15/2)/a^7 - (3003*(a*x^(1/3) + b)^(15/2) - 20790*(a*x^(1/3) + b)^(13/2)*b + 61425*(a*x^(1/3) + b)^(11/2)*b^2 - 100100*(a*x^(1/3) + b)^(9/2)*b^3 + 96525*(a*x^(1/3) + b)^(7/2)*b^4 - 54054*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6)/a^7)

$$3.179 \quad \int \frac{(bx^{2/3} + ax)^{3/2}}{x} dx$$

Optimal. Leaf size=84

$$\frac{16b^2(ax + bx^{2/3})^{5/2}}{105a^3x^{5/3}} - \frac{8b(ax + bx^{2/3})^{5/2}}{21a^2x^{4/3}} + \frac{2(ax + bx^{2/3})^{5/2}}{3ax}$$

[Out] $(16*b^2*(b*x^{(2/3)} + a*x)^{(5/2)})/(105*a^3*x^{(5/3)}) - (8*b*(b*x^{(2/3)} + a*x)^{(5/2)})/(21*a^2*x^{(4/3)}) + (2*(b*x^{(2/3)} + a*x)^{(5/2)})/(3*a*x)$

Rubi [A] time = 0.138779, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$\frac{16b^2(ax + bx^{2/3})^{5/2}}{105a^3x^{5/3}} - \frac{8b(ax + bx^{2/3})^{5/2}}{21a^2x^{4/3}} + \frac{2(ax + bx^{2/3})^{5/2}}{3ax}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]

[Out] $(16*b^2*(b*x^{(2/3)} + a*x)^{(5/2)})/(105*a^3*x^{(5/3)}) - (8*b*(b*x^{(2/3)} + a*x)^{(5/2)})/(21*a^2*x^{(4/3)}) + (2*(b*x^{(2/3)} + a*x)^{(5/2)})/(3*a*x)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^{2/3} + ax)^{3/2}}{x} dx &= \frac{2(bx^{2/3} + ax)^{5/2}}{3ax} - \frac{(4b) \int \frac{(bx^{2/3} + ax)^{3/2}}{x^{4/3}} dx}{9a} \\
&= -\frac{8b(bx^{2/3} + ax)^{5/2}}{21a^2x^{4/3}} + \frac{2(bx^{2/3} + ax)^{5/2}}{3ax} + \frac{(8b^2) \int \frac{(bx^{2/3} + ax)^{3/2}}{x^{5/3}} dx}{63a^2} \\
&= \frac{16b^2(bx^{2/3} + ax)^{5/2}}{105a^3x^{5/3}} - \frac{8b(bx^{2/3} + ax)^{5/2}}{21a^2x^{4/3}} + \frac{2(bx^{2/3} + ax)^{5/2}}{3ax}
\end{aligned}$$

Mathematica [A] time = 0.0579474, size = 63, normalized size = 0.75

$$\frac{2(a\sqrt[3]{x} + b)^2(35a^2x^{2/3} - 20ab\sqrt[3]{x} + 8b^2)\sqrt{ax + bx^{2/3}}}{105a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]

[Out] (2*(b + a*x^(1/3))^2*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*Sqrt[b*x^(2/3) + a*x])/(105*a^3*x^(1/3))

Maple [A] time = 0.004, size = 48, normalized size = 0.6

$$\frac{2}{105xa^3} \left(bx^{\frac{2}{3}} + ax\right)^{\frac{3}{2}} (b + a\sqrt[3]{x}) (35x^{2/3}a^2 - 20\sqrt[3]{x}ab + 8b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(3/2)/x,x)

[Out] 2/105*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(35*x^(2/3)*a^2-20*x^(1/3)*a*b+8*b^2)/x/a^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)

[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)

Giac [A] time = 1.17726, size = 155, normalized size = 1.85

$$-\frac{16b^{\frac{9}{2}}}{105a^3} + \frac{2 \left(\frac{3 \left(15 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} - 42 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b + 35 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^2 \right) b}{a^2} + \frac{35 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} - 135 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b + 189 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^2 - 105 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^3}{a^2} \right)}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")

[Out] $-\frac{16}{105} b^{\frac{9}{2}} / a^3 + \frac{2}{105} (3(15(a*x^{\frac{1}{3}} + b)^{\frac{7}{2}} - 42(a*x^{\frac{1}{3}} + b)^{\frac{5}{2}} * b + 35(a*x^{\frac{1}{3}} + b)^{\frac{3}{2}} * b^2) * b / a^2 + (35(a*x^{\frac{1}{3}} + b)^{\frac{9}{2}} - 135(a*x^{\frac{1}{3}} + b)^{\frac{7}{2}} * b + 189(a*x^{\frac{1}{3}} + b)^{\frac{5}{2}} * b^2 - 105(a*x^{\frac{1}{3}} + b)^{\frac{3}{2}} * b^3) / a^2) / a$

$$3.180 \quad \int \frac{(bx^{2/3}+ax)^{3/2}}{x^2} dx$$

Optimal. Leaf size=78

$$-6b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right) + \frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}} + \frac{2(ax+bx^{2/3})^{3/2}}{x}$$

[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]

Rubi [A] time = 0.136998, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2021, 2029, 206}

$$-6b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right) + \frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}} + \frac{2(ax+bx^{2/3})^{3/2}}{x}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]

[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^{2/3} + ax)^{3/2}}{x^2} dx &= \frac{2(bx^{2/3} + ax)^{3/2}}{x} + b \int \frac{\sqrt{bx^{2/3} + ax}}{x^{4/3}} dx \\
&= \frac{6b\sqrt{bx^{2/3} + ax}}{\sqrt[3]{x}} + \frac{2(bx^{2/3} + ax)^{3/2}}{x} + b^2 \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx \\
&= \frac{6b\sqrt{bx^{2/3} + ax}}{\sqrt[3]{x}} + \frac{2(bx^{2/3} + ax)^{3/2}}{x} - (6b^2) \text{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right) \\
&= \frac{6b\sqrt{bx^{2/3} + ax}}{\sqrt[3]{x}} + \frac{2(bx^{2/3} + ax)^{3/2}}{x} - 6b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0689394, size = 88, normalized size = 1.13

$$\frac{2\sqrt{ax + bx^{2/3}} \left(\sqrt{a\sqrt[3]{x} + b} (a\sqrt[3]{x} + 4b) - 3b^{3/2} \tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x} + b}}{\sqrt{b}}\right) \right)}{\sqrt[3]{x}\sqrt{a\sqrt[3]{x} + b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

Maple [A] time = 0.004, size = 69, normalized size = 0.9

$$-2 \frac{(bx^{2/3} + ax)^{3/2}}{x(b + a\sqrt[3]{x})^{3/2}} \left(3b^{3/2} \text{Artanh}\left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}}\right) - (b + a\sqrt[3]{x})^{3/2} - 3b\sqrt{b + a\sqrt[3]{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(2/3)+a*x)^(3/2)/x^2,x)`

[Out] $-2*(b*x^{(2/3)+a*x})^{(3/2)}*(3*b^{(3/2)}*\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)})-(b+a*x^{(1/3)})^{(3/2)}-3*b*(b+a*x^{(1/3)})^{(1/2)})/x/(b+a*x^{(1/3)})^{(3/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)`

[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)

Giac [A] time = 1.21538, size = 112, normalized size = 1.44

$$\frac{6b^2 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} + 6\sqrt{ax^{\frac{1}{3}}+bb} - \frac{2\left(3b^2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4\sqrt{-bb^{\frac{3}{2}}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")

[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-b)*b^(3/2))/sqrt(-b)

$$3.181 \quad \int \frac{(bx^{2/3} + ax)^{3/2}}{x^3} dx$$

Optimal. Leaf size=113

$$\frac{3a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}} - \frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}} - \frac{3a\sqrt{ax+bx^{2/3}}}{4x} - \frac{(ax+bx^{2/3})^{3/2}}{x^2}$$

[Out] $(-3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4*x) - (3*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8*b*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/x^2 + (3*a^3*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(8*b^{(3/2)})$

Rubi [A] time = 0.183797, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}} - \frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}} - \frac{3a\sqrt{ax+bx^{2/3}}}{4x} - \frac{(ax+bx^{2/3})^{3/2}}{x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x^{(2/3)} + a*x)^{(3/2)}/x^3, x]$

[Out] $(-3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4*x) - (3*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(8*b*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/x^2 + (3*a^3*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(8*b^{(3/2)})$

Rule 2020

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p / (c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j)) / (c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \ || \ \text{GtQ}[c, 0]) \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m+j*p+1, 0]$

Rule 2025

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Simp}[c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)} / (a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1)) / (a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, m, p\}, x]$

&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 209

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(bx^{2/3} + ax)^{3/2}}{x^3} dx &= -\frac{(bx^{2/3} + ax)^{3/2}}{x^2} + \frac{1}{2}a \int \frac{\sqrt{bx^{2/3} + ax}}{x^2} dx \\
 &= -\frac{3a\sqrt{bx^{2/3} + ax}}{4x} - \frac{(bx^{2/3} + ax)^{3/2}}{x^2} + \frac{1}{8}a^2 \int \frac{1}{x\sqrt{bx^{2/3} + ax}} dx \\
 &= -\frac{3a\sqrt{bx^{2/3} + ax}}{4x} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{8bx^{2/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{x^2} - \frac{a^3 \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{16b} \\
 &= -\frac{3a\sqrt{bx^{2/3} + ax}}{4x} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{8bx^{2/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{x^2} + \frac{(3a^3) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{8b} \\
 &= -\frac{3a\sqrt{bx^{2/3} + ax}}{4x} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{8bx^{2/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{x^2} + \frac{3a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{8b^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 0.0525169, size = 61, normalized size = 0.54

$$\frac{6a^3 (a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{5}{2}, 4; \frac{7}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{5b^4 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3, x]

[Out] $(6a^3(b + a^{1/3})^2 \sqrt{bx^{2/3} + ax} \operatorname{Hypergeometric2F1}[5/2, 4, 7/2, 1 + (a^{1/3})/b]) / (5b^4 x^{1/3})$

Maple [A] time = 0.01, size = 93, normalized size = 0.8

$$\frac{1}{8x^2} \left(bx^{\frac{2}{3}} + ax \right)^{\frac{3}{2}} \left(3b^{7/2} \sqrt{b + a\sqrt[3]{x}} - 8b^{5/2} (b + a\sqrt[3]{x})^{3/2} - 3b^{3/2} (b + a\sqrt[3]{x})^{5/2} + 3 \operatorname{Arctanh} \left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}} \right) ba^3 x \right) b^{-\frac{5}{2}} (b + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(2/3)+a*x)^(3/2)/x^3,x)`

[Out] $1/8*(b*x^{(2/3)+a*x})^{(3/2)}*(3*b^{(7/2)}*(b+a*x^{(1/3)})^{(1/2)}-8*b^{(5/2)}*(b+a*x^{(1/3)})^{(3/2)}-3*b^{(3/2)}*(b+a*x^{(1/3)})^{(5/2)}+3*\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)})*b*a^3*x)/x^2/(b+a*x^{(1/3)})^{(3/2)}/b^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)

[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)

Giac [A] time = 1.24507, size = 124, normalized size = 1.1

$$\frac{3a^4 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb}} + \frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4 + 8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b - 3\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}a^4b^2}{a^3bx}$$

$8a$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")

[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a^4*b^2)/(a^3*b*x))/a

$$3.182 \quad \int \frac{(bx^{2/3} + ax)^{3/2}}{x^4} dx$$

Optimal. Leaf size=203

$$\frac{21a^5\sqrt{ax + bx^{2/3}}}{512b^4x^{2/3}} - \frac{7a^4\sqrt{ax + bx^{2/3}}}{256b^3x} + \frac{7a^3\sqrt{ax + bx^{2/3}}}{320b^2x^{4/3}} - \frac{21a^6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}} - \frac{3a^2\sqrt{ax + bx^{2/3}}}{160bx^{5/3}} - \frac{3a\sqrt{ax + bx^{2/3}}}{20x^2}$$

[Out] $(-3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(20*x^2) - (3*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(160*b*x^{(5/3)}) + (7*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(320*b^2*x^{(4/3)}) - (7*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(256*b^3*x) + (21*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(512*b^4*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/(2*x^3) - (21*a^6*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(512*b^{(9/2)})$

Rubi [A] time = 0.34037, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^5\sqrt{ax + bx^{2/3}}}{512b^4x^{2/3}} - \frac{7a^4\sqrt{ax + bx^{2/3}}}{256b^3x} + \frac{7a^3\sqrt{ax + bx^{2/3}}}{320b^2x^{4/3}} - \frac{21a^6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}} - \frac{3a^2\sqrt{ax + bx^{2/3}}}{160bx^{5/3}} - \frac{3a\sqrt{ax + bx^{2/3}}}{20x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x^{(2/3)} + a*x)^{(3/2)}/x^4, x]$

[Out] $(-3*a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(20*x^2) - (3*a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(160*b*x^{(5/3)}) + (7*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(320*b^2*x^{(4/3)}) - (7*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(256*b^3*x) + (21*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(512*b^4*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/(2*x^3) - (21*a^6*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(512*b^{(9/2)})$

Rule 2020

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*x^p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^{2/3} + ax)^{3/2}}{x^4} dx &= -\frac{(bx^{2/3} + ax)^{3/2}}{2x^3} + \frac{1}{4}a \int \frac{\sqrt{bx^{2/3} + ax}}{x^3} dx \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3} + \frac{1}{40}a^2 \int \frac{1}{x^2\sqrt{bx^{2/3} + ax}} dx \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{160bx^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3} - \frac{(7a^3) \int \frac{1}{x^{5/3}\sqrt{bx^{2/3} + ax}} dx}{320b} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3} + ax}}{320b^2x^{4/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3} + \frac{(7a^4) \int \frac{1}{x^{4/3}\sqrt{bx^{2/3} + ax}} dx}{384b^2} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3} + ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3} + ax}}{256b^3x} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3} - \frac{(7a^5) \int \frac{1}{x^{1/3}\sqrt{bx^{2/3} + ax}} dx}{512b^3} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3} + ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3} + ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3} + ax}}{512b^4x^{2/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3} + ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3} + ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3} + ax}}{512b^4x^{2/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{2x^3}
\end{aligned}$$

Mathematica [C] time = 0.0463573, size = 61, normalized size = 0.3

$$\frac{6a^6 (a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{5}{2}, 7; \frac{7}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{5b^7 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]

[Out] (-6*a^6*(b + a*x^(1/3))^2*sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))

Maple [A] time = 0.012, size = 139, normalized size = 0.7

$$\frac{1}{2560x^3} \left(bx^{\frac{2}{3}} + ax \right)^{\frac{3}{2}} \left(105b^{9/2} (b + a\sqrt[3]{x})^{11/2} - 595b^{11/2} (b + a\sqrt[3]{x})^{9/2} + 1386b^{13/2} (b + a\sqrt[3]{x})^{7/2} - 1686b^{15/2} (b + a\sqrt[3]{x})^{5/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(3/2)/x^4,x)

[Out] 1/2560*(b*x^(2/3)+a*x)^(3/2)*(105*b^(9/2)*(b+a*x^(1/3))^(11/2)-595*b^(11/2)*(b+a*x^(1/3))^(9/2)+1386*b^(13/2)*(b+a*x^(1/3))^(7/2)-1686*b^(15/2)*(b+a*x^(1/3))^(5/2)-595*b^(17/2)*(b+a*x^(1/3))^(3/2)+105*b^(19/2)*(b+a*x^(1/3))^(1/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^6*x^2/x^3/(b+a*x^(1/3))^(3/2)/b^(17/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)

[Out] Timed out

Giac [A] time = 1.25095, size = 193, normalized size = 0.95

$$\frac{105 a^7 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^7 - 595 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^7 b + 1386 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^7 b^2 - 1686 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^7 b^3 - 595 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^7 b^4 + 105 \sqrt{ax^{\frac{1}{3}}+b} a^7 b^5}{a^6 b^4 x^2}$$

2560 a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")

[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2)/a

$$3.183 \quad \int \frac{(bx^{2/3} + ax)^{3/2}}{x^5} dx$$

Optimal. Leaf size=291

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}} + \frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x} - \frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}} + \frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}} - \frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2} + \frac{13a^3\sqrt{ax+bx^{2/3}}}{2688b^2x^{7/3}}$$

[Out] $-(a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16*x^3) - (a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(224*b*x^{(8/3)}) + (13*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2688*b^2*x^{(7/3)}) - (143*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(26880*b^3*x^2) + (429*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(71680*b^4*x^{(5/3)}) - (143*a^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(20480*b^5*x^{(4/3)}) + (143*a^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16384*b^6*x) - (429*a^8*\text{Sqrt}[b*x^{(2/3)} + a*x])/(32768*b^7*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/(3*x^4) + (429*a^9*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(32768*b^{(15/2)})$

Rubi [A] time = 0.521562, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}} + \frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x} - \frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}} + \frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}} - \frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2} + \frac{13a^3\sqrt{ax+bx^{2/3}}}{2688b^2x^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]

[Out] $-(a*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16*x^3) - (a^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(224*b*x^{(8/3)}) + (13*a^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2688*b^2*x^{(7/3)}) - (143*a^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(26880*b^3*x^2) + (429*a^5*\text{Sqrt}[b*x^{(2/3)} + a*x])/(71680*b^4*x^{(5/3)}) - (143*a^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(20480*b^5*x^{(4/3)}) + (143*a^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(16384*b^6*x) - (429*a^8*\text{Sqrt}[b*x^{(2/3)} + a*x])/(32768*b^7*x^{(2/3)}) - (b*x^{(2/3)} + a*x)^{(3/2)}/(3*x^4) + (429*a^9*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/(32768*b^{(15/2)})$

Rule 2020

Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a*x^j + b*x^n)^p)/(c*(m+j*p+1)), x] - Dist[(b*p*(n-j))/(c^n*(m+j*p+1)), Int[(c*x)^(m+n)*(a*x^j + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers

$Q[j, n] \parallel \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m + j*p + 1, 0]$

Rule 2025

$\text{Int}[(c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{j-1} \cdot (c \cdot x)^{m-j+1} \cdot (a \cdot x^j + b \cdot x^n)^{p+1}) / (a \cdot (m + j \cdot p + 1)), x] - \text{Dist}[(b \cdot (m + n \cdot p + n - j + 1)) / (a \cdot c^{n-j} \cdot (m + j \cdot p + 1)), \text{Int}[(c \cdot x)^{m+n-j} \cdot (a \cdot x^j + b \cdot x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, m, p\}, x] \ \&\& \ \text{!IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \ \&\& \ \text{LtQ}[m + j \cdot p + 1, 0]$

Rule 2029

$\text{Int}[x^m / \text{Sqrt}[a \cdot x^j + b \cdot x^n], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a \cdot x^2), x], x, x^{(j/2)} / \text{Sqrt}[a \cdot x^j + b \cdot x^n]], x] /;$ $\text{FreeQ}\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{(bx^{2/3} + ax)^{3/2}}{x^5} dx &= -\frac{(bx^{2/3} + ax)^{3/2}}{3x^4} + \frac{1}{6}a \int \frac{\sqrt{bx^{2/3} + ax}}{x^4} dx \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} + \frac{1}{96}a^2 \int \frac{1}{x^3\sqrt{bx^{2/3} + ax}} dx \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(13a^3) \int \frac{1}{x^{8/3}\sqrt{bx^{2/3} + ax}} dx}{1344b} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} + \frac{(143a^4) \int \frac{1}{x^{7/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{6/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{5/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{4/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{3/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{x^{1/3}\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3} + ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3} + ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3} + ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3} + ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3} + ax}}{71680b^4x^{5/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{3x^4} - \frac{(143a^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{16128b^2}
\end{aligned}$$

Mathematica [C] time = 0.0501321, size = 61, normalized size = 0.21

$$\frac{6a^9 (a\sqrt[3]{x} + b)^2 \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{5}{2}, 10; \frac{7}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{5b^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]

[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))

Maple [A] time = 0.014, size = 181, normalized size = 0.6

$$\frac{1}{3440640 x^4} \left(b x^{\frac{2}{3}} + a x \right)^{\frac{3}{2}} \left(45045 b^{\frac{31}{2}} \sqrt{b + a \sqrt[3]{x}} - 390390 b^{\frac{29}{2}} (b + a \sqrt[3]{x})^{3/2} - 2633274 b^{\frac{27}{2}} (b + a \sqrt[3]{x})^{5/2} + 4349826 b^{\frac{25}{2}} (b + a \sqrt[3]{x})^{7/2} - 4685824 b^{\frac{23}{2}} (b + a \sqrt[3]{x})^{9/2} + 3317886 b^{\frac{21}{2}} (b + a \sqrt[3]{x})^{11/2} - 1495494 b^{\frac{19}{2}} (b + a \sqrt[3]{x})^{13/2} + 390390 b^{\frac{17}{2}} (b + a \sqrt[3]{x})^{15/2} - 45045 b^{\frac{15}{2}} (b + a \sqrt[3]{x})^{17/2} + 45045 \operatorname{arctanh} \left(\frac{(b + a \sqrt[3]{x})^{1/2}}{b^{1/2}} \right) b^7 a^9 x^3 \right) / x^4 (b + a \sqrt[3]{x})^{3/2} / b^{29/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(2/3)+a*x)^(3/2)/x^5,x)`

[Out] `1/3440640*(b*x^(2/3)+a*x)^(3/2)*(45045*b^(31/2)*(b+a*x^(1/3))^(1/2)-390390*b^(29/2)*(b+a*x^(1/3))^(3/2)-2633274*b^(27/2)*(b+a*x^(1/3))^(5/2)+4349826*b^(25/2)*(b+a*x^(1/3))^(7/2)-4685824*b^(23/2)*(b+a*x^(1/3))^(9/2)+3317886*b^(21/2)*(b+a*x^(1/3))^(11/2)-1495494*b^(19/2)*(b+a*x^(1/3))^(13/2)+390390*b^(17/2)*(b+a*x^(1/3))^(15/2)-45045*b^(15/2)*(b+a*x^(1/3))^(17/2)+45045*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^9*x^3)/x^4/(b+a*x^(1/3))^(3/2)/b^(29/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)

[Out] Timed out

Giac [A] time = 1.372, size = 262, normalized size = 0.9

$$\frac{45045 a^{10} \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb^7}} + \frac{45045 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{10} - 390390 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{10} b + 1495494 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{10} b^2 - 3317886 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{10} b^3 + 4685824 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{10} b^4}{a^9 b^7 x^3}$$

3440640 a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/3440640*(45045*a^{10}*\arctan(\sqrt{a*x^{(1/3)}+b}/\sqrt{-b}))/(\sqrt{-b}*b^7) \\ & + (45045*(a*x^{(1/3)}+b)^{(17/2)}*a^{10} - 390390*(a*x^{(1/3)}+b)^{(15/2)}*a^{10}*b \\ & + 1495494*(a*x^{(1/3)}+b)^{(13/2)}*a^{10}*b^2 - 3317886*(a*x^{(1/3)}+b)^{(11/2)} \\ & *a^{10}*b^3 + 4685824*(a*x^{(1/3)}+b)^{(9/2)}*a^{10}*b^4 - 4349826*(a*x^{(1/3)}+b \\ &)^{(7/2)}*a^{10}*b^5 + 2633274*(a*x^{(1/3)}+b)^{(5/2)}*a^{10}*b^6 + 390390*(a*x^{(1/3)} \\ & + b)^{(3/2)}*a^{10}*b^7 - 45045*\sqrt{a*x^{(1/3)}+b}*a^{10}*b^8)/(a^9*b^7*x^3) \\ & /a \end{aligned}$$

$$3.184 \quad \int \frac{(bx^{2/3} + ax)^{3/2}}{x^6} dx$$

Optimal. Leaf size=379

$$\frac{12597a^{11}\sqrt{ax + bx^{2/3}}}{2097152b^{10}x^{2/3}} - \frac{4199a^{10}\sqrt{ax + bx^{2/3}}}{1048576b^9x} + \frac{4199a^9\sqrt{ax + bx^{2/3}}}{1310720b^8x^{4/3}} - \frac{12597a^8\sqrt{ax + bx^{2/3}}}{4587520b^7x^{5/3}} + \frac{4199a^7\sqrt{ax + bx^{2/3}}}{1720320b^6x^2} - \frac{4199a^6\sqrt{ax + bx^{2/3}}}{184320b^5x^{8/3}}$$

[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x])/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x) + (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(2097152*b^(21/2))

Rubi [A] time = 0.718415, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{11}\sqrt{ax + bx^{2/3}}}{2097152b^{10}x^{2/3}} - \frac{4199a^{10}\sqrt{ax + bx^{2/3}}}{1048576b^9x} + \frac{4199a^9\sqrt{ax + bx^{2/3}}}{1310720b^8x^{4/3}} - \frac{12597a^8\sqrt{ax + bx^{2/3}}}{4587520b^7x^{5/3}} + \frac{4199a^7\sqrt{ax + bx^{2/3}}}{1720320b^6x^2} - \frac{4199a^6\sqrt{ax + bx^{2/3}}}{184320b^5x^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6, x]

[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x])/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x) + (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(2097152*b^(21/2))

Rule 2020

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^{2/3} + ax)^{3/2}}{x^6} dx &= -\frac{(bx^{2/3} + ax)^{3/2}}{4x^5} + \frac{1}{8}a \int \frac{\sqrt{bx^{2/3} + ax}}{x^5} dx \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{(bx^{2/3} + ax)^{3/2}}{4x^5} + \frac{1}{176}a^2 \int \frac{1}{x^4\sqrt{bx^{2/3} + ax}} dx \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{4x^5} - \frac{(19a^3) \int \frac{1}{x^{11/3}\sqrt{bx^{2/3} + ax}} dx}{3520b} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{(bx^{2/3} + ax)^{3/2}}{4x^5} + \frac{(323a^4) \int \frac{1}{x^{10/3}\sqrt{bx^{2/3} + ax}} dx}{63360b^2} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} - \frac{(bx^{2/3} + ax)^{3/2}}{4x^5} - \frac{(323a^4) \int \frac{1}{x^{10/3}\sqrt{bx^{2/3} + ax}} dx}{63360b^2} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}} \\
&= -\frac{3a\sqrt{bx^{2/3} + ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3} + ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3} + ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3} + ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3} + ax}}{157696b^4x^{8/3}}
\end{aligned}$$

Mathematica [C] time = 0.0609208, size = 61, normalized size = 0.16

$$-\frac{6a^{12}(a\sqrt[3]{x} + b)^2\sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{5}{2}, 13; \frac{7}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{5b^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]

[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13, 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))

Maple [A] time = 0.016, size = 223, normalized size = 0.6

$$\frac{1}{2422210560 x^5} \left(b x^{\frac{2}{3}} + a x \right)^{\frac{3}{2}} \left(14549535 (b + a \sqrt[3]{x})^{23/2} b^{21/2} - 169744575 (b + a \sqrt[3]{x})^{21/2} b^{23/2} + 904981077 (b + a \sqrt[3]{x})^{19} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(2/3)+a*x)^(3/2)/x^6,x)

[Out] 1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(14549535*(b+a*x^(1/3))^(23/2)*b^(21/2)-169744575*(b+a*x^(1/3))^(21/2)*b^(23/2)+904981077*(b+a*x^(1/3))^(19/2)*b^(25/2)-2913648309*(b+a*x^(1/3))^(17/2)*b^(27/2)+6303782342*(b+a*x^(1/3))^(15/2)*b^(29/2)-9643633350*(b+a*x^(1/3))^(13/2)*b^(31/2)+10677769530*(b+a*x^(1/3))^(11/2)*b^(33/2)-8598579770*(b+a*x^(1/3))^(9/2)*b^(35/2)+4975837515*(b+a*x^(1/3))^(7/2)*b^(37/2)-2001671595*(b+a*x^(1/3))^(5/2)*b^(39/2)-169744575*(b+a*x^(1/3))^(3/2)*b^(41/2)+14549535*(b+a*x^(1/3))^(1/2)*b^(43/2)-14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^12*x^4/x^5/(b+a*x^(1/3))^(3/2)/b^(41/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)

[Out] Timed out

Giac [A] time = 1.44672, size = 331, normalized size = 0.87

$$\frac{14549535 a^{13} \arctan\left(\frac{\sqrt{\frac{1}{ax^3+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb^{10}}} + \frac{14549535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}} a^{13} - 169744575 \left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} a^{13} b + 904981077 \left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} a^{13} b^2 - 2913648309 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{13} b^3 + 6303782342 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{13} b^4 - 9643633350 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{13} b^5 + 10677769530 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{13} b^6 - 8598579770 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{13} b^7 + 4975837515 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{13} b^8 - 14549535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{13} b^9}{\sqrt{-bb^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")

[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^(1/3) + b)^(7/2)*a^13*b^8 - 14549535*(a*x^(1/3) + b)^(5/2)*a^13*b^9)

$$\frac{(1/3 + b)^{7/2} a^{13} b^8 - 2001671595 (a x^{1/3} + b)^{5/2} a^{13} b^9 - 169744575 (a x^{1/3} + b)^{3/2} a^{13} b^{10} + 14549535 \sqrt{a x^{1/3} + b} a^{13} b^{11}}{(a^{12} b^{10} x^4) a}$$

$$3.185 \quad \int \frac{x^4}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=401

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}} + \frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}} - \frac{131072b^9x\sqrt{ax+bx^{2/3}}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{ax+bx^{2/3}}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{ax+bx^{2/3}}}{557175a^8} + \frac{1171456b^6x^2\sqrt{ax+bx^{2/3}}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{ax+bx^{2/3}}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{ax+bx^{2/3}}}{137655a^5} - \frac{9152b^3x^3\sqrt{ax+bx^{2/3}}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{ax+bx^{2/3}}}{1725a^3} - \frac{52bx^{11/3}\sqrt{ax+bx^{2/3}}}{225a^2} + \frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}$$

[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (65536*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*x^(2/3) + a*x])/(9*a)

Rubi [A] time = 0.728113, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}} + \frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}} - \frac{131072b^9x\sqrt{ax+bx^{2/3}}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{ax+bx^{2/3}}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{ax+bx^{2/3}}}{557175a^8} + \frac{1171456b^6x^2\sqrt{ax+bx^{2/3}}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{ax+bx^{2/3}}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{ax+bx^{2/3}}}{137655a^5} - \frac{9152b^3x^3\sqrt{ax+bx^{2/3}}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{ax+bx^{2/3}}}{1725a^3} - \frac{52bx^{11/3}\sqrt{ax+bx^{2/3}}}{225a^2} + \frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[b*x^(2/3) + a*x], x]

[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (65536*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*x^(2/3) + a*x])/(9*a)

$x^{(2/3) + a*x}]/(9*a)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{bx^{2/3} + ax}} dx &= \frac{2x^4\sqrt{bx^{2/3} + ax}}{9a} - \frac{(26b) \int \frac{x^{11/3}}{\sqrt{bx^{2/3} + ax}} dx}{27a} \\
&= -\frac{52bx^{11/3}\sqrt{bx^{2/3} + ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3} + ax}}{9a} + \frac{(208b^2) \int \frac{x^{10/3}}{\sqrt{bx^{2/3} + ax}} dx}{225a^2} \\
&= \frac{416b^2x^{10/3}\sqrt{bx^{2/3} + ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3} + ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3} + ax}}{9a} - \frac{(4576b^3) \int \frac{x^3}{\sqrt{bx^{2/3} + ax}} dx}{5175a^3} \\
&= -\frac{9152b^3x^3\sqrt{bx^{2/3} + ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3} + ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3} + ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3} + ax}}{9a} + \frac{(18304b^4) \int \frac{x^2}{\sqrt{bx^{2/3} + ax}} dx}{1725a^3} \\
&= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3} + ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3} + ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3} + ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3} + ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3} + ax}}{9a} \\
&= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3} + ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3} + ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3} + ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3} + ax}}{1725a^3} \\
&= \frac{1171456b^6x^2\sqrt{bx^{2/3} + ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3} + ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3} + ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3} + ax}}{36225a^4} \\
&= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3} + ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3} + ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3} + ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3} + ax}}{137655a^5} \\
&= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3} + ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3} + ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3} + ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3} + ax}}{260015a^6} \\
&= -\frac{131072b^9x\sqrt{bx^{2/3} + ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3} + ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3} + ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3} + ax}}{3900225a^7} \\
&= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3} + ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3} + ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3} + ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3} + ax}}{557175a^8} \\
&= -\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3} + ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3} + ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3} + ax}}{185725a^9} \\
&= \frac{8388608b^{12}\sqrt{bx^{2/3} + ax}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3} + ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3} + ax}}{334305a^{10}} \\
&= \frac{8388608b^{12}\sqrt{bx^{2/3} + ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3} + ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3} + ax}}{2340135a^{11}}
\end{aligned}$$

Mathematica [A] time = 0.153033, size = 185, normalized size = 0.46

$$2\sqrt{ax + bx^{2/3}} (1410864a^{11}b^2x^{11/3} - 1478048a^{10}b^3x^{10/3} + 1555840a^9b^4x^3 - 1647360a^8b^5x^{8/3} + 1757184a^7b^6x^{7/3} - 1892352a^6b^7x^{2/3} + 1892352a^5b^8x^{1/3} - 1892352a^4b^9x^{2/3} + 1892352a^3b^{10}x^{5/3} - 1892352a^2b^{11}x^{8/3} + 1892352ab^{12}x^{11/3} - 1892352b^{13}x^{14/3})$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]

[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a^14*x^(1/3))

Maple [A] time = 0.004, size = 167, normalized size = 0.4

$$\frac{2}{11700675 a^{14}} \sqrt[3]{x} (b + a \sqrt[3]{x}) (1300075 x^{13/3} a^{13} - 1352078 x^4 a^{12} b + 1410864 x^{11/3} a^{11} b^2 - 1478048 x^{10/3} a^{10} b^3 + 1555840 x^7 a^9 b^4 - 1647360 x^{8/3} a^8 b^5 + 1757184 x^{7/3} a^7 b^6 - 1892352 x^2 a^6 b^7 + 2064384 x^{5/3} a^5 b^8 - 2293760 x^{4/3} a^4 b^9 + 2621440 x a^3 b^{10} - 3145728 x^{2/3} a^2 b^{11} + 4194304 x^{1/3} a b^{12} - 8388608 b^{13}) / (b x^{2/3} + a x)^{1/2} / a^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^(2/3)+a*x)^(1/2),x)

[Out] 2/11700675*x^(1/3)*(b+a*x^(1/3))*(1300075*x^(13/3)*a^13-1352078*x^4*a^12*b+1410864*x^(11/3)*a^11*b^2-1478048*x^(10/3)*a^10*b^3+1555840*x^3*a^9*b^4-1647360*x^(8/3)*a^8*b^5+1757184*x^(7/3)*a^7*b^6-1892352*x^2*a^6*b^7+2064384*x^(5/3)*a^5*b^8-2293760*x^(4/3)*a^4*b^9+2621440*x*a^3*b^10-3145728*x^(2/3)*a^2*b^11+4194304*x^(1/3)*a*b^12-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.14363, size = 278, normalized size = 0.69

$$\frac{16777216 b^{\frac{27}{2}}}{11700675 a^{14}} + \frac{2 \left(1300075 \left(ax^{\frac{1}{3}} + b \right)^{\frac{27}{2}} - 18253053 \left(ax^{\frac{1}{3}} + b \right)^{\frac{25}{2}} b + 119041650 \left(ax^{\frac{1}{3}} + b \right)^{\frac{23}{2}} b^2 - 478056150 \left(ax^{\frac{1}{3}} + b \right)^{\frac{21}{2}} b^3 + 1320944625 \left(ax^{\frac{1}{3}} + b \right)^{\frac{19}{2}} b^4 - 2657429775 \left(ax^{\frac{1}{3}} + b \right)^{\frac{17}{2}} b^5 + 4015671660 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} b^6 - 4633467300 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} b^7 + 4106936925 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} b^8 - 2788660875 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} b^9 + 1434168450 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b^{10} - 547591590 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^{11} + 152108775 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^{12} - 35102025 \sqrt{ax^{\frac{1}{3}} + b} b^{13} \right)}{a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13/a^14

$$3.186 \quad \int \frac{x^3}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=313

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}} - \frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8} + \frac{20480b^6x\sqrt{ax+bx^{2/3}}}{46189a^7}$$

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8) + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rubi [A] time = 0.53092, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}} - \frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8} + \frac{20480b^6x\sqrt{ax+bx^{2/3}}}{46189a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x], x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8) + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

```

Rule 2002

```

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]

```

Rule 2014

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{bx^{2/3} + ax}} dx &= \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} - \frac{(20b) \int \frac{x^{8/3}}{\sqrt{bx^{2/3} + ax}} dx}{21a} \\
&= -\frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} + \frac{(120b^2) \int \frac{x^{7/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^2} \\
&= \frac{720b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} - \frac{(1920b^3) \int \frac{x^2}{\sqrt{bx^{2/3} + ax}} dx}{2261a^3} \\
&= -\frac{768b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} + \frac{(256b^4) \int \frac{x}{\sqrt{bx^{2/3} + ax}} dx}{2261a^4} \\
&= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} \\
&= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} \\
&= \frac{20480b^6x\sqrt{bx^{2/3} + ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^4} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} \\
&= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3} + ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^5} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} \\
&= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3} + ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{46189a^6} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a} \\
&= -\frac{262144b^9\sqrt{bx^{2/3} + ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3} + ax}}{46189a^7} - \frac{40bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^2} \\
&= -\frac{262144b^9\sqrt{bx^{2/3} + ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3} + ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{323323a^8} + \frac{2x^3\sqrt{bx^{2/3} + ax}}{7a}
\end{aligned}$$

Mathematica [A] time = 0.115059, size = 148, normalized size = 0.47

$$\frac{2\sqrt{ax + bx^{2/3}} (51480a^8b^2x^{8/3} - 54912a^7b^3x^{7/3} + 59136a^6b^4x^2 - 64512a^5b^5x^{5/3} + 71680a^4b^6x^{4/3} + 98304a^2b^8x^{2/3} - 81920a^3b^7x + 71680a^4b^6x^{4/3} - 64512a^5b^5x^{5/3})}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x], x]

[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3))

$$\frac{+ 59136a^6b^4x^2 - 54912a^7b^3x^{(7/3)} + 51480a^8b^2x^{(8/3)} - 48620a^9b^1x^3 + 46189a^{10}x^{(10/3)}}{(323323a^{11}x^{(1/3)})}$$

Maple [A] time = 0.003, size = 134, normalized size = 0.4

$$\frac{2}{323323a^{11}} \sqrt[3]{x} (b + a\sqrt[3]{x}) (46189x^{10/3}a^{10} - 48620x^3a^9b + 51480x^{8/3}a^8b^2 - 54912x^{7/3}a^7b^3 + 59136x^2a^6b^4 - 64512x^{5/3}a^5b^5 + 71680x^{4/3}a^4b^6 - 81920x^1a^3b^7 + 98304x^{2/3}a^2b^8 - 131072x^{1/3}ab^9 + 262144b^{10}) / (b^2x^{2/3} + a^2x^{1/2}) / a^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^(2/3)+a*x)^(1/2), x)

[Out] 2/323323*x^(1/3)*(b+a*x^(1/3))*(46189*x^(10/3)*a^10-48620*x^3*a^9*b+51480*x^(8/3)*a^8*b^2-54912*x^(7/3)*a^7*b^3+59136*x^2*a^6*b^4-64512*x^(5/3)*a^5*b^5+71680*x^(4/3)*a^4*b^6-81920*x*a^3*b^7+98304*x^(2/3)*a^2*b^8-131072*x^(1/3)*a*b^9+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2), x)

[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.15723, size = 221, normalized size = 0.71

$$\frac{524288 b^{\frac{21}{2}}}{323323 a^{11}} + \frac{2 \left(46189 \left(ax^{\frac{1}{3}} + b \right)^{\frac{21}{2}} - 510510 \left(ax^{\frac{1}{3}} + b \right)^{\frac{19}{2}} b + 2567565 \left(ax^{\frac{1}{3}} + b \right)^{\frac{17}{2}} b^2 - 7759752 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} b^3 + 15668730 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} b^4 - 2222110 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} b^5 + 22632610 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} b^6 - 16628040 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b^7 + 8729721 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^8 - 3233230 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^9 + 969969 \sqrt{ax^{\frac{1}{3}} + b} b^{10} \right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2), x, algorithm="giac")

[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11

$$3.187 \quad \int \frac{x^2}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=225

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}} + \frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5} - \frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4} + \frac{336b^2x^2\sqrt{ax+bx^{2/3}}}{429a^3} - \frac{28b^2x^{5/3}\sqrt{ax+bx^{2/3}}}{65a^2} + \frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6) + (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3) - (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a)

Rubi [A] time = 0.34578, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}} + \frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5} - \frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4} + \frac{336b^2x^2\sqrt{ax+bx^{2/3}}}{429a^3} - \frac{28b^2x^{5/3}\sqrt{ax+bx^{2/3}}}{65a^2} + \frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[b*x^(2/3) + a*x], x]

[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6) + (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3) - (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a)

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{bx^{2/3} + ax}} dx &= \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} - \frac{(14b) \int \frac{x^{5/3}}{\sqrt{bx^{2/3} + ax}} dx}{15a} \\
&= -\frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(56b^2) \int \frac{x^{4/3}}{\sqrt{bx^{2/3} + ax}} dx}{65a^2} \\
&= \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} - \frac{(112b^3) \int \frac{x}{\sqrt{bx^{2/3} + ax}} dx}{143a^3} \\
&= -\frac{224b^3x\sqrt{bx^{2/3} + ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(896b^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{143a^3} \\
&= \frac{256b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3} + ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(896b^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{143a^3} \\
&= -\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3} + ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(896b^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{143a^3} \\
&= \frac{2048b^6\sqrt{bx^{2/3} + ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3} + ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(896b^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{143a^3} \\
&= \frac{2048b^6\sqrt{bx^{2/3} + ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3} + ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3} + ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3} + ax}}{5a} + \frac{(896b^4) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{143a^3}
\end{aligned}$$

Mathematica [A] time = 0.0879218, size = 111, normalized size = 0.49

$$\frac{2\sqrt{ax + bx^{2/3}} (504a^5b^2x^{5/3} - 560a^4b^3x^{4/3} - 768a^2b^5x^{2/3} + 640a^3b^4x - 462a^6bx^2 + 429a^7x^{7/3} + 1024ab^6\sqrt[3]{x} - 2048b^7)}{2145a^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]

[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))

Maple [A] time = 0.005, size = 101, normalized size = 0.5

$$\frac{2}{2145 a^8} \sqrt[3]{x} (b + a \sqrt[3]{x}) (429 x^{7/3} a^7 - 462 x^2 a^6 b + 504 x^{5/3} a^5 b^2 - 560 x^{4/3} a^4 b^3 + 640 x a^3 b^4 - 768 x^{2/3} a^2 b^5 + 1024 \sqrt[3]{x} a b^6 - 2145 a^7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^(2/3)+a*x)^(1/2),x)

[Out] 2/2145*x^(1/3)*(b+a*x^(1/3))*(429*x^(7/3)*a^7-462*x^2*a^6*b+504*x^(5/3)*a^5*b^2-560*x^(4/3)*a^4*b^3+640*x*a^3*b^4-768*x^(2/3)*a^2*b^5+1024*x^(1/3)*a*b^6-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)`

[Out] `Integral(x**2/sqrt(a*x + b*x**(2/3)), x)`

Giac [A] time = 1.16623, size = 165, normalized size = 0.73

$$\frac{4096 b^{\frac{15}{2}}}{2145 a^8} + \frac{2 \left(429 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} - 3465 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} b + 12285 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} b^2 - 25025 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} b^3 + 32175 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b^4 \right)}{2145 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")`

[Out] `4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8`

$$3.188 \quad \int \frac{x}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=137

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}} - \frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3} - \frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2} + \frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

[Out] $(-128*b^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(105*a^4) + (256*b^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(105*a^5*x^{(1/3)}) + (32*b^2*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(35*a^3) - (16*b*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^2) + (2*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(3*a)$

Rubi [A] time = 0.179859, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2016, 2002, 2014}

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}} - \frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3} - \frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2} + \frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/\text{Sqrt}[b*x^{(2/3)} + a*x], x]$

[Out] $(-128*b^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(105*a^4) + (256*b^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(105*a^5*x^{(1/3)}) + (32*b^2*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(35*a^3) - (16*b*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^2) + (2*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(3*a)$

Rule 2016

$\text{Int}[\left((c_{.})*(x_{.})^{(m_{.})}*((a_{.})*(x_{.})^{(j_{.})} + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x_{\text{Symbol}}\right) \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[m+j*p+1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2002

$\text{Int}[\left((a_{.})*(x_{.})^{(j_{.})} + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x_{\text{Symbol}}\right) \rightarrow \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(a*(j*p+1)*x^{(j-1)}), x] - \text{Dist}[(b*(n*p+n-j+1))/(a*$

$(j*p + 1)$), Int $[x^{(n - j)}*(a*x^j + b*x^n)^p, x], x]$ /; FreeQ $[{a, b, j, n, p}, x]$ && !IntegerQ $[p]$ && NeQ $[n, j]$ && ILtQ $[Simplify[(n*p + n - j + 1)/(n - j)], 0]$ && NeQ $[j*p + 1, 0]$

Rule 2014

Int $[((c_.)*(x_.))^{(m_.)}*((a_.)*(x_.)^{(j_.)} + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol]$:> -Simp $[(c^{(j - 1)}*(c*x)^{(m - j + 1)}*(a*x^j + b*x^n)^{(p + 1)})/(a*(n - j)*(p + 1)), x]$ /; FreeQ $[{a, b, c, j, m, n, p}, x]$ && !IntegerQ $[p]$ && NeQ $[n, j]$ && EqQ $[m + n*p + n - j + 1, 0]$ && (IntegerQ $[j]$ || GtQ $[c, 0]$)

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{bx^{2/3} + ax}} dx &= \frac{2x\sqrt{bx^{2/3} + ax}}{3a} - \frac{(8b) \int \frac{x^{2/3}}{\sqrt{bx^{2/3} + ax}} dx}{9a} \\ &= -\frac{16bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3} + ax}}{3a} + \frac{(16b^2) \int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}} dx}{21a^2} \\ &= \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3} + ax}}{3a} - \frac{(64b^3) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{105a^3} \\ &= -\frac{128b^3\sqrt{bx^{2/3} + ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3} + ax}}{3a} + \frac{(128b^4) \int \frac{1}{\sqrt[3]{bx^{2/3} + ax}} dx}{315a^4} \\ &= -\frac{128b^3\sqrt{bx^{2/3} + ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3} + ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3} + ax}}{3a} \end{aligned}$$

Mathematica [A] time = 0.063737, size = 74, normalized size = 0.54

$$\frac{2\sqrt{ax + bx^{2/3}}(48a^2b^2x^{2/3} - 40a^3bx + 35a^4x^{4/3} - 64ab^3\sqrt[3]{x} + 128b^4)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate $[x/\text{Sqrt}[b*x^{(2/3)} + a*x], x]$

[Out] $(2*\text{Sqrt}[b*x^{(2/3)} + a*x]*(128*b^4 - 64*a*b^3*x^{(1/3)} + 48*a^2*b^2*x^{(2/3)} - 40*a^3*b*x + 35*a^4*x^{(4/3)}))/(105*a^5*x^{(1/3)})$

Maple [A] time = 0.003, size = 68, normalized size = 0.5

$$\frac{2}{105 a^5} \sqrt[3]{x} (b + a \sqrt[3]{x}) (35 x^{4/3} a^4 - 40 x a^3 b + 48 x^{2/3} a^2 b^2 - 64 \sqrt[3]{x} a b^3 + 128 b^4) \frac{1}{\sqrt{b x^{2/3} + a x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*x^(2/3)+a*x)^(1/2),x)`

[Out] $\frac{2}{105} x^{1/3} (b + a x^{1/3}) (35 x^{4/3} a^4 - 40 x a^3 b + 48 x^{2/3} a^2 b^2 - 64 x^{1/3} a b^3 + 128 b^4) / (b x^{2/3} + a x)^{1/2} / a^5$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a x + b x^{2/3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/sqrt(a*x + b*x^(2/3)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a x + b x^{2/3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.13309, size = 108, normalized size = 0.79

$$-\frac{256 b^2}{105 a^5} + \frac{2 \left(35 \left(a x^{\frac{1}{3}} + b \right)^{\frac{9}{2}} - 180 \left(a x^{\frac{1}{3}} + b \right)^{\frac{7}{2}} b + 378 \left(a x^{\frac{1}{3}} + b \right)^{\frac{5}{2}} b^2 - 420 \left(a x^{\frac{1}{3}} + b \right)^{\frac{3}{2}} b^3 + 315 \sqrt{a x^{\frac{1}{3}} + b} b^4 \right)}{105 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) + b)*b^4)/a^5

$$3.189 \quad \int \frac{1}{\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=47

$$\frac{2\sqrt{ax+bx^{2/3}}}{a} - \frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))

Rubi [A] time = 0.0495725, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2002, 2014}

$$\frac{2\sqrt{ax+bx^{2/3}}}{a} - \frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*x^(2/3) + a*x], x]

[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[(((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{bx^{2/3} + ax}} dx = \frac{2\sqrt{bx^{2/3} + ax}}{a} - \frac{(2b) \int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3} + ax}} dx}{3a}$$

$$= \frac{2\sqrt{bx^{2/3} + ax}}{a} - \frac{4b\sqrt{bx^{2/3} + ax}}{a^2\sqrt[3]{x}}$$

Mathematica [A] time = 0.0274036, size = 36, normalized size = 0.77

$$\frac{2(a\sqrt[3]{x} - 2b)\sqrt{ax + bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]

[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$2 \frac{\sqrt[3]{x} (b + a\sqrt[3]{x}) (a\sqrt[3]{x} - 2b)}{\sqrt{bx^{2/3} + axa^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^(2/3)+a*x)^(1/2),x)

[Out] 2*x^(1/3)*(b+a*x^(1/3))*(a*x^(1/3)-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)

Giac [A] time = 1.1424, size = 49, normalized size = 1.04

$$\frac{4b^{\frac{3}{2}}}{a^2} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} - 3\sqrt{ax^{\frac{1}{3}} + bb}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2

$$3.190 \quad \int \frac{1}{x\sqrt{bx^{2/3}+ax}} dx$$

Optimal. Leaf size=61

$$\frac{3a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}} - \frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(b*x^{(2/3)}) + (3*a*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/b^{(3/2)}$

Rubi [A] time = 0.0934507, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2029, 206}

$$\frac{3a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}} - \frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[b*x^{(2/3)} + a*x]),x]$

[Out] $(-3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(b*x^{(2/3)}) + (3*a*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(1/3)})/\text{Sqrt}[b*x^{(2/3)} + a*x]])/b^{(3/2)}$

Rule 2025

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m+j*p+1, 0]

Rule 2029

$\text{Int}[(x_*)^{(m_*)}/\text{Sqrt}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[-2/(n-j), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /;$ FreeQ[{a, b, j, n}, x] && EqQ[m, j/2-1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{bx^{2/3} + ax}} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{bx^{2/3}} - \frac{a \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{2b} \\ &= -\frac{3\sqrt{bx^{2/3} + ax}}{bx^{2/3}} + \frac{(3a) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{b} \\ &= -\frac{3\sqrt{bx^{2/3} + ax}}{bx^{2/3}} + \frac{3a \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0868025, size = 90, normalized size = 1.48

$$\frac{6a\sqrt[3]{x}(a\sqrt[3]{x} + b) \left(\frac{\tanh^{-1}\left(\sqrt{\frac{a\sqrt[3]{x}}{b} + 1}\right)}{2\sqrt{\frac{a\sqrt[3]{x}}{b} + 1}} - \frac{b}{2a\sqrt[3]{x}} \right)}{b^2\sqrt{x^{2/3}}(a\sqrt[3]{x} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]

[Out] (6*a*(b + a*x^(1/3))*x^(1/3)*(-b/(2*a*x^(1/3)) + ArcTanh[Sqrt[1 + (a*x^(1/3))/b]]/(2*Sqrt[1 + (a*x^(1/3))/b]))/(b^2*Sqrt[(b + a*x^(1/3))*x^(2/3)])

Maple [A] time = 0.007, size = 61, normalized size = 1.

$$3 \frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{bx^{2/3} + axb^{5/2}}} \left(\text{Artanh}\left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}}\right) ba\sqrt[3]{x} - \sqrt{b + a\sqrt[3]{x}b^{3/2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^(2/3)+a*x)^(1/2),x)`

[Out] $3*(b+a*x^{(1/3)})^{(1/2)}*(\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)})*b*a*x^{(1/3)}-(b+a*x^{(1/3)})^{(1/2)}*b^{(3/2)})/(b*x^{(2/3)+a*x}^{(1/2)}/b^{(5/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)`

[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)

Giac [A] time = 1.15623, size = 69, normalized size = 1.13

$$\frac{3 \left(\frac{a^2 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb}} + \frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+ba}}}{bx^{\frac{1}{3}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3) + b)*a/(b*x^(1/3)))/a

$$3.191 \quad \int \frac{1}{x^2 \sqrt{bx^{2/3} + ax}} dx$$

Optimal. Leaf size=153

$$\frac{105a^3 \sqrt{ax + bx^{2/3}}}{64b^4 x^{2/3}} - \frac{35a^2 \sqrt{ax + bx^{2/3}}}{32b^3 x} - \frac{105a^4 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt[3]{x}}{\sqrt{ax + bx^{2/3}}}\right)}{64b^{9/2}} + \frac{7a \sqrt{ax + bx^{2/3}}}{8b^2 x^{4/3}} - \frac{3 \sqrt{ax + bx^{2/3}}}{4bx^{5/3}}$$

[Out] $(-3 \sqrt{bx^{2/3} + ax}) / (4bx^{5/3}) + (7a \sqrt{bx^{2/3} + ax}) / (8b^2 x^{4/3}) - (35a^2 \sqrt{bx^{2/3} + ax}) / (32b^3 x) + (105a^3 \sqrt{bx^{2/3} + ax}) / (64b^4 x^{2/3}) - (105a^4 \operatorname{ArcTanh}[(\sqrt{b} x^{1/3}) / \sqrt{bx^{2/3} + ax}]) / (64b^{9/2})$

Rubi [A] time = 0.239309, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2029, 206}

$$\frac{105a^3 \sqrt{ax + bx^{2/3}}}{64b^4 x^{2/3}} - \frac{35a^2 \sqrt{ax + bx^{2/3}}}{32b^3 x} - \frac{105a^4 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt[3]{x}}{\sqrt{ax + bx^{2/3}}}\right)}{64b^{9/2}} + \frac{7a \sqrt{ax + bx^{2/3}}}{8b^2 x^{4/3}} - \frac{3 \sqrt{ax + bx^{2/3}}}{4bx^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]

[Out] $(-3 \sqrt{bx^{2/3} + ax}) / (4bx^{5/3}) + (7a \sqrt{bx^{2/3} + ax}) / (8b^2 x^{4/3}) - (35a^2 \sqrt{bx^{2/3} + ax}) / (32b^3 x) + (105a^3 \sqrt{bx^{2/3} + ax}) / (64b^4 x^{2/3}) - (105a^4 \operatorname{ArcTanh}[(\sqrt{b} x^{1/3}) / \sqrt{bx^{2/3} + ax}]) / (64b^{9/2})$

Rule 2025

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],

x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \sqrt{bx^{2/3} + ax}} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} - \frac{(7a) \int \frac{1}{x^{5/3} \sqrt{bx^{2/3} + ax}} dx}{8b} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3} + ax}}{8b^2 x^{4/3}} + \frac{(35a^2) \int \frac{1}{x^{4/3} \sqrt{bx^{2/3} + ax}} dx}{48b^2} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3} + ax}}{8b^2 x^{4/3}} - \frac{35a^2 \sqrt{bx^{2/3} + ax}}{32b^3 x} - \frac{(35a^3) \int \frac{1}{x \sqrt{bx^{2/3} + ax}} dx}{64b^3} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3} + ax}}{8b^2 x^{4/3}} - \frac{35a^2 \sqrt{bx^{2/3} + ax}}{32b^3 x} + \frac{105a^3 \sqrt{bx^{2/3} + ax}}{64b^4 x^{2/3}} + \frac{(35a^4) \int \frac{1}{x^{2/3} \sqrt{bx^{2/3} + ax}} dx}{128b^4} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3} + ax}}{8b^2 x^{4/3}} - \frac{35a^2 \sqrt{bx^{2/3} + ax}}{32b^3 x} + \frac{105a^3 \sqrt{bx^{2/3} + ax}}{64b^4 x^{2/3}} - \frac{(105a^4) \text{Subst}\left(\int \frac{1}{1 - \frac{\sqrt{b} \sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}}\right)}{64b^4} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3} + ax}}{8b^2 x^{4/3}} - \frac{35a^2 \sqrt{bx^{2/3} + ax}}{32b^3 x} + \frac{105a^3 \sqrt{bx^{2/3} + ax}}{64b^4 x^{2/3}} - \frac{105a^4 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{64b^{9/2}}
 \end{aligned}$$

Mathematica [C] time = 0.0545172, size = 48, normalized size = 0.31

$$\frac{6a^4 \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{1}{2}, 5; \frac{3}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^5 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]

[Out] (-6*a^4*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (a*x^(1/3))/b])/(b^5*x^(1/3))

Maple [A] time = 0.007, size = 126, normalized size = 0.8

$$-\frac{1}{64x^2}\sqrt{b+a\sqrt[3]{x}}\left(-56b^{7/2}x^{4/3}\sqrt{b+a\sqrt[3]{x}a}+70b^{5/2}x^{5/3}\sqrt{b+a\sqrt[3]{x}a^2}+105\operatorname{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)x^{7/3}a^4b+48\sqrt{b+a\sqrt[3]{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x)`

[Out] $-1/64*(b+a*x^{(1/3)})^{(1/2)}*(-56*b^{(7/2)}*x^{(4/3)}*(b+a*x^{(1/3)})^{(1/2)}*a+70*b^{(5/2)}*x^{(5/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^2+105*\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)}))*x^{(7/3)}*a^4*b+48*(b+a*x^{(1/3)})^{(1/2)}*b^{(9/2)}*x-105*b^{(3/2)}*x^2*(b+a*x^{(1/3)})^{(1/2)}*a^3)/x^2/(b*x^{(2/3)}+a*x)^{(1/2)}/b^{(11/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)

Giac [A] time = 1.18958, size = 147, normalized size = 0.96

$$\frac{105a^5 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5 - 385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b + 511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2 - 279\sqrt{ax^{\frac{1}{3}}+b}a^5b^3}{a^4b^4x^{\frac{4}{3}}}$$

$64a$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3) + b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3))/a

$$3.192 \quad \int \frac{1}{x^3 \sqrt{bx^{2/3} + ax}} dx$$

Optimal. Leaf size=241

$$-\frac{1287a^6 \sqrt{ax + bx^{2/3}}}{1024b^7 x^{2/3}} + \frac{429a^5 \sqrt{ax + bx^{2/3}}}{512b^6 x} - \frac{429a^4 \sqrt{ax + bx^{2/3}}}{640b^5 x^{4/3}} + \frac{1287a^3 \sqrt{ax + bx^{2/3}}}{2240b^4 x^{5/3}} - \frac{143a^2 \sqrt{ax + bx^{2/3}}}{280b^3 x^2} + \frac{1287a^7 \operatorname{arctanh}\left(\frac{\sqrt{b} x^{1/3}}{\sqrt{bx^{2/3} + ax}}\right)}{1024b^{15/2}}$$

[Out] $(-3 \sqrt{bx^{2/3} + ax}) / (7 b^{8/3} x) + (13 a \sqrt{bx^{2/3} + ax}) / (28 b^2 x^{7/3}) - (143 a^2 \sqrt{bx^{2/3} + ax}) / (280 b^3 x^2) + (1287 a^3 \sqrt{bx^{2/3} + ax}) / (2240 b^4 x^{5/3}) - (429 a^4 \sqrt{bx^{2/3} + ax}) / (640 b^5 x^{4/3}) + (429 a^5 \sqrt{bx^{2/3} + ax}) / (512 b^6 x) - (1287 a^6 \sqrt{bx^{2/3} + ax}) / (1024 b^7 x^{2/3}) + (1287 a^7 \operatorname{ArcTanh}[(\sqrt{b} x^{1/3}) / \sqrt{bx^{2/3} + ax}]) / (1024 b^{15/2})$

Rubi [A] time = 0.407532, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2029, 206}

$$-\frac{1287a^6 \sqrt{ax + bx^{2/3}}}{1024b^7 x^{2/3}} + \frac{429a^5 \sqrt{ax + bx^{2/3}}}{512b^6 x} - \frac{429a^4 \sqrt{ax + bx^{2/3}}}{640b^5 x^{4/3}} + \frac{1287a^3 \sqrt{ax + bx^{2/3}}}{2240b^4 x^{5/3}} - \frac{143a^2 \sqrt{ax + bx^{2/3}}}{280b^3 x^2} + \frac{1287a^7 \operatorname{arctanh}\left(\frac{\sqrt{b} x^{1/3}}{\sqrt{bx^{2/3} + ax}}\right)}{1024b^{15/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]

[Out] $(-3 \sqrt{bx^{2/3} + ax}) / (7 b^{8/3} x) + (13 a \sqrt{bx^{2/3} + ax}) / (28 b^2 x^{7/3}) - (143 a^2 \sqrt{bx^{2/3} + ax}) / (280 b^3 x^2) + (1287 a^3 \sqrt{bx^{2/3} + ax}) / (2240 b^4 x^{5/3}) - (429 a^4 \sqrt{bx^{2/3} + ax}) / (640 b^5 x^{4/3}) + (429 a^5 \sqrt{bx^{2/3} + ax}) / (512 b^6 x) - (1287 a^6 \sqrt{bx^{2/3} + ax}) / (1024 b^7 x^{2/3}) + (1287 a^7 \operatorname{ArcTanh}[(\sqrt{b} x^{1/3}) / \sqrt{bx^{2/3} + ax}]) / (1024 b^{15/2})$

Rule 2025

Int[((c_.)(x_))^(m_.)*((a_.)(x_)^(j_.) + (b_.)(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
 [-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
 Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
 Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \sqrt{bx^{2/3} + ax}} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} - \frac{(13a) \int \frac{1}{x^{8/3} \sqrt{bx^{2/3} + ax}} dx}{14b} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} + \frac{(143a^2) \int \frac{1}{x^{7/3} \sqrt{bx^{2/3} + ax}} dx}{168b^2} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} - \frac{(429a^3) \int \frac{1}{x^2 \sqrt{bx^{2/3} + ax}} dx}{560b^3} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} + \frac{(429a^4) \int \frac{1}{x^{5/3} \sqrt{bx^{2/3} + ax}} dx}{640b^4} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3} + ax}}{640b^5x^{4/3}} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3} + ax}}{640b^5x^{4/3}} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3} + ax}}{640b^5x^{4/3}} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3} + ax}}{640b^5x^{4/3}} \\
 &= -\frac{3\sqrt{bx^{2/3} + ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3} + ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3} + ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3} + ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3} + ax}}{640b^5x^{4/3}}
 \end{aligned}$$

Mathematica [C] time = 0.0558736, size = 48, normalized size = 0.2

$$\frac{6a^7 \sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{1}{2}, 8; \frac{3}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^8 \sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]

[Out] (6*a^7*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 8, 3/2, 1 + (a*x^(1/3))/b])/(b^8*x^(1/3))

Maple [A] time = 0.006, size = 188, normalized size = 0.8

$$-\frac{1}{35840x^4} \sqrt{b + a\sqrt[3]{x}} \left(24024 b^{7/2} x^{10/3} \sqrt{b + a\sqrt[3]{xa^4}} + 45045 b^{3/2} x^4 \sqrt{b + a\sqrt[3]{xa^6}} - 45045 \operatorname{Artanh} \left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}} \right) x^{13/3} a^7 b - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x)

[Out] -1/35840/x^4*(b+a*x^(1/3))^(1/2)*(24024*b^(7/2)*x^(10/3)*(b+a*x^(1/3))^(1/2)*a^4+45045*b^(3/2)*x^4*(b+a*x^(1/3))^(1/2)*a^6-45045*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*x^(13/3)*a^7*b-16640*b^(13/2)*x^(7/3)*(b+a*x^(1/3))^(1/2)*a-30030*b^(5/2)*x^(11/3)*(b+a*x^(1/3))^(1/2)*a^5-20592*b^(9/2)*x^3*(b+a*x^(1/3))^(1/2)*a^3+18304*b^(11/2)*x^(8/3)*(b+a*x^(1/3))^(1/2)*a^2+15360*(b+a*x^(1/3))^(1/2)*b^(15/2)*x^2)/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{ax + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)

Giac [A] time = 1.26854, size = 216, normalized size = 0.9

$$\frac{45045 a^8 \arctan\left(\frac{\sqrt{\frac{1}{ax^{\frac{1}{3}}+b}}}{\sqrt{-b}}\right)}{\sqrt{-bb^7}} + \frac{45045 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^8 - 300300 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^8 b + 849849 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^8 b^2 - 1317888 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^8 b^3 + 1200199 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^8 b^4 - 6315}{a^7 b^7 x^{\frac{7}{3}}}$$

35840 a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84

$$\frac{9849(a^8x^{1/3} + b)^{9/2} - 1317888(a^8x^{1/3} + b)^{7/2} + 1200199(a^8x^{1/3} + b)^{5/2} - 631540(a^8x^{1/3} + b)^{3/2} + 169995\sqrt{a^8x^{1/3} + b}}{a^7b^7x^{7/3}}$$

$$3.193 \quad \int \frac{1}{x^4 \sqrt{bx^{2/3} + ax}} dx$$

Optimal. Leaf size=329

$$\frac{138567a^9 \sqrt{ax + bx^{2/3}}}{131072b^{10}x^{2/3}} - \frac{46189a^8 \sqrt{ax + bx^{2/3}}}{65536b^9x} + \frac{46189a^7 \sqrt{ax + bx^{2/3}}}{81920b^8x^{4/3}} - \frac{138567a^6 \sqrt{ax + bx^{2/3}}}{286720b^7x^{5/3}} + \frac{46189a^5 \sqrt{ax + bx^{2/3}}}{107520b^6x^2} - \frac{46189a^4 \sqrt{ax + bx^{2/3}}}{896b^4x^{8/3}} + \frac{46189a^3 \sqrt{ax + bx^{2/3}}}{107520b^5x^{7/3}} - \frac{46189a^2 \sqrt{ax + bx^{2/3}}}{960b^3x^3} + \frac{46189a \sqrt{ax + bx^{2/3}}}{60b^2x^{10/3}} - \frac{46189 \sqrt{ax + bx^{2/3}}}{131072b^{10}x^{2/3}}$$

[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x])/(107520*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2) - (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) - (138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2 1/2))

Rubi [A] time = 0.577139, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2029, 206}

$$\frac{138567a^9 \sqrt{ax + bx^{2/3}}}{131072b^{10}x^{2/3}} - \frac{46189a^8 \sqrt{ax + bx^{2/3}}}{65536b^9x} + \frac{46189a^7 \sqrt{ax + bx^{2/3}}}{81920b^8x^{4/3}} - \frac{138567a^6 \sqrt{ax + bx^{2/3}}}{286720b^7x^{5/3}} + \frac{46189a^5 \sqrt{ax + bx^{2/3}}}{107520b^6x^2} - \frac{46189a^4 \sqrt{ax + bx^{2/3}}}{896b^4x^{8/3}} + \frac{46189a^3 \sqrt{ax + bx^{2/3}}}{107520b^5x^{7/3}} - \frac{46189a^2 \sqrt{ax + bx^{2/3}}}{960b^3x^3} + \frac{46189a \sqrt{ax + bx^{2/3}}}{60b^2x^{10/3}} - \frac{46189 \sqrt{ax + bx^{2/3}}}{131072b^{10}x^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]

[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x])/(107520*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2) - (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) - (138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2 1/2))

Rule 2025


```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2029

```

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{bx^{2/3} + ax}} dx &= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} - \frac{(19a) \int \frac{1}{x^{11/3} \sqrt{bx^{2/3} + ax}} dx}{20b} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} + \frac{(323a^2) \int \frac{1}{x^{10/3} \sqrt{bx^{2/3} + ax}} dx}{360b^2} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} - \frac{(323a^3) \int \frac{1}{x^3 \sqrt{bx^{2/3} + ax}} dx}{384b^3} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} + \frac{(4199a^4) \int \frac{1}{x^{8/3} \sqrt{bx^{2/3} + ax}} dx}{5376b^4} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}} \\
&= -\frac{3\sqrt{bx^{2/3} + ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3} + ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3} + ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3} + ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3} + ax}}{10752b^5x^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.0547898, size = 48, normalized size = 0.15

$$-\frac{6a^{10}\sqrt{ax + bx^{2/3}} {}_2F_1\left(\frac{1}{2}, 11; \frac{3}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*sqrt[b*x^(2/3) + a*x]),x]

[Out] $(-6*a^{10}*\text{Sqrt}[b*x^{(2/3)} + a*x]*\text{Hypergeometric2F1}[1/2, 11, 3/2, 1 + (a*x^{(1/3)})/b])/ (b^{11}*x^{(1/3)})$

Maple [A] time = 0.007, size = 248, normalized size = 0.8

$$-\frac{1}{13762560x^6}\sqrt{b+a\sqrt[3]{x}}\left(-4358144b^{19/2}x^{10/3}\sqrt{b+a\sqrt[3]{xa}}+4630528b^{17/2}x^{11/3}\sqrt{b+a\sqrt[3]{xa^2}}+5374720b^{13/2}x^{13/3}\sqrt{b+a\sqrt[3]{xa^3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x)`

[Out] $-1/13762560*(b+a*x^{(1/3)})^{(1/2)}*(-4358144*b^{(19/2)}*x^{(10/3)}*(b+a*x^{(1/3)})^{(1/2)}*a+4630528*b^{(17/2)}*x^{(11/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^2+5374720*b^{(13/2)}*x^{(13/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^3-5912192*b^{(11/2)}*x^{(14/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^4-7759752*b^{(7/2)}*x^{(16/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^5+9699690*b^{(5/2)}*x^{(17/3)}*(b+a*x^{(1/3)})^{(1/2)}*a^6+14549535*\text{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)})*x^{(19/3)}*a^{10}+4128768*(b+a*x^{(1/3)})^{(1/2)}*b^{(21/2)}*x^3-4961280*b^{(15/2)}*x^4+6651216*b^{(9/2)}*x^5*(b+a*x^{(1/3)})^{(1/2)}*a^6-14549535*b^{(3/2)}*x^6*(b+a*x^{(1/3)})^{(1/2)}*a^9)/x^6/(b*x^{(2/3)}+a*x)^{(1/2)}/b^{(23/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.35374, size = 285, normalized size = 0.87

$$\frac{14549535 a^{11} \arctan\left(\frac{\sqrt{\frac{1}{ax^3+b}}}{\sqrt{-b}}\right)}{\sqrt{-b} b^{10}} + \frac{14549535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} a^{11} - 140645505 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{11} b + 609140532 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{11} b^2 - 1554721740 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{11} b^3 + 2585198330 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{11} b^4 - 2918514950 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{11} b^5 + 2255541300 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{11} b^6 - 1168982220 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{11} b^7 + 382331775 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^{11} b^8 - 68025825 \sqrt{ax^{\frac{1}{3}}+b} a^{11} b^9}{a^{10} b^{10} x^{\frac{10}{3}}}$$

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 2918514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3) + b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(10/3))/a

$$3.194 \quad \int \frac{x^4}{(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=336

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}} - \frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9} + \frac{40960b^6x\sqrt{ax+bx^{2/3}}}{4199a^8}$$

[Out] $(-6*x^4)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) - (524288*b^9*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{11}) + (1048576*b^{10}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{12}*x^{(1/3)}) + (393216*b^8*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{10}) - (327680*b^7*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^9) + (40960*b^6*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^8) - (36864*b^5*x^{(4/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^7) + (33792*b^4*x^{(5/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^6) - (16896*b^3*x^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2261*a^5) + (15840*b^2*x^{(7/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2261*a^4) - (880*b*x^{(8/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(133*a^3) + (44*x^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(7*a^2)$

Rubi [A] time = 0.599225, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2015, 2016, 2002, 2014}

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}} - \frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9} + \frac{40960b^6x\sqrt{ax+bx^{2/3}}}{4199a^8}$$

Antiderivative was successfully verified.

[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2), x]

[Out] $(-6*x^4)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) - (524288*b^9*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{11}) + (1048576*b^{10}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{12}*x^{(1/3)}) + (393216*b^8*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^{10}) - (327680*b^7*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(29393*a^9) + (40960*b^6*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^8) - (36864*b^5*x^{(4/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^7) + (33792*b^4*x^{(5/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(4199*a^6) - (16896*b^3*x^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2261*a^5) + (15840*b^2*x^{(7/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2261*a^4) - (880*b*x^{(8/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(133*a^3) + (44*x^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(7*a^2)$

Rule 2015

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(bx^{2/3} + ax)^{3/2}} dx &= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{22 \int \frac{x^3}{\sqrt{bx^{2/3} + ax}} dx}{a} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{44x^3\sqrt{bx^{2/3} + ax}}{7a^2} - \frac{(440b) \int \frac{x^{8/3}}{\sqrt{bx^{2/3} + ax}} dx}{21a^2} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{880bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3} + ax}}{7a^2} + \frac{(2640b^2) \int \frac{x^{7/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3} + ax}}{7a^2} - \frac{(42240b^3) \int \frac{x^{6/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3} + ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3} + ax}}{7a^2} - \frac{(42240b^3) \int \frac{x^{5/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{x^{4/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{x^{3/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{x^{2/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{x^{1/3}}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{524288b^9\sqrt{bx^{2/3} + ax}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{133a^3} \\
&= -\frac{6x^4}{a\sqrt{bx^{2/3} + ax}} - \frac{524288b^9\sqrt{bx^{2/3} + ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3} + ax}}{29393a^{12}\sqrt[3]{x}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3} + ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3} + ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3} + ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3} + ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3} + ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3} + ax}}{2261a^4} - \frac{(42240b^3) \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{133a^3}
\end{aligned}$$

Mathematica [A] time = 0.132949, size = 161, normalized size = 0.48

$$\frac{2\sqrt[3]{x} (5720a^9b^2x^3 - 6864a^8b^3x^{8/3} + 8448a^7b^4x^{7/3} - 10752a^6b^5x^2 + 14336a^5b^6x^{5/3} - 20480a^4b^7x^{4/3} - 65536a^2b^9x^{2/3} + 393216b^8\sqrt[3]{x}\sqrt{bx^{2/3} + ax})}{29393a^{12}\sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]

[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3*2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 - 4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.005, size = 143, normalized size = 0.4

$$\frac{2x}{29393a^{12}}(b + a\sqrt[3]{x})(4199x^{11/3}a^{11} - 4862x^{10/3}a^{10}b + 5720x^3a^9b^2 - 6864x^{8/3}a^8b^3 + 8448x^{7/3}a^7b^4 - 10752x^2a^6b^5 + 14336x^{5/3}a^5b^6 - 20480x^{4/3}a^4b^7 + 32768x^3a^3b^8 - 65536x^{2/3}a^2b^9 + 262144x^{1/3}ab^{10} + 524288b^{11})/(b*x^{2/3} + a*x)^{3/2}/a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^(2/3)+a*x)^(3/2),x)

[Out] 2/29393*x*(b+a*x^(1/3))*(4199*x^(11/3)*a^11-4862*x^(10/3)*a^10*b+5720*x^3*a^9*b^2-6864*x^(8/3)*a^8*b^3+8448*x^(7/3)*a^7*b^4-10752*x^2*a^6*b^5+14336*x^(5/3)*a^5*b^6-20480*x^(4/3)*a^4*b^7+32768*x*a^3*b^8-65536*x^(2/3)*a^2*b^9+262144*x^(1/3)*a*b^10+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.23366, size = 289, normalized size = 0.86

$$\frac{1048576 b^{\frac{21}{2}}}{29393 a^{12}} + \frac{6 b^{11}}{\sqrt{a x^{\frac{1}{3}} + b a^{12}}} + \frac{2 \left(4199 \left(a x^{\frac{1}{3}} + b \right)^{\frac{21}{2}} a^{240} - 51051 \left(a x^{\frac{1}{3}} + b \right)^{\frac{19}{2}} a^{240} b + 285285 \left(a x^{\frac{1}{3}} + b \right)^{\frac{17}{2}} a^{240} b^2 - 969969 \left(a x^{\frac{1}{3}} + b \right)^{\frac{15}{2}} a^{240} b^3 + 2238390 \left(a x^{\frac{1}{3}} + b \right)^{\frac{13}{2}} a^{240} b^4 - 3703518 \left(a x^{\frac{1}{3}} + b \right)^{\frac{11}{2}} a^{240} b^5 + 4526522 \left(a x^{\frac{1}{3}} + b \right)^{\frac{9}{2}} a^{240} b^6 - 4157010 \left(a x^{\frac{1}{3}} + b \right)^{\frac{7}{2}} a^{240} b^7 + 2909907 \left(a x^{\frac{1}{3}} + b \right)^{\frac{5}{2}} a^{240} b^8 - 1616615 \left(a x^{\frac{1}{3}} + b \right)^{\frac{3}{2}} a^{240} b^9 + 969969 \sqrt{a x^{\frac{1}{3}} + b} a^{240} b^{10} \right)}{a^{252}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b + 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 1616615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^10)/a^252

$$3.195 \quad \int \frac{x^3}{(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=248

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}} + \frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6} - \frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}$$

[Out] $(-6*x^3)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) + (32768*b^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2145*a^8) - (65536*b^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2145*a^9*x^{(1/3)}) - (8192*b^5*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(715*a^7) + (4096*b^4*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(429*a^6) - (3584*b^3*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(429*a^5) + (5376*b^2*x^{(4/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(715*a^4) - (448*b*x^{(5/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(65*a^3) + (32*x^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(5*a^2)$

Rubi [A] time = 0.413968, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2015, 2016, 2002, 2014}

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}} + \frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6} - \frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2), x]

[Out] $(-6*x^3)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) + (32768*b^6*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2145*a^8) - (65536*b^7*\text{Sqrt}[b*x^{(2/3)} + a*x])/(2145*a^9*x^{(1/3)}) - (8192*b^5*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(715*a^7) + (4096*b^4*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(429*a^6) - (3584*b^3*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(429*a^5) + (5376*b^2*x^{(4/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(715*a^4) - (448*b*x^{(5/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(65*a^3) + (32*x^2*\text{Sqrt}[b*x^{(2/3)} + a*x])/(5*a^2)$

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -

j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(bx^{2/3} + ax)^{3/2}} dx &= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{16 \int \frac{x^2}{\sqrt{bx^{2/3} + ax}} dx}{a} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{32x^2\sqrt{bx^{2/3} + ax}}{5a^2} - \frac{(224b) \int \frac{x^{5/3}}{\sqrt{bx^{2/3} + ax}} dx}{15a^2} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} - \frac{448bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3} + ax}}{5a^2} + \frac{(896b^2) \int \frac{x^{4/3}}{\sqrt{bx^{2/3} + ax}} dx}{65a^3} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3} + ax}}{5a^2} - \frac{(1792b^3)}{1} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} - \frac{3584b^3x\sqrt{bx^{2/3} + ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3} + ax}}{5a^2} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3} + ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3} + ax}}{65a^3} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3} + ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3} + ax}}{715a^4} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{32768b^6\sqrt{bx^{2/3} + ax}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3} + ax}}{429a^5} \\
&= -\frac{6x^3}{a\sqrt{bx^{2/3} + ax}} + \frac{32768b^6\sqrt{bx^{2/3} + ax}}{2145a^8} - \frac{65536b^7\sqrt{bx^{2/3} + ax}}{2145a^9\sqrt[3]{x}} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3} + ax}}{429a^6}
\end{aligned}$$

Mathematica [A] time = 0.0885618, size = 122, normalized size = 0.49

$$\frac{2(672a^6b^2x^{7/3} - 896a^5b^3x^2 + 1280a^4b^4x^{5/3} - 2048a^3b^5x^{4/3} + 4096a^2b^6x - 528a^7bx^{8/3} + 429a^8x^3 - 16384ab^7x^{2/3} - 32768b^8)}{2145a^9\sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2), x]

[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) - 528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.006, size = 110, normalized size = 0.4

$$\frac{2x}{2145a^9} (b + a\sqrt[3]{x}) (429x^{8/3}a^8 - 528x^{7/3}a^7b + 672x^2a^6b^2 - 896x^{5/3}a^5b^3 + 1280x^{4/3}a^4b^4 - 2048xa^3b^5 + 4096x^{2/3}a^2b^6 - 16384x^{1/3}ab^7 + 32768b^8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^(2/3)+a*x)^(3/2),x)`

[Out] $\frac{2}{2145}x(b+ax^{1/3})(429x^{8/3}a^8-528x^{7/3}a^7b+672x^2a^6b^2-896x^{5/3}a^5b^3+1280x^{4/3}a^4b^4-2048xa^3b^5+4096x^{2/3}a^2b^6-16384x^{1/3}ab^7-32768b^8)/(b*x^{2/3}+a*x)^{3/2}/a^9$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)

Giac [A] time = 1.21474, size = 220, normalized size = 0.89

$$\frac{65536 b^{\frac{15}{2}}}{2145 a^9} - \frac{6 b^8}{\sqrt{ax^{\frac{1}{3}} + ba^9}} + \frac{2 \left(429 \left(ax^{\frac{1}{3}} + b \right)^{\frac{15}{2}} a^{126} - 3960 \left(ax^{\frac{1}{3}} + b \right)^{\frac{13}{2}} a^{126} b + 16380 \left(ax^{\frac{1}{3}} + b \right)^{\frac{11}{2}} a^{126} b^2 - 40040 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} a^{126} b^3 + 64350 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} a^{126} b^4 - 72072 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} a^{126} b^5 + 60060 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} a^{126} b^6 - 51480 \sqrt{ax^{\frac{1}{3}} + b} a^{126} b^7 \right)}{a^{135}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 64350*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5 + 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b^7)/a^135

$$3.196 \quad \int \frac{x^2}{(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=160

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}} - \frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4} - \frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3} + \frac{20x\sqrt{ax+bx^{2/3}}}{3a^2} - \frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

[Out] $(-6*x^2)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) - (256*b^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^5) + (512*b^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^6*x^{(1/3)}) + (64*b^2*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(7*a^4) - (160*b*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^3) + (20*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(3*a^2)$

Rubi [A] time = 0.242094, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2015, 2016, 2002, 2014}

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}} - \frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4} - \frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3} + \frac{20x\sqrt{ax+bx^{2/3}}}{3a^2} - \frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2), x]

[Out] $(-6*x^2)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) - (256*b^3*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^5) + (512*b^4*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^6*x^{(1/3)}) + (64*b^2*x^{(1/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(7*a^4) - (160*b*x^{(2/3)}*\text{Sqrt}[b*x^{(2/3)} + a*x])/(21*a^3) + (20*x*\text{Sqrt}[b*x^{(2/3)} + a*x])/(3*a^2)$

Rule 2015

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(bx^{2/3} + ax)^{3/2}} dx &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} + \frac{10 \int \frac{x}{\sqrt{bx^{2/3} + ax}} dx}{a} \\
 &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} + \frac{20x\sqrt{bx^{2/3} + ax}}{3a^2} - \frac{(80b) \int \frac{x^{2/3}}{\sqrt{bx^{2/3} + ax}} dx}{9a^2} \\
 &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} - \frac{160bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3} + ax}}{3a^2} + \frac{(160b^2) \int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}} dx}{21a^3} \\
 &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3} + ax}}{3a^2} - \frac{(128b^3) \int \frac{\sqrt{bx^{2/3}}}{\sqrt{bx^{2/3} + ax}} dx}{21a^4} \\
 &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} - \frac{256b^3\sqrt{bx^{2/3} + ax}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3} + ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}}}{3a^2} \\
 &= -\frac{6x^2}{a\sqrt{bx^{2/3} + ax}} - \frac{256b^3\sqrt{bx^{2/3} + ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3} + ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3} + ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}}}{21a^3}
 \end{aligned}$$

Mathematica [A] time = 0.0714595, size = 85, normalized size = 0.53

$$\frac{32a^3b^2x^{4/3} - 64a^2b^3x - 20a^4bx^{5/3} + 14a^5x^2 + 256ab^4x^{2/3} + 512b^5\sqrt[3]{x}}{21a^6\sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2), x]

[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) - 20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.006, size = 77, normalized size = 0.5

$$\frac{2x}{21a^6} (b + a\sqrt[3]{x}) (7x^{5/3}a^5 - 10x^{4/3}a^4b + 16xa^3b^2 - 32x^{2/3}a^2b^3 + 128\sqrt[3]{xab^4} + 256b^5) \left(bx^{\frac{2}{3}} + ax \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^(2/3)+a*x)^(3/2), x)

[Out] 2/21*x*(b+a*x^(1/3))*(7*x^(5/3)*a^5-10*x^(4/3)*a^4*b+16*x*a^3*b^2-32*x^(2/3)*a^2*b^3+128*x^(1/3)*a*b^4+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)

Giac [A] time = 1.14521, size = 151, normalized size = 0.94

$$-\frac{512 b^{\frac{9}{2}}}{21 a^6} + \frac{6 b^5}{\sqrt{ax^{\frac{1}{3}} + ba^6}} + \frac{2 \left(7 \left(ax^{\frac{1}{3}} + b \right)^{\frac{9}{2}} a^{48} - 45 \left(ax^{\frac{1}{3}} + b \right)^{\frac{7}{2}} a^{48} b + 126 \left(ax^{\frac{1}{3}} + b \right)^{\frac{5}{2}} a^{48} b^2 - 210 \left(ax^{\frac{1}{3}} + b \right)^{\frac{3}{2}} a^{48} b^3 + 315 \sqrt{ax^{\frac{1}{3}} + b} a^{48} b^4 \right)}{21 a^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3) + b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*a^48*b^4)/a^54

$$3.197 \quad \int \frac{x}{(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=68

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}} + \frac{8\sqrt{ax+bx^{2/3}}}{a^2} - \frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

[Out] $(-6*x)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) + (8*\text{Sqrt}[b*x^{(2/3)} + a*x])/a^2 - (16*b*\text{Sqrt}[b*x^{(2/3)} + a*x])/(a^3*x^{(1/3)})$

Rubi [A] time = 0.0838576, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2015, 2002, 2014}

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}} + \frac{8\sqrt{ax+bx^{2/3}}}{a^2} - \frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(b*x^{(2/3)} + a*x)^{(3/2)}, x]$

[Out] $(-6*x)/(a*\text{Sqrt}[b*x^{(2/3)} + a*x]) + (8*\text{Sqrt}[b*x^{(2/3)} + a*x])/a^2 - (16*b*\text{Sqrt}[b*x^{(2/3)} + a*x])/(a^3*x^{(1/3)})$

Rule 2015

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, j, m, n\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[j] \parallel \text{GtQ}[c, 0])$

Rule 2002

$\text{Int}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(a*(j*p+1)*x^{(j-1)}), x] - \text{Dist}[(b*(n*p+n-j+1))/(a*(j*p+1)), \text{Int}[x^{(n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, j, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[j*p+1, 0]$

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(bx^{2/3} + ax)^{3/2}} dx &= -\frac{6x}{a\sqrt{bx^{2/3} + ax}} + \frac{4 \int \frac{1}{\sqrt{bx^{2/3} + ax}} dx}{a} \\ &= -\frac{6x}{a\sqrt{bx^{2/3} + ax}} + \frac{8\sqrt{bx^{2/3} + ax}}{a^2} - \frac{(8b) \int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3} + ax}} dx}{3a^2} \\ &= -\frac{6x}{a\sqrt{bx^{2/3} + ax}} + \frac{8\sqrt{bx^{2/3} + ax}}{a^2} - \frac{16b\sqrt{bx^{2/3} + ax}}{a^3\sqrt[3]{x}} \end{aligned}$$

Mathematica [A] time = 0.0503076, size = 60, normalized size = 0.88

$$\frac{2(a^2x^{2/3} - 4ab\sqrt[3]{x} - 8b^2)\sqrt{ax + bx^{2/3}}}{a^3\sqrt[3]{x}(a\sqrt[3]{x} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2), x]

[Out] (2*(-8*b^2 - 4*a*b*x^(1/3) + a^2*x^(2/3))*Sqrt[b*x^(2/3) + a*x])/(a^3*(b + a*x^(1/3))*x^(1/3))

Maple [A] time = 0.005, size = 45, normalized size = 0.7

$$2 \frac{x(b + a\sqrt[3]{x})(x^{2/3}a^2 - 4\sqrt[3]{x}ab - 8b^2)}{(bx^{2/3} + ax)^{3/2} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^(2/3)+a*x)^(3/2), x)

[Out] $2*x*(b+a*x^{(1/3)})*(x^{(2/3)}*a^2-4*x^{(1/3)}*a*b-8*b^2)/(b*x^{(2/3)}+a*x)^{(3/2)}/a^3$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x/(a*x + b*x^(2/3))^(3/2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**(2/3)+a*x)**(3/2),x)`

[Out] `Integral(x/(a*x + b*x**(2/3))**(3/2), x)`

Giac [A] time = 1.17889, size = 81, normalized size = 1.19

$$\frac{16b^{\frac{3}{2}}}{a^3} - \frac{6b^2}{\sqrt{ax^{\frac{1}{3}} + ba^3}} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}}a^6 - 6\sqrt{ax^{\frac{1}{3}} + ba^6}b\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9

$$3.198 \quad \int \frac{1}{(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=60

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}} - \frac{6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)

Rubi [A] time = 0.0560228, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2006, 2029, 206}

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}} - \frac{6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(2/3) + a*x)^(-3/2), x]

[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)

Rule 2006

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(a*x^j + b*x^n)^(p+1)/(a*(n-j)*(p+1)*x^(j-1)), x] + Dist[(n*p + n - j + 1)/(a*(n-j)*(p+1)), Int[(a*x^j + b*x^n)^(p+1)/x^j, x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[-2/(n-j), Subst[Int[1/(1-a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(bx^{2/3} + ax)^{3/2}} dx &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3} + ax}} + \frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{b} \\ &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3} + ax}} - \frac{6 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{b} \\ &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3} + ax}} - \frac{6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3} + ax}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.0259092, size = 45, normalized size = 0.75

$$\frac{6\sqrt[3]{x} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b\sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2), x]
```

```
[Out] (6*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (a*x^(1/3))/b])/(b*Sqrt[b*x^(2/3) + a*x])
```

Maple [A] time = 0.006, size = 55, normalized size = 0.9

$$6 \frac{x(b + a\sqrt[3]{x})}{(bx^{2/3} + ax)^{3/2} b^{5/2}} \left(b^{3/2} - \operatorname{Artanh}\left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}}\right) b\sqrt{b + a\sqrt[3]{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x)^(3/2), x)
```


[Out] $6*x*(b+a*x^{(1/3)})*(b^{(3/2)}-\operatorname{arctanh}((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)}))*b*(b+a*x^{(1/3)})^{(1/2)}/(b*x^{(2/3)}+a*x)^{(3/2)}/b^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x + b*x^(2/3))^(3/2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**(2/3)+a*x)**(3/2),x)`

[Out] `Integral((a*x + b*x**(2/3))**(-3/2), x)`

Giac [A] time = 1.16026, size = 96, normalized size = 1.6

$$\frac{6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-bb}} - \frac{6\left(\sqrt{b} \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\right)}{\sqrt{-bb^{\frac{3}{2}}}} + \frac{6}{\sqrt{ax^{\frac{1}{3}}+bb}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)

$$3.199 \quad \int \frac{1}{x(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=146

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}} + \frac{105a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}} + \frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x} - \frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}} + \frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(8*b^(9/2))

Rubi [A] time = 0.240522, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}} + \frac{105a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}} + \frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x} - \frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}} + \frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]

[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(8*b^(9/2))

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c
*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(bx^{2/3} + ax)^{3/2}} dx &= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} + \frac{7 \int \frac{1}{x^{5/3}\sqrt{bx^{2/3} + ax}} dx}{b} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} - \frac{7\sqrt{bx^{2/3} + ax}}{b^2x^{4/3}} - \frac{(35a) \int \frac{1}{x^{4/3}\sqrt{bx^{2/3} + ax}} dx}{6b^2} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} - \frac{7\sqrt{bx^{2/3} + ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3} + ax}}{4b^3x} + \frac{(35a^2) \int \frac{1}{x\sqrt{bx^{2/3} + ax}} dx}{8b^3} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} - \frac{7\sqrt{bx^{2/3} + ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3} + ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3} + ax}}{8b^4x^{2/3}} - \frac{(35a^3) \int \frac{1}{x^{2/3}\sqrt{bx^{2/3} + ax}} dx}{16b^4} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} - \frac{7\sqrt{bx^{2/3} + ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3} + ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3} + ax}}{8b^4x^{2/3}} + \frac{(105a^3) \text{Subst}\left(\int \frac{1}{\sqrt{bx^{2/3} + ax}} dx\right)}{8b^4} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3} + ax}} - \frac{7\sqrt{bx^{2/3} + ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3} + ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3} + ax}}{8b^4x^{2/3}} + \frac{105a^3 \tanh^{-1}\left(\frac{\sqrt{bx^{2/3} + ax}}{\sqrt{bx^{2/3}}}\right)}{8b^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.0565704, size = 48, normalized size = 0.33

$$\frac{6a^3 \sqrt[3]{x} {}_2F_1\left(-\frac{1}{2}, 4; \frac{1}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^4 \sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]

[Out] (-6*a^3*x^(1/3)*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (a*x^(1/3))/b])/(b^4*sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.013, size = 88, normalized size = 0.6

$$-\frac{1}{8} (b + a\sqrt[3]{x}) \left(105 \sqrt{b} b x a^3 + 35 x^{2/3} b^{3/2} a^2 - 14 \sqrt[3]{x} b^{5/2} a + 8 b^{7/2} - 105 \operatorname{Arctanh} \left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}} \right) \sqrt{b + a\sqrt[3]{x} x a^3} \right) (b x^{2/3} + a x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^(2/3)+a*x)^(3/2),x)

[Out] -1/8*(b+a*x^(1/3))*(105*b^(1/2)*x*a^3+35*x^(2/3)*b^(3/2)*a^2-14*x^(1/3)*b^(5/2)*a+8*b^(7/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/2)*x*a^3)/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)

Giac [A] time = 1.23205, size = 142, normalized size = 0.97

$$\frac{105 a^3 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8 \sqrt{-bb^4}} - \frac{6 a^3}{\sqrt{ax^{\frac{1}{3}} + bb^4}} - \frac{57 \left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} a^3 - 136 \left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^3 b + 87 \sqrt{ax^{\frac{1}{3}} + ba^3 b^2}}{8 a^3 b^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqrt(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3) + b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)

$$3.200 \quad \int \frac{1}{x^2(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=236

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}} - \frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x} + \frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}} - \frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}} - \frac{9009a^6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}} +$$

[Out] $6/(b*x^{(5/3)}*Sqrt[b*x^{(2/3)} + a*x]) - (13*Sqrt[b*x^{(2/3)} + a*x])/(2*b^2*x^{(7/3)}) + (143*a*Sqrt[b*x^{(2/3)} + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^{(2/3)} + a*x])/(160*b^4*x^{(5/3)}) + (3003*a^3*Sqrt[b*x^{(2/3)} + a*x])/(320*b^5*x^{(4/3)}) - (3003*a^4*Sqrt[b*x^{(2/3)} + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^{(2/3)} + a*x])/(512*b^7*x^{(2/3)}) - (9009*a^6*ArcTanh[(Sqrt[b]*x^{(1/3)})/Sqrt[b*x^{(2/3)} + a*x]])/(512*b^{(15/2)})$

Rubi [A] time = 0.410442, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}} - \frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x} + \frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}} - \frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}} - \frac{9009a^6 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}} +$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]

[Out] $6/(b*x^{(5/3)}*Sqrt[b*x^{(2/3)} + a*x]) - (13*Sqrt[b*x^{(2/3)} + a*x])/(2*b^2*x^{(7/3)}) + (143*a*Sqrt[b*x^{(2/3)} + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^{(2/3)} + a*x])/(160*b^4*x^{(5/3)}) + (3003*a^3*Sqrt[b*x^{(2/3)} + a*x])/(320*b^5*x^{(4/3)}) - (3003*a^4*Sqrt[b*x^{(2/3)} + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^{(2/3)} + a*x])/(512*b^7*x^{(2/3)}) - (9009*a^6*ArcTanh[(Sqrt[b]*x^{(1/3)})/Sqrt[b*x^{(2/3)} + a*x]])/(512*b^{(15/2)})$

Rule 2023

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]

]

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
    + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
  t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
  && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
    + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
  x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (bx^{2/3} + ax)^{3/2}} dx &= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} + \frac{13 \int \frac{1}{x^{8/3} \sqrt{bx^{2/3} + ax}} dx}{b} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} - \frac{(143a) \int \frac{1}{x^{7/3} \sqrt{bx^{2/3} + ax}} dx}{12b^2} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} + \frac{(429a^2) \int \frac{1}{x^2 \sqrt{bx^{2/3} + ax}} dx}{40b^3} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} - \frac{(3003a^3) \int \frac{1}{x} dx}{320b^5} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} + \frac{3003a^3 \sqrt{bx^{2/3} + ax}}{320b^5 x^4} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} + \frac{3003a^3 \sqrt{bx^{2/3} + ax}}{320b^5 x^4} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} + \frac{3003a^3 \sqrt{bx^{2/3} + ax}}{320b^5 x^4} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} + \frac{3003a^3 \sqrt{bx^{2/3} + ax}}{320b^5 x^4} \\
&= \frac{6}{bx^{5/3} \sqrt{bx^{2/3} + ax}} - \frac{13 \sqrt{bx^{2/3} + ax}}{2b^2 x^{7/3}} + \frac{143a \sqrt{bx^{2/3} + ax}}{20b^3 x^2} - \frac{1287a^2 \sqrt{bx^{2/3} + ax}}{160b^4 x^{5/3}} + \frac{3003a^3 \sqrt{bx^{2/3} + ax}}{320b^5 x^4}
\end{aligned}$$

Mathematica [C] time = 0.0578095, size = 48, normalized size = 0.2

$$\frac{6a^6 \sqrt[3]{x} {}_2F_1\left(-\frac{1}{2}, 7; \frac{1}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^7 \sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)), x]

[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.017, size = 126, normalized size = 0.5

$$-\frac{1}{2560x} (b + a\sqrt[3]{x}) \left(45045 \sqrt{b + a\sqrt[3]{x}} \operatorname{Artanh} \left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}} \right) x^2 a^6 - 1664 b^{11/2} \sqrt[3]{xa} + 2288 b^{9/2} x^{2/3} a^2 - 3432 b^{7/2} x a^3 + 6006 b^{5/2} x^2 a^4 - 15015 b^{3/2} x^3 a^5 - 45045 x^4 a^6 + 1280 b^{13/2} x^5 \right) / (b x^{2/3} + a x)^{3/2} / b^{15/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^(2/3)+a*x)^(3/2), x)

[Out] -1/2560*(b+a*x^(1/3))*(45045*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*x^2*a^6-1664*b^(11/2)*x^(1/3)*a+2288*b^(9/2)*x^(2/3)*a^2-3432*b^(7/2)*x*a^3+6006*b^(5/2)*x^2*a^4-15015*b^(3/2)*x^3*a^5-45045*x^4*a^6+1280*b^(13/2))/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2), x)

[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)

Giac [A] time = 1.29799, size = 211, normalized size = 0.89

$$\frac{9009 a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512 \sqrt{-bb^7}} + \frac{6 a^6}{\sqrt{ax^{\frac{1}{3}} + bb^7}} + \frac{29685 \left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^6 - 163095 \left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} a^6 b + 364194 \left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} a^6 b^2 - 416094 \left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} a^6 b^3 + 246505 \left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^6 b^4 - 62475 \sqrt{ax^{\frac{1}{3}} + b} a^6 b^5}{2560 a^6 b^7 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2), x, algorithm="giac")

[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(sqrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 416094*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)

$$3.201 \quad \int \frac{1}{x^3(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=324

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}} + \frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x} - \frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}} + \frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}} - \frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}$$

[Out] $6/(b*x^{(8/3)}*Sqrt[b*x^{(2/3)} + a*x]) - (19*Sqrt[b*x^{(2/3)} + a*x])/(3*b^2*x^{(10/3)}) + (323*a*Sqrt[b*x^{(2/3)} + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^{(2/3)} + a*x])/(224*b^4*x^{(8/3)}) + (20995*a^3*Sqrt[b*x^{(2/3)} + a*x])/(2688*b^5*x^{(7/3)}) - (46189*a^4*Sqrt[b*x^{(2/3)} + a*x])/(5376*b^6*x^2) + (138567*a^5*Sqrt[b*x^{(2/3)} + a*x])/(14336*b^7*x^{(5/3)}) - (46189*a^6*Sqrt[b*x^{(2/3)} + a*x])/(4096*b^8*x^{(4/3)}) + (230945*a^7*Sqrt[b*x^{(2/3)} + a*x])/(16384*b^9*x) - (692835*a^8*Sqrt[b*x^{(2/3)} + a*x])/(32768*b^{10}*x^{(2/3)}) + (692835*a^9*ArcTanh[(Sqrt[b]*x^{(1/3)})/Sqrt[b*x^{(2/3)} + a*x]])/(32768*b^{(21/2)})$

Rubi [A] time = 0.597076, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}} + \frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x} - \frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}} + \frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}} - \frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)), x]

[Out] $6/(b*x^{(8/3)}*Sqrt[b*x^{(2/3)} + a*x]) - (19*Sqrt[b*x^{(2/3)} + a*x])/(3*b^2*x^{(10/3)}) + (323*a*Sqrt[b*x^{(2/3)} + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^{(2/3)} + a*x])/(224*b^4*x^{(8/3)}) + (20995*a^3*Sqrt[b*x^{(2/3)} + a*x])/(2688*b^5*x^{(7/3)}) - (46189*a^4*Sqrt[b*x^{(2/3)} + a*x])/(5376*b^6*x^2) + (138567*a^5*Sqrt[b*x^{(2/3)} + a*x])/(14336*b^7*x^{(5/3)}) - (46189*a^6*Sqrt[b*x^{(2/3)} + a*x])/(4096*b^8*x^{(4/3)}) + (230945*a^7*Sqrt[b*x^{(2/3)} + a*x])/(16384*b^9*x) - (692835*a^8*Sqrt[b*x^{(2/3)} + a*x])/(32768*b^{10}*x^{(2/3)}) + (692835*a^9*ArcTanh[(Sqrt[b]*x^{(1/3)})/Sqrt[b*x^{(2/3)} + a*x]])/(32768*b^{(21/2)})$

Rule 2023

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)

```

*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]

```

Rule 2025

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2029

```

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (bx^{2/3} + ax)^{3/2}} dx &= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} + \frac{19 \int \frac{1}{x^{11/3} \sqrt{bx^{2/3} + ax}} dx}{b} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} - \frac{(323a) \int \frac{1}{x^{10/3} \sqrt{bx^{2/3} + ax}} dx}{18b^2} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} + \frac{(1615a^2) \int \frac{1}{x^3 \sqrt{bx^{2/3} + ax}} dx}{96b^3} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} - \frac{(20995a^3) \int \frac{1}{x^3 \sqrt{bx^{2/3} + ax}} dx}{1344b^4} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}} \\
&= \frac{6}{bx^{8/3} \sqrt{bx^{2/3} + ax}} - \frac{19 \sqrt{bx^{2/3} + ax}}{3b^2 x^{10/3}} + \frac{323a \sqrt{bx^{2/3} + ax}}{48b^3 x^3} - \frac{1615a^2 \sqrt{bx^{2/3} + ax}}{224b^4 x^{8/3}} + \frac{20995a^3 \sqrt{bx^{2/3} + ax}}{2688b^5 x^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.0566937, size = 48, normalized size = 0.15

$$\frac{6a^9 \sqrt[3]{x} {}_2F_1\left(-\frac{1}{2}, 10; \frac{1}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^{10} \sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]

[Out] $(-6*a^9*x^{(1/3)}*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^{(1/3)})/b])/(b^{10}*Sqrt[b*x^{(2/3)} + a*x])$

Maple [A] time = 0.018, size = 159, normalized size = 0.5

$$\frac{1}{688128 x^2} (b + a \sqrt[3]{x}) \left(-537472 b^{11/2} x^{4/3} a^4 + 739024 b^{9/2} x^{5/3} a^5 - 1108536 b^{7/2} x^2 a^6 + 1939938 b^{5/2} x^{7/3} a^7 - 4849845 b^{3/2} x^3 a^8 + 14549535 b^{1/2} x^4 a^9 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x)

[Out] $1/688128*(b+a*x^{(1/3)})*(-537472*b^{(11/2)}*x^{(4/3)}*a^4+739024*b^{(9/2)}*x^{(5/3)}*a^5-1108536*b^{(7/2)}*x^2*a^6+1939938*b^{(5/2)}*x^{(7/3)}*a^7-4849845*b^{(3/2)}*x^{(8/3)}*a^8-14549535*x^3*a^9*b^{(1/2)}+272384*b^{(17/2)}*x^{(1/3)}*a-330752*b^{(15/2)}*x^{(2/3)}*a^2+413440*b^{(13/2)}*x*a^3-229376*b^{(19/2)}+14549535*(b+a*x^{(1/3)})^{(1/2)}*arctanh((b+a*x^{(1/3)})^{(1/2)}/b^{(1/2)})*x^3*a^9)/x^2/(b*x^{(2/3)}+a*x)^{(3/2)}/b^{(21/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)

Giac [A] time = 1.36586, size = 279, normalized size = 0.86

$$\frac{692835 a^9 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768 \sqrt{-b} b^{10}} - \frac{6 a^9}{\sqrt{ax^{\frac{1}{3}} + b} b^{10}} - \frac{10420767 \left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}} a^9 - 88937058 \left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} a^9 b + 334408914 \left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} a^9 b^2 - 724860666 \left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^9 b^3 + 993296384 \left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} a^9 b^4 - 884769030 \left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} a^9 b^5 + 503730990 \left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} a^9 b^6 - 169799070 \left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^9 b^7 + 26738145 \sqrt{ax^{\frac{1}{3}} + b} a^9 b^8}{(a^9 b^{10} x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 + 26738145*sqrt(a*x^(1/3) + b)*a^9*b^8/(a^9*b^10*x^3)

$$3.202 \quad \int \frac{1}{x^4(bx^{2/3}+ax)^{3/2}} dx$$

Optimal. Leaf size=412

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}} - \frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x} + \frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}} - \frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}} + \frac{482885a^7\sqrt{ax+bx^{2/3}}}{49152b^9x^2}$$

[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (3380195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(2097152*b^(27/2))

Rubi [A] time = 0.83999, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2029, 206}

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}} - \frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x} + \frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}} - \frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}} + \frac{482885a^7\sqrt{ax+bx^{2/3}}}{49152b^9x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]

[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (3380195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(2097152*b^(27/2))

)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(2097152*b^(27/2))

Rule 2023

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

Mathematica [C] time = 0.0589611, size = 48, normalized size = 0.12

$$\frac{6a^{12}\sqrt[3]{x} {}_2F_1\left(-\frac{1}{2}, 13; \frac{1}{2}; \frac{\sqrt[3]{xa}}{b} + 1\right)}{b^{13}\sqrt{ax + bx^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)), x]

[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*Sqrt[b*x^(2/3) + a*x])

Maple [A] time = 0.025, size = 192, normalized size = 0.5

$$-\frac{1}{69206016x^3} (b + a\sqrt[3]{x}) \left(17301504b^{\frac{25}{2}} + 1673196525\sqrt{b + a\sqrt[3]{x}} \operatorname{Artanh}\left(\frac{\sqrt{b + a\sqrt[3]{x}}}{\sqrt{b}}\right) x^4 a^{12} - 19660800b^{23/2}\sqrt[3]{xa} + 22609920b^{21/2}x^{2/3}a^2 - 26378240b^{19/2}xa^3 + 31324160b^{17/2}x^{4/3}a^4 - 38036480b^{15/2}x^{5/3}a^5 + 47545600b^{13/2}x^2a^6 - 61809280b^{11/2}x^{7/3}a^7 + 84987760b^{9/2}x^{8/3}a^8 - 127481640b^{7/2}x^3a^9 + 223092870b^{5/2}x^{10/3}a^{10} - 557732175b^{3/2}x^{11/3}a^{11} - 1673196525x^4a^{12}b^{1/2} \right) / x^3 / (b*x^{2/3} + a*x)^{3/2} / b^{27/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^(2/3)+a*x)^(3/2), x)

[Out] -1/69206016*(b+a*x^(1/3))*(17301504*b^(25/2)+1673196525*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*x^4*a^12-19660800*b^(23/2)*x^(1/3)*a+22609920*b^(21/2)*x^(2/3)*a^2-26378240*b^(19/2)*x*a^3+31324160*b^(17/2)*x^(4/3)*a^4-38036480*b^(15/2)*x^(5/3)*a^5+47545600*b^(13/2)*x^2*a^6-61809280*b^(11/2)*x^(7/3)*a^7+84987760*b^(9/2)*x^(8/3)*a^8-127481640*b^(7/2)*x^3*a^9+223092870*b^(5/2)*x^(10/3)*a^10-557732175*b^(3/2)*x^(11/3)*a^11-1673196525*x^4*a^12*b^(1/2))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax + bx^{\frac{2}{3}})^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.51867, size = 348, normalized size = 0.84

$$\frac{50702925 a^{12} \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152 \sqrt{-bb^{13}}} + \frac{6 a^{12}}{\sqrt{ax^{\frac{1}{3}} + bb^{13}}} + \frac{1257960429 \left(ax^{\frac{1}{3}} + b\right)^{\frac{23}{2}} a^{12} - 14537792973 \left(ax^{\frac{1}{3}} + b\right)^{\frac{21}{2}} a^{12} b + 7666}{\sqrt{ax^{\frac{1}{3}} + bb^{13}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")

[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13) + 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b

$$\begin{aligned}
&)^{(23/2)} * a^{12} - 14537792973 * (a * x^{(1/3)} + b)^{(21/2)} * a^{12} * b + 76667241519 * (a * \\
& x^{(1/3)} + b)^{(19/2)} * a^{12} * b^2 - 243717614415 * (a * x^{(1/3)} + b)^{(17/2)} * a^{12} * b^3 \\
& + 519393101810 * (a * x^{(1/3)} + b)^{(15/2)} * a^{12} * b^4 - 780150847218 * (a * x^{(1/3)} + \\
& b)^{(13/2)} * a^{12} * b^5 + 844265343246 * (a * x^{(1/3)} + b)^{(11/2)} * a^{12} * b^6 - 659969 \\
& 685518 * (a * x^{(1/3)} + b)^{(9/2)} * a^{12} * b^7 + 366679446705 * (a * x^{(1/3)} + b)^{(7/2)} * \\
& a^{12} * b^8 - 138840292305 * (a * x^{(1/3)} + b)^{(5/2)} * a^{12} * b^9 + 32660709939 * (a * x^{(\\
& 1/3)} + b)^{(3/2)} * a^{12} * b^{10} - 3724872723 * \text{sqrt}(a * x^{(1/3)} + b) * a^{12} * b^{11} / (a^{12} \\
& * b^{13} * x^4)
\end{aligned}$$

3.203 $\int x^2 (ax^2 + bx^3) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

[Out] (a*x^5)/5 + (b*x^6)/6

Rubi [A] time = 0.0088074, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3), x]

[Out] (a*x^5)/5 + (b*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 (ax^2 + bx^3) dx &= \int (ax^4 + bx^5) dx \\ &= \frac{ax^5}{5} + \frac{bx^6}{6} \end{aligned}$$

Mathematica [A] time = 0.0011068, size = 17, normalized size = 1.

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3),x]

[Out] (a*x^5)/5 + (b*x^6)/6

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a*x^2),x)

[Out] 1/5*a*x^5+1/6*b*x^6

Maxima [A] time = 0.994284, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")

[Out] 1/6*b*x^6 + 1/5*a*x^5

Fricas [A] time = 1.06124, size = 31, normalized size = 1.82

$$\frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")

[Out] $1/6*x^6*b + 1/5*x^5*a$

Sympy [A] time = 0.116661, size = 12, normalized size = 0.71

$$\frac{ax^5}{5} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a*x**2),x)`

[Out] $a*x**5/5 + b*x**6/6$

Giac [A] time = 1.14835, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $1/6*b*x^6 + 1/5*a*x^5$

3.204 $\int x(ax^2 + bx^3) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

[Out] (a*x^4)/4 + (b*x^5)/5

Rubi [A] time = 0.0079168, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x(ax^2 + bx^3) dx &= \int (ax^3 + bx^4) dx \\ &= \frac{ax^4}{4} + \frac{bx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.0010626, size = 17, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x^2),x)

[Out] 1/4*a*x^4+1/5*b*x^5

Maxima [A] time = 0.988774, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")

[Out] 1/5*b*x^5 + 1/4*a*x^4

Fricas [A] time = 1.02401, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")

[Out] $1/5*x^5*b + 1/4*x^4*a$

Sympy [A] time = 0.142072, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**3+a*x**2),x)`

[Out] $a*x**4/4 + b*x**5/5$

Giac [A] time = 1.10421, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $1/5*b*x^5 + 1/4*a*x^4$

3.205

$$\int (ax^2 + bx^3) dx$$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

[Out] (a*x^3)/3 + (b*x^4)/4

Rubi [A] time = 0.0029354, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[a*x^2 + b*x^3,x]

[Out] (a*x^3)/3 + (b*x^4)/4

Rubi steps

$$\int (ax^2 + bx^3) dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

Mathematica [A] time = 0.0000287, size = 17, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[a*x^2 + b*x^3,x]

[Out] (a*x^3)/3 + (b*x^4)/4

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^3+a*x^2,x)

[Out] 1/3*a*x^3+1/4*b*x^4

Maxima [A] time = 0.98957, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")

[Out] 1/4*b*x^4 + 1/3*a*x^3

Fricas [A] time = 1.09005, size = 31, normalized size = 1.82

$$\frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")

[Out] 1/4*x^4*b + 1/3*x^3*a

Sympy [A] time = 0.217801, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A] time = 1.15425, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

$$3.206 \quad \int \frac{ax^2+bx^3}{x} dx$$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

[Out] (a*x^2)/2 + (b*x^3)/3

Rubi [A] time = 0.006029, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3}{x} dx &= \int (ax + bx^2) dx \\ &= \frac{ax^2}{2} + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0011616, size = 17, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)/x,x)

[Out] 1/2*a*x^2+1/3*b*x^3

Maxima [A] time = 0.985271, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")

[Out] 1/3*b*x^3 + 1/2*a*x^2

Fricas [A] time = 0.958315, size = 31, normalized size = 1.82

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")

[Out] $\frac{1}{3}bx^3 + \frac{1}{2}ax^2$

Sympy [A] time = 0.08106, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)/x,x)`

[Out] $a*x**2/2 + b*x**3/3$

Giac [A] time = 1.19771, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)/x,x, algorithm="giac")`

[Out] $\frac{1}{3}bx^3 + \frac{1}{2}ax^2$

$$3.207 \quad \int \frac{ax^2 + bx^3}{x^2} dx$$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

[Out] a*x + (b*x^2)/2

Rubi [A] time = 0.0035858, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/x^2,x]

[Out] a*x + (b*x^2)/2

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3}{x^2} dx &= \int (a + bx) dx \\ &= ax + \frac{bx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.000439, size = 12, normalized size = 1.

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)/x^2,x]

[Out] a*x + (b*x^2)/2

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)/x^2,x)

[Out] a*x+1/2*b*x^2

Maxima [A] time = 0.974904, size = 14, normalized size = 1.17

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*x

Fricas [A] time = 0.801145, size = 23, normalized size = 1.92

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")

[Out] $1/2*b*x^2 + a*x$

Sympy [A] time = 0.100523, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)/x**2,x)`

[Out] $a*x + b*x**2/2$

Giac [A] time = 1.13288, size = 14, normalized size = 1.17

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")`

[Out] $1/2*b*x^2 + a*x$

3.208 $\int x^2 (ax^2 + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{b^2x^9}{9}$$

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9

Rubi [A] time = 0.0314118, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 (ax^2 + bx^3)^2 dx &= \int x^6 (a + bx)^2 dx \\ &= \int (a^2 x^6 + 2abx^7 + b^2 x^8) dx \\ &= \frac{a^2 x^7}{7} + \frac{1}{4} abx^8 + \frac{b^2 x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0023497, size = 30, normalized size = 1.

$$\frac{a^2 x^7}{7} + \frac{1}{4} abx^8 + \frac{b^2 x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{a^2 x^7}{7} + \frac{abx^8}{4} + \frac{b^2 x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a*x^2)^2,x)

[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9

Maxima [A] time = 0.986406, size = 32, normalized size = 1.07

$$\frac{1}{9} b^2 x^9 + \frac{1}{4} abx^8 + \frac{1}{7} a^2 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{9}b^2x^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$

Fricas [A] time = 0.574869, size = 55, normalized size = 1.83

$$\frac{1}{9}x^9b^2 + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9b^2 + \frac{1}{4}x^8b*a + \frac{1}{7}x^7a^2$

Sympy [A] time = 0.122817, size = 24, normalized size = 0.8

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9$

Giac [A] time = 1.1433, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $\frac{1}{9}b^2x^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$

3.209 $\int x (ax^2 + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{b^2x^8}{8}$$

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8

Rubi [A] time = 0.0170845, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 43}

$$\frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int x(ax^2 + bx^3)^2 dx &= \int x^5(a + bx)^2 dx \\
 &= \int (a^2x^5 + 2abx^6 + b^2x^7) dx \\
 &= \frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{b^2x^8}{8}
 \end{aligned}$$

Mathematica [A] time = 0.001907, size = 30, normalized size = 1.

$$\frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x^2)^2,x)

[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8

Maxima [A] time = 0.979917, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{8}b^2x^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$

Fricas [A] time = 0.617166, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2 + \frac{2}{7}x^7ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $\frac{1}{8}x^8b^2 + \frac{2}{7}x^7b*a + \frac{1}{6}x^6a^2$

Sympy [A] time = 0.242338, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8$

Giac [A] time = 1.13299, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $\frac{1}{8}b^2x^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$

3.210 $\int (ax^2 + bx^3)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{b^2x^7}{7}$$

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7

Rubi [A] time = 0.0134057, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 43}

$$\frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}\int (ax^2 + bx^3)^2 dx &= \int x^4(a + bx)^2 dx \\ &= \int (a^2x^4 + 2abx^5 + b^2x^6) dx \\ &= \frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0015421, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^2,x)

[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7

Maxima [A] time = 0.993275, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5$

Fricas [A] time = 0.626779, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2 + \frac{1}{3}x^6ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $1/7*x^7*b^2 + 1/3*x^6*b*a + 1/5*x^5*a^2$

Sympy [A] time = 0.122, size = 24, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7$

Giac [A] time = 1.15384, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5$

$$3.211 \quad \int \frac{(ax^2+bx^3)^2}{x} dx$$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6

Rubi [A] time = 0.0174214, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3)^2}{x} dx &= \int x^3(a + bx)^2 dx \\ &= \int (a^2x^3 + 2abx^4 + b^2x^5) dx \\ &= \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6} \end{aligned}$$

Mathematica [A] time = 0.0016031, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6

Maple [A] time = 0.002, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^2/x,x)

[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6

Maxima [A] time = 1.01225, size = 32, normalized size = 1.07

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")

[Out] $1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4$

Fricas [A] time = 0.704881, size = 55, normalized size = 1.83

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")`

[Out] $1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4$

Sympy [A] time = 0.239738, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**2/x,x)`

[Out] $a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6$

Giac [A] time = 1.14748, size = 32, normalized size = 1.07

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")`

[Out] $1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4$

$$3.212 \quad \int \frac{(ax^2+bx^3)^2}{x^2} dx$$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5

Rubi [A] time = 0.0163855, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^2/x^2, x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3)^2}{x^2} dx &= \int x^2(a + bx)^2 dx \\ &= \int (a^2x^2 + 2abx^3 + b^2x^4) dx \\ &= \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.0015831, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5

Maple [A] time = 0., size = 25, normalized size = 0.8

$$\frac{x^3a^2}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^2/x^2,x)

[Out] 1/3*x^3*a^2+1/2*a*b*x^4+1/5*b^2*x^5

Maxima [A] time = 0.993497, size = 32, normalized size = 1.07

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")

[Out] $\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$

Fricas [A] time = 0.71121, size = 55, normalized size = 1.83

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")`

[Out] $\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$

Sympy [A] time = 0.219798, size = 24, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**2/x**2,x)`

[Out] $a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5$

Giac [A] time = 1.20632, size = 32, normalized size = 1.07

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")`

[Out] $\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$

$$3.213 \quad \int \frac{x^6}{ax^2+bx^3} dx$$

Optimal. Leaf size=57

$$\frac{a^2x^2}{2b^3} - \frac{a^3x}{b^4} + \frac{a^4 \log(a+bx)}{b^5} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

[Out] $-\frac{(a^3x)}{b^4} + \frac{(a^2x^2)}{(2b^3)} - \frac{(ax^3)}{(3b^2)} + \frac{x^4}{(4b)} + \frac{(a^4 \text{Log}[a + bx])}{b^5}$

Rubi [A] time = 0.0355117, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2x^2}{2b^3} - \frac{a^3x}{b^4} + \frac{a^4 \log(a+bx)}{b^5} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3),x]

[Out] $-\frac{(a^3x)}{b^4} + \frac{(a^2x^2)}{(2b^3)} - \frac{(ax^3)}{(3b^2)} + \frac{x^4}{(4b)} + \frac{(a^4 \text{Log}[a + bx])}{b^5}$

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
 [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
 x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
 Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^6}{ax^2 + bx^3} dx &= \int \frac{x^4}{a + bx} dx \\ &= \int \left(-\frac{a^3}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a + bx)} \right) dx \\ &= -\frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4 \log(a + bx)}{b^5} \end{aligned}$$

Mathematica [A] time = 0.0045484, size = 57, normalized size = 1.

$$\frac{a^2x^2}{2b^3} - \frac{a^3x}{b^4} + \frac{a^4 \log(a + bx)}{b^5} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3), x]

[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log[a + b*x])/b^5

Maple [A] time = 0.003, size = 52, normalized size = 0.9

$$-\frac{xa^3}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4 \ln(bx + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^3+a*x^2), x)

[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5

Maxima [A] time = 0.961211, size = 70, normalized size = 1.23

$$\frac{a^4 \log(bx + a)}{b^5} + \frac{3b^3x^4 - 4ab^2x^3 + 6a^2bx^2 - 12a^3x}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4

Fricas [A] time = 0.734538, size = 117, normalized size = 2.05

$$\frac{3b^4x^4 - 4ab^3x^3 + 6a^2b^2x^2 - 12a^3bx + 12a^4 \log(bx + a)}{12b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")

[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x + a))/b^5

Sympy [A] time = 0.721893, size = 49, normalized size = 0.86

$$\frac{a^4 \log(a + bx)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**3+a*x**2),x)

[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2) + x**4/(4*b)

Giac [A] time = 1.14571, size = 72, normalized size = 1.26

$$\frac{a^4 \log(|bx + a|)}{b^5} + \frac{3b^3x^4 - 4ab^2x^3 + 6a^2bx^2 - 12a^3x}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")

[Out] $a^4 \log(\text{abs}(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4$

$$3.214 \quad \int \frac{x^5}{ax^2+bx^3} dx$$

Optimal. Leaf size=44

$$\frac{a^2x}{b^3} - \frac{a^3 \log(a+bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4

Rubi [A] time = 0.0270089, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2x}{b^3} - \frac{a^3 \log(a+bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3), x]

[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2 + bx^3} dx &= \int \frac{x^3}{a + bx} dx \\
&= \int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a + bx)} \right) dx \\
&= \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \log(a + bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.003585, size = 44, normalized size = 1.

$$\frac{a^2x}{b^3} - \frac{a^3 \log(a + bx)}{b^4} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3),x]

[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4

Maple [A] time = 0.002, size = 41, normalized size = 0.9

$$\frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \ln(bx + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a*x^2),x)

[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4

Maxima [A] time = 0.971426, size = 57, normalized size = 1.3

$$-\frac{a^3 \log(bx + a)}{b^4} + \frac{2b^2x^3 - 3abx^2 + 6a^2x}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $-a^3 \log(bx + a)/b^4 + 1/6(2b^2x^3 - 3abx^2 + 6a^2x)/b^3$

Fricas [A] time = 0.872357, size = 92, normalized size = 2.09

$$\frac{2b^3x^3 - 3ab^2x^2 + 6a^2bx - 6a^3 \log(bx + a)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $1/6(2b^3x^3 - 3ab^2x^2 + 6a^2bx - 6a^3 \log(bx + a))/b^4$

Sympy [A] time = 0.486927, size = 37, normalized size = 0.84

$$-\frac{a^3 \log(a + bx)}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**3+a*x**2),x)`

[Out] $-a**3 \log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)$

Giac [A] time = 1.12933, size = 58, normalized size = 1.32

$$-\frac{a^3 \log(|bx + a|)}{b^4} + \frac{2b^2x^3 - 3abx^2 + 6a^2x}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $-a^3 \log(\text{abs}(bx + a))/b^4 + 1/6(2b^2x^3 - 3abx^2 + 6a^2x)/b^3$

$$3.215 \quad \int \frac{x^4}{ax^2+bx^3} dx$$

Optimal. Leaf size=31

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

[Out] $-\frac{(a*x)}{b^2} + \frac{x^2}{2*b} + \frac{(a^2*\text{Log}[a + b*x])}{b^3}$

Rubi [A] time = 0.0205414, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(a*x^2 + b*x^3), x]$

[Out] $-\frac{(a*x)}{b^2} + \frac{x^2}{2*b} + \frac{(a^2*\text{Log}[a + b*x])}{b^3}$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{ax^2 + bx^3} dx &= \int \frac{x^2}{a + bx} dx \\ &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a + bx)} \right) dx \\ &= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a + bx)}{b^3} \end{aligned}$$

Mathematica [A] time = 0.003362, size = 31, normalized size = 1.

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3),x]

[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3

Maple [A] time = 0.001, size = 30, normalized size = 1.

$$-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \ln(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x^2),x)

[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3

Maxima [A] time = 0.977369, size = 39, normalized size = 1.26

$$\frac{a^2 \log(bx + a)}{b^3} + \frac{bx^2 - 2ax}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $a^2 \log(bx + a)/b^3 + 1/2*(bx^2 - 2ax)/b^2$

Fricas [A] time = 0.689718, size = 68, normalized size = 2.19

$$\frac{b^2 x^2 - 2 abx + 2 a^2 \log(bx + a)}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*\log(b*x + a))/b^3$

Sympy [A] time = 0.624748, size = 26, normalized size = 0.84

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**3+a*x**2),x)`

[Out] $a**2*\log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)$

Giac [A] time = 1.15661, size = 41, normalized size = 1.32

$$\frac{a^2 \log(|bx + a|)}{b^3} + \frac{bx^2 - 2ax}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $a^2*\log(\text{abs}(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2$

$$3.216 \quad \int \frac{x^3}{ax^2+bx^3} dx$$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

[Out] x/b - (a*Log[a + b*x])/b^2

Rubi [A] time = 0.0163976, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3),x]

[Out] x/b - (a*Log[a + b*x])/b^2

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax^2 + bx^3} dx &= \int \frac{x}{a + bx} dx \\ &= \int \left(\frac{1}{b} - \frac{a}{b(a + bx)} \right) dx \\ &= \frac{x}{b} - \frac{a \log(a + bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.0025374, size = 18, normalized size = 1.

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3),x]

[Out] x/b - (a*Log[a + b*x])/b^2

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{x}{b} - \frac{a \ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x^2),x)

[Out] x/b-a*ln(b*x+a)/b^2

Maxima [A] time = 0.989485, size = 24, normalized size = 1.33

$$\frac{x}{b} - \frac{a \log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $x/b - a \cdot \log(b \cdot x + a) / b^2$

Fricas [A] time = 0.83147, size = 38, normalized size = 2.11

$$\frac{bx - a \log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $(b \cdot x - a \cdot \log(b \cdot x + a)) / b^2$

Sympy [A] time = 0.647865, size = 14, normalized size = 0.78

$$-\frac{a \log(a + bx)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**3+a*x**2),x)`

[Out] $-a \cdot \log(a + b \cdot x) / b^2 + x / b$

Giac [A] time = 1.15236, size = 26, normalized size = 1.44

$$\frac{x}{b} - \frac{a \log(|bx + a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $x/b - a \cdot \log(\text{abs}(b \cdot x + a)) / b^2$

$$3.217 \quad \int \frac{x^2}{ax^2+bx^3} dx$$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

[Out] Log[a + b*x]/b

Rubi [A] time = 0.0072511, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3), x]

[Out] Log[a + b*x]/b

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{ax^2 + bx^3} dx = \int \frac{1}{a + bx} dx = \frac{\log(a + bx)}{b}$$

Mathematica [A] time = 0.0009937, size = 10, normalized size = 1.

$$\frac{\log(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3),x]

[Out] Log[a + b*x]/b

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x^2),x)

[Out] ln(b*x+a)/b

Maxima [A] time = 1.09744, size = 14, normalized size = 1.4

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] log(b*x + a)/b

Fricas [A] time = 0.761894, size = 22, normalized size = 2.2

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.229684, size = 7, normalized size = 0.7

$$\frac{\log(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A] time = 1.15313, size = 15, normalized size = 1.5

$$\frac{\log(|bx + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

$$3.218 \quad \int \frac{x}{ax^2+bx^3} dx$$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

[Out] Log[x]/a - Log[a + b*x]/a

Rubi [A] time = 0.0074465, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1584, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3),x]

[Out] Log[x]/a - Log[a + b*x]/a

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{ax^2 + bx^3} dx &= \int \frac{1}{x(a + bx)} dx \\ &= \frac{\int \frac{1}{x} dx}{a} - \frac{b \int \frac{1}{a+bx} dx}{a} \\ &= \frac{\log(x)}{a} - \frac{\log(a + bx)}{a} \end{aligned}$$

Mathematica [A] time = 0.0035121, size = 18, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log(a + bx)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3), x]

[Out] Log[x]/a - Log[a + b*x]/a

Maple [A] time = 0.005, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{a} - \frac{\ln(bx + a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x^2), x)

[Out] ln(x)/a-ln(b*x+a)/a

Maxima [A] time = 0.978949, size = 24, normalized size = 1.33

$$-\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $-\log(b*x + a)/a + \log(x)/a$

Fricas [A] time = 0.781125, size = 38, normalized size = 2.11

$$\frac{\log(bx + a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")

[Out] $-(\log(b*x + a) - \log(x))/a$

Sympy [A] time = 0.542956, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a*x**2),x)

[Out] $(\log(x) - \log(a/b + x))/a$

Giac [A] time = 1.15968, size = 27, normalized size = 1.5

$$-\frac{\log(|bx + a|)}{a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")

[Out] $-\log(\text{abs}(b*x + a))/a + \log(\text{abs}(x))/a$

$$3.219 \quad \int \frac{1}{ax^2+bx^3} dx$$

Optimal. Leaf size=28

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Rubi [A] time = 0.0146112, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 44}

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3)^{-1}, x]$

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 44

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{ax^2 + bx^3} dx &= \int \frac{1}{x^2(a + bx)} dx \\ &= \int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a + bx)} \right) dx \\ &= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx)}{a^2} \end{aligned}$$

Mathematica [A] time = 0.0041116, size = 28, normalized size = 1.

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a + bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(-1),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2

Maple [A] time = 0.006, size = 29, normalized size = 1.

$$-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx + a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a*x^2),x)

[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2

Maxima [A] time = 1.01665, size = 38, normalized size = 1.36

$$\frac{b \log(bx + a)}{a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $b \cdot \log(bx + a)/a^2 - b \cdot \log(x)/a^2 - 1/(a \cdot x)$

Fricas [A] time = 0.795504, size = 61, normalized size = 2.18

$$\frac{bx \log(bx + a) - bx \log(x) - a}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $(b \cdot x \cdot \log(bx + a) - b \cdot x \cdot \log(x) - a)/(a^2 \cdot x)$

Sympy [A] time = 0.714865, size = 19, normalized size = 0.68

$$-\frac{1}{ax} + \frac{b \left(-\log(x) + \log\left(\frac{a}{b} + x\right) \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a*x**2),x)`

[Out] $-1/(a \cdot x) + b \cdot (-\log(x) + \log(a/b + x))/a^2$

Giac [A] time = 1.14083, size = 41, normalized size = 1.46

$$\frac{b \log(|bx + a|)}{a^2} - \frac{b \log(|x|)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $b \cdot \log(\text{abs}(bx + a))/a^2 - b \cdot \log(\text{abs}(x))/a^2 - 1/(a \cdot x)$

$$3.220 \quad \int \frac{1}{x(ax^2+bx^3)} dx$$

Optimal. Leaf size=42

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3$

Rubi [A] time = 0.0212892, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 44}

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3)),x]

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3$

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[
 ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
 & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x(ax^2 + bx^3)} dx &= \int \frac{1}{x^3(a + bx)} dx \\ &= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a + bx)} \right) dx \\ &= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a + bx)}{a^3} \end{aligned}$$

Mathematica [A] time = 0.0043671, size = 42, normalized size = 1.

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a + bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]

[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3

Maple [A] time = 0.007, size = 41, normalized size = 1.

$$-\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(bx + a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x^2),x)

[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3

Maxima [A] time = 1.04069, size = 54, normalized size = 1.29

$$-\frac{b^2 \log(bx + a)}{a^3} + \frac{b^2 \log(x)}{a^3} + \frac{2bx - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $-b^2 \log(bx + a)/a^3 + b^2 \log(x)/a^3 + 1/2(2bx - a)/(a^2 x^2)$

Fricas [A] time = 0.780534, size = 103, normalized size = 2.45

$$\frac{2b^2x^2 \log(bx + a) - 2b^2x^2 \log(x) - 2abx + a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $-1/2(2b^2x^2 \log(bx + a) - 2b^2x^2 \log(x) - 2abx + a^2)/(a^3x^2)$

Sympy [A] time = 1.03191, size = 31, normalized size = 0.74

$$\frac{-a + 2bx}{2a^2x^2} + \frac{b^2 \left(\log(x) - \log\left(\frac{a}{b} + x\right) \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**3+a*x**2),x)`

[Out] $(-a + 2bx)/(2a^2x^2) + b^2(\log(x) - \log(a/b + x))/a^3$

Giac [A] time = 1.15355, size = 61, normalized size = 1.45

$$-\frac{b^2 \log(|bx + a|)}{a^3} + \frac{b^2 \log(|x|)}{a^3} + \frac{2abx - a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")`

[Out] $-b^2 \log(\text{abs}(bx + a))/a^3 + b^2 \log(\text{abs}(x))/a^3 + 1/2(2abx - a^2)/(a^3 x^2)$

$$3.221 \quad \int \frac{1}{x^2(ax^2+bx^3)} dx$$

Optimal. Leaf size=56

$$-\frac{b^2}{a^3x} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log(a+bx)}{a^4} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

[Out] $-1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a + b*x])/a^4$

Rubi [A] time = 0.0267151, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 44}

$$-\frac{b^2}{a^3x} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log(a+bx)}{a^4} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a*x^2 + b*x^3)),x]`

[Out] $-1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a + b*x])/a^4$

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
  & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
  + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(ax^2 + bx^3)} dx &= \int \frac{1}{x^4(a + bx)} dx \\ &= \int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a + bx)} \right) dx \\ &= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2}{a^3x} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log(a + bx)}{a^4} \end{aligned}$$

Mathematica [A] time = 0.0048084, size = 56, normalized size = 1.

$$-\frac{b^2}{a^3x} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log(a + bx)}{a^4} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]

[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a + b*x])/a^4

Maple [A] time = 0.006, size = 53, normalized size = 1.

$$-\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2}{xa^3} - \frac{b^3 \ln(x)}{a^4} + \frac{b^3 \ln(bx + a)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a*x^2),x)

[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4

Maxima [A] time = 0.971695, size = 69, normalized size = 1.23

$$\frac{b^3 \log(bx + a)}{a^4} - \frac{b^3 \log(x)}{a^4} - \frac{6b^2x^2 - 3abx + 2a^2}{6a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")

[Out] $b^3 \log(bx + a)/a^4 - b^3 \log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(a^3*x^3)$

Fricas [A] time = 0.725751, size = 126, normalized size = 2.25

$$\frac{6b^3x^3 \log(bx + a) - 6b^3x^3 \log(x) - 6ab^2x^2 + 3a^2bx - 2a^3}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")

[Out] $1/6*(6*b^3*x^3*\log(bx + a) - 6*b^3*x^3*\log(x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)/(a^4*x^3)$

Sympy [A] time = 1.20975, size = 44, normalized size = 0.79

$$-\frac{2a^2 - 3abx + 6b^2x^2}{6a^3x^3} + \frac{b^3(-\log(x) + \log(\frac{a}{b} + x))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a*x**2),x)

[Out] $-(2*a**2 - 3*a*b*x + 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-\log(x) + \log(a/b + x))/a**4$

Giac [A] time = 1.20949, size = 76, normalized size = 1.36

$$\frac{b^3 \log(|bx + a|)}{a^4} - \frac{b^3 \log(|x|)}{a^4} - \frac{6ab^2x^2 - 3a^2bx + 2a^3}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")


```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*  
b*x + 2*a^3)/(a^4*x^3)
```

$$3.222 \quad \int \frac{x^8}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)} + \frac{3a^2x}{b^4} - \frac{4a^3 \log(a+bx)}{b^5} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Log[a + b*x])/b^5

Rubi [A] time = 0.040053, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$-\frac{a^4}{b^5(a+bx)} + \frac{3a^2x}{b^4} - \frac{4a^3 \log(a+bx)}{b^5} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x^2 + b*x^3)^2, x]

[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Log[a + b*x])/b^5

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^8}{(ax^2 + bx^3)^2} dx &= \int \frac{x^4}{(a + bx)^2} dx \\ &= \int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a + bx)^2} - \frac{4a^3}{b^4(a + bx)} \right) dx \\ &= \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2} - \frac{a^4}{b^5(a + bx)} - \frac{4a^3 \log(a + bx)}{b^5} \end{aligned}$$

Mathematica [A] time = 0.0200056, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx} + 9a^2bx - 12a^3 \log(a + bx) - 3ab^2x^2 + b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]

[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x])/ (3*b^5)

Maple [A] time = 0.006, size = 57, normalized size = 1.

$$3 \frac{a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2} - \frac{a^4}{b^5(bx + a)} - 4 \frac{a^3 \ln(bx + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a*x^2)^2,x)

[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5

Maxima [A] time = 0.976477, size = 80, normalized size = 1.38

$$-\frac{a^4}{b^6x + ab^5} - \frac{4a^3 \log(bx + a)}{b^5} + \frac{b^2x^3 - 3abx^2 + 9a^2x}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $-a^4/(b^6x + ab^5) - 4a^3\log(bx + a)/b^5 + 1/3(b^2x^3 - 3abx^2 + 9a^2x)/b^4$

Fricas [A] time = 0.67851, size = 155, normalized size = 2.67

$$\frac{b^4x^4 - 2ab^3x^3 + 6a^2b^2x^2 + 9a^3bx - 3a^4 - 12(a^3bx + a^4)\log(bx + a)}{3(b^6x + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $1/3(b^4x^4 - 2ab^3x^3 + 6a^2b^2x^2 + 9a^3bx - 3a^4 - 12(a^3bx + a^4)\log(bx + a))/(b^6x + ab^5)$

Sympy [A] time = 1.19462, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5 + b^6x} - \frac{4a^3\log(a + bx)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**3+a*x**2)**2,x)

[Out] $-a**4/(a*b**5 + b**6*x) - 4*a**3*\log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2/b**3 + x**3/(3*b**2)$

Giac [A] time = 1.15278, size = 84, normalized size = 1.45

$$-\frac{4a^3\log(|bx + a|)}{b^5} - \frac{a^4}{(bx + a)b^5} + \frac{b^4x^3 - 3ab^3x^2 + 9a^2b^2x}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3*x^2 + 9*a^2*b^2*x)/b^6
```

$$3.223 \quad \int \frac{x^7}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=46

$$\frac{a^3}{b^4(a+bx)} + \frac{3a^2 \log(a+bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

[Out] $(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4$

Rubi [A] time = 0.0301855, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a^3}{b^4(a+bx)} + \frac{3a^2 \log(a+bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x^2 + b*x^3)^2,x]

[Out] $(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^7}{(ax^2 + bx^3)^2} dx &= \int \frac{x^3}{(a + bx)^2} dx \\ &= \int \left(-\frac{2a}{b^3} + \frac{x}{b^2} - \frac{a^3}{b^3(a + bx)^2} + \frac{3a^2}{b^3(a + bx)} \right) dx \\ &= -\frac{2ax}{b^3} + \frac{x^2}{2b^2} + \frac{a^3}{b^4(a + bx)} + \frac{3a^2 \log(a + bx)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.0136239, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx} + 6a^2 \log(a + bx) - 4abx + b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]

[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)

Maple [A] time = 0.006, size = 45, normalized size = 1.

$$-2 \frac{ax}{b^3} + \frac{x^2}{2b^2} + \frac{a^3}{b^4(bx + a)} + 3 \frac{a^2 \ln(bx + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x^3+a*x^2)^2,x)

[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4

Maxima [A] time = 0.993047, size = 63, normalized size = 1.37

$$\frac{a^3}{b^5x + ab^4} + \frac{3a^2 \log(bx + a)}{b^4} + \frac{bx^2 - 4ax}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3

Fricas [A] time = 0.75775, size = 132, normalized size = 2.87

$$\frac{b^3x^3 - 3ab^2x^2 - 4a^2bx + 2a^3 + 6(a^2bx + a^3)\log(bx + a)}{2(b^5x + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x + a))/(b^5*x + a*b^4)

Sympy [A] time = 0.969929, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2 \log(a + bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**3+a*x**2)**2,x)

[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b**2)

Giac [A] time = 1.23639, size = 65, normalized size = 1.41

$$\frac{3a^2 \log(|bx + a|)}{b^4} + \frac{a^3}{(bx + a)b^4} + \frac{b^2x^2 - 4abx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")


```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

$$3.224 \quad \int \frac{x^6}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a \log(a+bx)}{b^3} + \frac{x}{b^2}$$

[Out] $x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3$

Rubi [A] time = 0.023916, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a \log(a+bx)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3)^2,x]

[Out] $x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n_.], x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(ax^2 + bx^3)^2} dx &= \int \frac{x^2}{(a + bx)^2} dx \\ &= \int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a + bx)^2} - \frac{2a}{b^2(a + bx)} \right) dx \\ &= \frac{x}{b^2} - \frac{a^2}{b^3(a + bx)} - \frac{2a \log(a + bx)}{b^3} \end{aligned}$$

Mathematica [A] time = 0.0122328, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx} - 2a \log(a + bx) + bx}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]

[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3

Maple [A] time = 0.006, size = 34, normalized size = 1.

$$\frac{x}{b^2} - \frac{a^2}{b^3(bx + a)} - 2 \frac{a \ln(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^3+a*x^2)^2,x)

[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3

Maxima [A] time = 0.991994, size = 49, normalized size = 1.48

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2a \log(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3

Fricas [A] time = 0.780113, size = 97, normalized size = 2.94

$$\frac{b^2x^2 + abx - a^2 - 2(abx + a^2)\log(bx + a)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)

Sympy [A] time = 0.558101, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a \log(a + bx)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**3+a*x**2)**2,x)

[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2

Giac [A] time = 1.16743, size = 46, normalized size = 1.39

$$\frac{x}{b^2} - \frac{2a \log(|bx + a|)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)

$$3.225 \quad \int \frac{x^5}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2

Rubi [A] time = 0.0180811, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 43}

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3)^2,x]

[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(ax^2 + bx^3)^2} dx &= \int \frac{x}{(a + bx)^2} dx \\
 &= \int \left(-\frac{a}{b(a + bx)^2} + \frac{1}{b(a + bx)} \right) dx \\
 &= \frac{a}{b^2(a + bx)} + \frac{\log(a + bx)}{b^2}
 \end{aligned}$$

Mathematica [A] time = 0.0063073, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx} + \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]

[Out] (a/(a + b*x) + Log[a + b*x])/b^2

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{a}{b^2(bx + a)} + \frac{\ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a*x^2)^2,x)

[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2

Maxima [A] time = 0.983391, size = 35, normalized size = 1.52

$$\frac{a}{b^3x + ab^2} + \frac{\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2

Fricas [A] time = 0.765177, size = 62, normalized size = 2.7

$$\frac{(bx + a) \log(bx + a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)

Sympy [A] time = 0.527788, size = 20, normalized size = 0.87

$$\frac{a}{ab^2 + b^3x} + \frac{\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**3+a*x**2)**2,x)

[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2

Giac [A] time = 1.19752, size = 32, normalized size = 1.39

$$\frac{\log(|bx + a|)}{b^2} + \frac{a}{(bx + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)

$$3.226 \quad \int \frac{x^4}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

[Out] -(1/(b*(a + b*x)))

Rubi [A] time = 0.0073729, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3)^2,x]

[Out] -(1/(b*(a + b*x)))

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(ax^2 + bx^3)^2} dx &= \int \frac{1}{(a + bx)^2} dx \\ &= -\frac{1}{b(a + bx)} \end{aligned}$$

Mathematica [A] time = 0.0026073, size = 12, normalized size = 1.

$$-\frac{1}{b(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]

[Out] -(1/(b*(a + b*x)))

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$-\frac{1}{b(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x^2)^2,x)

[Out] -1/b/(b*x+a)

Maxima [A] time = 0.988879, size = 18, normalized size = 1.5

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] -1/(b^2*x + a*b)

Fricas [A] time = 0.789475, size = 24, normalized size = 2.

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] -1/(b^2*x + a*b)

Sympy [A] time = 0.920552, size = 10, normalized size = 0.83

$$-\frac{1}{ab + b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**3+a*x**2)**2,x)

[Out] -1/(a*b + b**2*x)

Giac [A] time = 1.1492, size = 16, normalized size = 1.33

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] -1/((b*x + a)*b)

$$3.227 \quad \int \frac{x^3}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=29

$$-\frac{\log(a+bx)}{a^2} + \frac{\log(x)}{a^2} + \frac{1}{a(a+bx)}$$

[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2

Rubi [A] time = 0.0188643, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 44}

$$-\frac{\log(a+bx)}{a^2} + \frac{\log(x)}{a^2} + \frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3)^2,x]

[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(ax^2 + bx^3)^2} dx &= \int \frac{1}{x(a + bx)^2} dx \\
&= \int \left(\frac{1}{a^2x} - \frac{b}{a(a + bx)^2} - \frac{b}{a^2(a + bx)} \right) dx \\
&= \frac{1}{a(a + bx)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0099306, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx} - \log(a + bx) + \log(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]

[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2

Maple [A] time = 0.006, size = 30, normalized size = 1.

$$\frac{1}{a(bx + a)} + \frac{\ln(x)}{a^2} - \frac{\ln(bx + a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x^2)^2,x)

[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2

Maxima [A] time = 1.01662, size = 38, normalized size = 1.31

$$\frac{1}{abx + a^2} - \frac{\log(bx + a)}{a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2

Fricas [A] time = 0.730418, size = 89, normalized size = 3.07

$$\frac{(bx + a) \log(bx + a) - (bx + a) \log(x) - a}{a^2bx + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)

Sympy [A] time = 1.03874, size = 22, normalized size = 0.76

$$\frac{1}{a^2 + abx} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**3+a*x**2)**2,x)

[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2

Giac [A] time = 1.14525, size = 42, normalized size = 1.45

$$-\frac{\log(|bx + a|)}{a^2} + \frac{\log(|x|)}{a^2} + \frac{1}{(bx + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)

$$3.228 \quad \int \frac{x^2}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=42

$$-\frac{b}{a^2(a+bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3} - \frac{1}{a^2x}$$

[Out] $-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^3$

Rubi [A] time = 0.0239151, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 44}

$$-\frac{b}{a^2(a+bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3)^2, x]

[Out] $-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^3$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
  & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
  + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(ax^2 + bx^3)^2} dx &= \int \frac{1}{x^2(a + bx)^2} dx \\
&= \int \left(\frac{1}{a^2 x^2} - \frac{2b}{a^3 x} + \frac{b^2}{a^2(a + bx)^2} + \frac{2b^2}{a^3(a + bx)} \right) dx \\
&= -\frac{1}{a^2 x} - \frac{b}{a^2(a + bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a + bx)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.0374035, size = 35, normalized size = 0.83

$$-\frac{a \left(\frac{b}{a+bx} + \frac{1}{x} \right) - 2b \log(a + bx) + 2b \log(x)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]

[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)

Maple [A] time = 0.009, size = 43, normalized size = 1.

$$-\frac{1}{a^2 x} - \frac{b}{a^2 (bx + a)} - 2 \frac{b \ln(x)}{a^3} + 2 \frac{b \ln(bx + a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x^2)^2,x)

[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3

Maxima [A] time = 1.00111, size = 61, normalized size = 1.45

$$-\frac{2bx + a}{a^2 bx^2 + a^3 x} + \frac{2b \log(bx + a)}{a^3} - \frac{2b \log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $-(2bx + a)/(a^2bx^2 + a^3x) + 2b \log(bx + a)/a^3 - 2b \log(x)/a^3$

Fricas [A] time = 0.725405, size = 138, normalized size = 3.29

$$-\frac{2abx + a^2 - 2(b^2x^2 + abx) \log(bx + a) + 2(b^2x^2 + abx) \log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $-(2abx + a^2 - 2(b^2x^2 + abx) \log(bx + a) + 2(b^2x^2 + abx) \log(x))/(a^3bx^2 + a^4x)$

Sympy [A] time = 1.24111, size = 36, normalized size = 0.86

$$-\frac{a + 2bx}{a^3x + a^2bx^2} + \frac{2b(-\log(x) + \log(\frac{a}{b} + x))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**3+a*x**2)**2,x)

[Out] $-(a + 2bx)/(a^3x + a^2bx^2) + 2b(-\log(x) + \log(a/b + x))/a^3$

Giac [A] time = 1.16753, size = 61, normalized size = 1.45

$$\frac{2b \log(|bx + a|)}{a^3} - \frac{2b \log(|x|)}{a^3} - \frac{2bx + a}{(bx^2 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $2b \log(\text{abs}(bx + a))/a^3 - 2b \log(\text{abs}(x))/a^3 - (2bx + a)/((bx^2 + ax) \cdot a^2)$

$$3.229 \quad \int \frac{x}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=58

$$\frac{b^2}{a^3(a+bx)} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a+bx)}{a^4} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

[Out] $-1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 - (3*b^2*Log[a + b*x])/a^4$

Rubi [A] time = 0.0315931, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 44}

$$\frac{b^2}{a^3(a+bx)} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a+bx)}{a^4} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3)^2,x]

[Out] $-1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 - (3*b^2*Log[a + b*x])/a^4$

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2 + bx^3)^2} dx &= \int \frac{1}{x^3(a + bx)^2} dx \\
&= \int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a + bx)^2} - \frac{3b^3}{a^4(a + bx)} \right) dx \\
&= -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a + bx)} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a + bx)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.0500844, size = 53, normalized size = 0.91

$$\frac{a \left(\frac{2b^2}{a+bx} - \frac{a}{x^2} + \frac{4b}{x} \right) - 6b^2 \log(a + bx) + 6b^2 \log(x)}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3)^2,x]

[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a + b*x])/(2*a^4)

Maple [A] time = 0.01, size = 57, normalized size = 1.

$$-\frac{1}{2a^2x^2} + 2\frac{b}{xa^3} + \frac{b^2}{a^3(bx+a)} + 3\frac{b^2 \ln(x)}{a^4} - 3\frac{b^2 \ln(bx+a)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x^2)^2,x)

[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4

Maxima [A] time = 1.01718, size = 86, normalized size = 1.48

$$\frac{6b^2x^2 + 3abx - a^2}{2(a^3bx^3 + a^4x^2)} - \frac{3b^2 \log(bx + a)}{a^4} + \frac{3b^2 \log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*\log(b*x + a)/a^4 + 3*b^2*\log(x)/a^4$

Fricas [A] time = 0.791073, size = 177, normalized size = 3.05

$$\frac{6ab^2x^2 + 3a^2bx - a^3 - 6(b^3x^3 + ab^2x^2)\log(bx + a) + 6(b^3x^3 + ab^2x^2)\log(x)}{2(a^4bx^3 + a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*\log(b*x + a) + 6*(b^3*x^3 + a*b^2*x^2)*\log(x))/(a^4*b*x^3 + a^5*x^2)$

Sympy [A] time = 0.759451, size = 54, normalized size = 0.93

$$\frac{-a^2 + 3abx + 6b^2x^2}{2a^4x^2 + 2a^3bx^3} + \frac{3b^2(\log(x) - \log(\frac{a}{b} + x))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a*x**2)**2,x)

[Out] $(-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(\log(x) - \log(a/b + x))/a**4$

Giac [A] time = 1.18427, size = 86, normalized size = 1.48

$$-\frac{3b^2\log(|bx + a|)}{a^4} + \frac{3b^2\log(|x|)}{a^4} + \frac{6ab^2x^2 + 3a^2bx - a^3}{2(bx + a)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3  
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

$$3.230 \quad \int \frac{1}{(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=69

$$-\frac{b^3}{a^4(a+bx)} - \frac{3b^2}{a^4x} - \frac{4b^3 \log(x)}{a^5} + \frac{4b^3 \log(a+bx)}{a^5} + \frac{b}{a^3x^2} - \frac{1}{3a^2x^3}$$

[Out] $-1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b^3*\text{Log}[x])/a^5 + (4*b^3*\text{Log}[a + b*x])/a^5$

Rubi [A] time = 0.0389185, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 44}

$$-\frac{b^3}{a^4(a+bx)} - \frac{3b^2}{a^4x} - \frac{4b^3 \log(x)}{a^5} + \frac{4b^3 \log(a+bx)}{a^5} + \frac{b}{a^3x^2} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(-2), x]

[Out] $-1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b^3*\text{Log}[x])/a^5 + (4*b^3*\text{Log}[a + b*x])/a^5$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^2 + bx^3)^2} dx &= \int \frac{1}{x^4(a + bx)^2} dx \\ &= \int \left(\frac{1}{a^2x^4} - \frac{2b}{a^3x^3} + \frac{3b^2}{a^4x^2} - \frac{4b^3}{a^5x} + \frac{b^4}{a^4(a + bx)^2} + \frac{4b^4}{a^5(a + bx)} \right) dx \\ &= -\frac{1}{3a^2x^3} + \frac{b}{a^3x^2} - \frac{3b^2}{a^4x} - \frac{b^3}{a^4(a + bx)} - \frac{4b^3 \log(x)}{a^5} + \frac{4b^3 \log(a + bx)}{a^5} \end{aligned}$$

Mathematica [A] time = 0.0528533, size = 66, normalized size = 0.96

$$-\frac{a(-2a^2bx+a^3+6ab^2x^2+12b^3x^3)}{x^3(a+bx)} - \frac{12b^3 \log(a + bx) + 12b^3 \log(x)}{3a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(-2), x]

[Out] -((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12*b^3*Log[x] - 12*b^3*Log[a + b*x])/(3*a^5)

Maple [A] time = 0.01, size = 68, normalized size = 1.

$$-\frac{1}{3x^3a^2} + \frac{b}{x^2a^3} - 3\frac{b^2}{a^4x} - \frac{b^3}{a^4(bx+a)} - 4\frac{b^3 \ln(x)}{a^5} + 4\frac{b^3 \ln(bx+a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a*x^2)^2, x)

[Out] -1/3/x^3/a^2+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln(b*x+a)/a^5

Maxima [A] time = 0.986677, size = 99, normalized size = 1.43

$$-\frac{12b^3x^3 + 6ab^2x^2 - 2a^2bx + a^3}{3(a^4bx^4 + a^5x^3)} + \frac{4b^3 \log(bx + a)}{a^5} - \frac{4b^3 \log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $-1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4*b^3*\log(b*x + a)/a^5 - 4*b^3*\log(x)/a^5$

Fricas [A] time = 0.8104, size = 204, normalized size = 2.96

$$\frac{12 ab^3 x^3 + 6 a^2 b^2 x^2 - 2 a^3 b x + a^4 - 12 (b^4 x^4 + ab^3 x^3) \log(bx + a) + 12 (b^4 x^4 + ab^3 x^3) \log(x)}{3 (a^5 b x^4 + a^6 x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $-1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*x^3)*\log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*\log(x))/(a^5*b*x^4 + a^6*x^3)$

Sympy [A] time = 1.76697, size = 66, normalized size = 0.96

$$-\frac{a^3 - 2a^2bx + 6ab^2x^2 + 12b^3x^3}{3a^5x^3 + 3a^4bx^4} + \frac{4b^3(-\log(x) + \log(\frac{a}{b} + x))}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**3+a*x**2)**2,x)

[Out] $-(a**3 - 2*a**2*b*x + 6*a*b**2*x**2 + 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b*x**4) + 4*b**3*(-\log(x) + \log(a/b + x))/a**5$

Giac [A] time = 1.16329, size = 99, normalized size = 1.43

$$\frac{4 b^3 \log(|bx + a|)}{a^5} - \frac{4 b^3 \log(|x|)}{a^5} - \frac{12 ab^3 x^3 + 6 a^2 b^2 x^2 - 2 a^3 b x + a^4}{3 (bx + a) a^5 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6  
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```


$$3.231 \quad \int \frac{1}{x(ax^2+bx^3)^2} dx$$

Optimal. Leaf size=84

$$-\frac{3b^2}{2a^4x^2} + \frac{b^4}{a^5(a+bx)} + \frac{4b^3}{a^5x} + \frac{5b^4 \log(x)}{a^6} - \frac{5b^4 \log(a+bx)}{a^6} + \frac{2b}{3a^3x^3} - \frac{1}{4a^2x^4}$$

[Out] $-1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6$

Rubi [A] time = 0.052285, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 44}

$$-\frac{3b^2}{2a^4x^2} + \frac{b^4}{a^5(a+bx)} + \frac{4b^3}{a^5x} + \frac{5b^4 \log(x)}{a^6} - \frac{5b^4 \log(a+bx)}{a^6} + \frac{2b}{3a^3x^3} - \frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3)^2), x]

[Out] $-1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6$

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x(ax^2 + bx^3)^2} dx &= \int \frac{1}{x^5(a + bx)^2} dx \\ &= \int \left(\frac{1}{a^2x^5} - \frac{2b}{a^3x^4} + \frac{3b^2}{a^4x^3} - \frac{4b^3}{a^5x^2} + \frac{5b^4}{a^6x} - \frac{b^5}{a^5(a + bx)^2} - \frac{5b^5}{a^6(a + bx)} \right) dx \\ &= -\frac{1}{4a^2x^4} + \frac{2b}{3a^3x^3} - \frac{3b^2}{2a^4x^2} + \frac{4b^3}{a^5x} + \frac{b^4}{a^5(a + bx)} + \frac{5b^4 \log(x)}{a^6} - \frac{5b^4 \log(a + bx)}{a^6} \end{aligned}$$

Mathematica [A] time = 0.0426332, size = 79, normalized size = 0.94

$$\frac{\frac{a(-10a^2b^2x^2 + 5a^3bx - 3a^4 + 30ab^3x^3 + 60b^4x^4)}{x^4(a+bx)} - 60b^4 \log(a + bx) + 60b^4 \log(x)}{12a^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3)^2), x]

[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)

Maple [A] time = 0.01, size = 79, normalized size = 0.9

$$-\frac{1}{4a^2x^4} + \frac{2b}{3a^3x^3} - \frac{3b^2}{2a^4x^2} + 4\frac{b^3}{a^5x} + \frac{b^4}{a^5(bx + a)} + 5\frac{b^4 \ln(x)}{a^6} - 5\frac{b^4 \ln(bx + a)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x^2)^2, x)

[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6

Maxima [A] time = 1.10895, size = 116, normalized size = 1.38

$$\frac{60b^4x^4 + 30ab^3x^3 - 10a^2b^2x^2 + 5a^3bx - 3a^4}{12(a^5bx^5 + a^6x^4)} - \frac{5b^4 \log(bx + a)}{a^6} + \frac{5b^4 \log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*b*x^5 + a^6*x^4) - 5*b^4*\log(b*x + a)/a^6 + 5*b^4*\log(x)/a^6$

Fricas [A] time = 0.677263, size = 231, normalized size = 2.75

$$\frac{60 ab^4x^4 + 30 a^2b^3x^3 - 10 a^3b^2x^2 + 5 a^4bx - 3 a^5 - 60 (b^5x^5 + ab^4x^4) \log (bx + a) + 60 (b^5x^5 + ab^4x^4) \log (x)}{12 (a^6bx^5 + a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 - 60*(b^5*x^5 + a*b^4*x^4)*\log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*\log(x))/(a^6*b*x^5 + a^7*x^4)$

Sympy [A] time = 1.7031, size = 80, normalized size = 0.95

$$\frac{-3a^4 + 5a^3bx - 10a^2b^2x^2 + 30ab^3x^3 + 60b^4x^4}{12a^6x^4 + 12a^5bx^5} + \frac{5b^4(\log(x) - \log(\frac{a}{b} + x))}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a*x**2)**2,x)

[Out] $(-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(\log(x) - \log(a/b + x))/a**6$

Giac [A] time = 1.14102, size = 116, normalized size = 1.38

$$-\frac{5b^4 \log(|bx + a|)}{a^6} + \frac{5b^4 \log(|x|)}{a^6} + \frac{60 ab^4x^4 + 30 a^2b^3x^3 - 10 a^3b^2x^2 + 5 a^4bx - 3 a^5}{12 (bx + a)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +  
30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

3.232 $\int x^2 \sqrt{ax^2 + bx^3} dx$

Optimal. Leaf size=105

$$-\frac{32a^3 (ax^2 + bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2 (ax^2 + bx^3)^{3/2}}{105b^3x^2} - \frac{4a (ax^2 + bx^3)^{3/2}}{21b^2x} + \frac{2 (ax^2 + bx^3)^{3/2}}{9b}$$

[Out] $(2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(3/2))/(21*b^2*x)$

Rubi [A] time = 0.119729, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$-\frac{32a^3 (ax^2 + bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2 (ax^2 + bx^3)^{3/2}}{105b^3x^2} - \frac{4a (ax^2 + bx^3)^{3/2}}{21b^2x} + \frac{2 (ax^2 + bx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]

[Out] $(2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(3/2))/(21*b^2*x)$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -

j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
  *(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
  j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{ax^2 + bx^3} dx &= \frac{2(ax^2 + bx^3)^{3/2}}{9b} - \frac{(2a) \int x \sqrt{ax^2 + bx^3} dx}{3b} \\ &= \frac{2(ax^2 + bx^3)^{3/2}}{9b} - \frac{4a(ax^2 + bx^3)^{3/2}}{21b^2x} + \frac{(8a^2) \int \sqrt{ax^2 + bx^3} dx}{21b^2} \\ &= \frac{2(ax^2 + bx^3)^{3/2}}{9b} + \frac{16a^2(ax^2 + bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2 + bx^3)^{3/2}}{21b^2x} - \frac{(16a^3) \int \frac{\sqrt{ax^2 + bx^3}}{x} dx}{105b^3} \\ &= \frac{2(ax^2 + bx^3)^{3/2}}{9b} - \frac{32a^3(ax^2 + bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2(ax^2 + bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2 + bx^3)^{3/2}}{21b^2x} \end{aligned}$$

Mathematica [A] time = 0.0281838, size = 53, normalized size = 0.5

$$\frac{2(x^2(a + bx))^{3/2}(24a^2bx - 16a^3 - 30ab^2x^2 + 35b^3x^3)}{315b^4x^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3], x]

[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315*b^4*x^3)

Maple [A] time = 0.004, size = 57, normalized size = 0.5

$$\frac{(2bx + 2a)(-35x^3b^3 + 30ab^2x^2 - 24a^2xb + 16a^3)}{315b^4x} \sqrt{bx^3 + ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a*x^2)^(1/2),x)`

[Out] $-2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(1/2)/b^4/x$

Maxima [A] time = 1.00444, size = 72, normalized size = 0.69

$$\frac{2(35b^4x^4 + 5ab^3x^3 - 6a^2b^2x^2 + 8a^3bx - 16a^4)\sqrt{bx+a}}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] $2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*\text{sqrt}(b*x + a)/b^4$

Fricas [A] time = 0.819516, size = 134, normalized size = 1.28

$$\frac{2(35b^4x^4 + 5ab^3x^3 - 6a^2b^2x^2 + 8a^3bx - 16a^4)\sqrt{bx^3+ax^2}}{315b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*\text{sqrt}(b*x^3 + a*x^2)/(b^4*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x^2(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)

Giac [A] time = 1.14418, size = 84, normalized size = 0.8

$$\frac{32 a^2 \operatorname{sgn}(x)}{315 b^4} + \frac{2 \left(35 (bx + a)^{\frac{9}{2}} - 135 (bx + a)^{\frac{7}{2}} a + 189 (bx + a)^{\frac{5}{2}} a^2 - 105 (bx + a)^{\frac{3}{2}} a^3 \right) \operatorname{sgn}(x)}{315 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*sgn(x)/b^4

3.233 $\int x\sqrt{ax^2 + bx^3} dx$

Optimal. Leaf size=80

$$\frac{16a^2(ax^2 + bx^3)^{3/2}}{105b^3x^3} - \frac{8a(ax^2 + bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2 + bx^3)^{3/2}}{7bx}$$

[Out] $(16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)$

Rubi [A] time = 0.073556, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2016, 2002, 2014}

$$\frac{16a^2(ax^2 + bx^3)^{3/2}}{105b^3x^3} - \frac{8a(ax^2 + bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2 + bx^3)^{3/2}}{7bx}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a*x^2 + b*x^3],x]

[Out] $(16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)$

Rule 2016

```
Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int x\sqrt{ax^2 + bx^3} dx &= \frac{2(ax^2 + bx^3)^{3/2}}{7bx} - \frac{(4a) \int \sqrt{ax^2 + bx^3} dx}{7b} \\ &= -\frac{8a(ax^2 + bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2 + bx^3)^{3/2}}{7bx} + \frac{(8a^2) \int \frac{\sqrt{ax^2 + bx^3}}{x} dx}{35b^2} \\ &= \frac{16a^2(ax^2 + bx^3)^{3/2}}{105b^3x^3} - \frac{8a(ax^2 + bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2 + bx^3)^{3/2}}{7bx} \end{aligned}$$

Mathematica [A] time = 0.0196859, size = 42, normalized size = 0.52

$$\frac{2(x^2(a + bx))^{3/2} (8a^2 - 12abx + 15b^2x^2)}{105b^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a*x^2 + b*x^3], x]

[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)

Maple [A] time = 0.003, size = 46, normalized size = 0.6

$$\frac{(2bx + 2a)(15b^2x^2 - 12abx + 8a^2)}{105b^3x} \sqrt{bx^3 + ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a*x^2)^(1/2), x)

[Out] 2/105*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2)*(b*x^3+a*x^2)^(1/2)/b^3/x

Maxima [A] time = 1.01651, size = 57, normalized size = 0.71

$$\frac{2(15b^3x^3 + 3ab^2x^2 - 4a^2bx + 8a^3)\sqrt{bx + a}}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3

Fricas [A] time = 0.81328, size = 111, normalized size = 1.39

$$\frac{2(15b^3x^3 + 3ab^2x^2 - 4a^2bx + 8a^3)\sqrt{bx^3 + ax^2}}{105b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b^3*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x*sqrt(x**2*(a + b*x)), x)

Giac [A] time = 1.15079, size = 68, normalized size = 0.85

$$-\frac{16a^{\frac{7}{2}}\operatorname{sgn}(x)}{105b^3} + \frac{2\left(15(bx+a)^{\frac{7}{2}} - 42(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2\right)\operatorname{sgn}(x)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)*sgn(x)/b^3

3.234 $\int \sqrt{ax^2 + bx^3} dx$

Optimal. Leaf size=52

$$\frac{2(ax^2 + bx^3)^{3/2}}{5bx^2} - \frac{4a(ax^2 + bx^3)^{3/2}}{15b^2x^3}$$

[Out] $(-4*a*(a*x^2 + b*x^3)^{(3/2)})/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^{(3/2)})/(5*b*x^2)$

Rubi [A] time = 0.0430886, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2002, 2014}

$$\frac{2(ax^2 + bx^3)^{3/2}}{5bx^2} - \frac{4a(ax^2 + bx^3)^{3/2}}{15b^2x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3], x]

[Out] $(-4*a*(a*x^2 + b*x^3)^{(3/2)})/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^{(3/2)})/(5*b*x^2)$

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\int \sqrt{ax^2 + bx^3} dx = \frac{2(ax^2 + bx^3)^{3/2}}{5bx^2} - \frac{(2a) \int \frac{\sqrt{ax^2 + bx^3}}{x} dx}{5b}$$

$$= -\frac{4a(ax^2 + bx^3)^{3/2}}{15b^2x^3} + \frac{2(ax^2 + bx^3)^{3/2}}{5bx^2}$$

Mathematica [A] time = 0.0139493, size = 31, normalized size = 0.6

$$\frac{2(x^2(a + bx))^{3/2}(3bx - 2a)}{15b^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3], x]

[Out] (2*(x^2*(a + b*x))^(3/2)*(-2*a + 3*b*x))/(15*b^2*x^3)

Maple [A] time = 0.003, size = 35, normalized size = 0.7

$$-\frac{(2bx + 2a)(-3bx + 2a)\sqrt{bx^3 + ax^2}}{15b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(1/2), x)

[Out] -2/15*(b*x+a)*(-3*b*x+2*a)*(b*x^3+a*x^2)^(1/2)/b^2/x

Maxima [A] time = 0.976278, size = 41, normalized size = 0.79

$$\frac{2(3b^2x^2 + abx - 2a^2)\sqrt{bx + a}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2), x, algorithm="maxima")

[Out] $2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*\sqrt{b*x + a}/b^2$

Fricas [A] time = 0.873243, size = 84, normalized size = 1.62

$$\frac{2(3b^2x^2 + abx - 2a^2)\sqrt{bx^3 + ax^2}}{15b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*\sqrt{b*x^3 + a*x^2}/(b^2*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^2 + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(sqrt(a*x**2 + b*x**3), x)`

Giac [A] time = 1.1859, size = 51, normalized size = 0.98

$$\frac{4a^{\frac{5}{2}}\operatorname{sgn}(x)}{15b^2} + \frac{2\left(3(bx+a)^{\frac{5}{2}} - 5(bx+a)^{\frac{3}{2}}a\right)\operatorname{sgn}(x)}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

[Out] $4/15*a^{(5/2)}*\operatorname{sgn}(x)/b^2 + 2/15*(3*(b*x + a)^{(5/2)} - 5*(b*x + a)^{(3/2)}*a)*\operatorname{sgn}(x)/b^2$

$$3.235 \quad \int \frac{\sqrt{ax^2+bx^3}}{x} dx$$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)

Rubi [A] time = 0.0358451, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$\frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]

[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
  *(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
  j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x} dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

Mathematica [A] time = 0.0090109, size = 23, normalized size = 0.92

$$\frac{2(x^2(a+bx))^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]

[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)

Maple [A] time = 0.001, size = 27, normalized size = 1.1

$$\frac{2bx + 2a}{3bx} \sqrt{bx^3 + ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(1/2)/x,x)

[Out] 2/3*(b*x+a)*(b*x^3+a*x^2)^(1/2)/b/x

Maxima [A] time = 1.00775, size = 16, normalized size = 0.64

$$\frac{2(bx + a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")

[Out] 2/3*(b*x + a)^(3/2)/b

Fricas [A] time = 0.823993, size = 55, normalized size = 2.2

$$\frac{2\sqrt{bx^3 + ax^2}(bx + a)}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")

[Out] $\frac{2}{3}\sqrt{b^3x^3 + a^2x^2}(bx + a)/(bx)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)

[Out] Integral(sqrt(x**2*(a + b*x))/x, x)

Giac [A] time = 1.17417, size = 34, normalized size = 1.36

$$\frac{2(bx + a)^{\frac{3}{2}}\operatorname{sgn}(x)}{3b} - \frac{2a^{\frac{3}{2}}\operatorname{sgn}(x)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")

[Out] $\frac{2}{3}(bx + a)^{3/2}\operatorname{sgn}(x)/b - \frac{2}{3}a^{3/2}\operatorname{sgn}(x)/b$

$$3.236 \quad \int \frac{\sqrt{ax^2+bx^3}}{x^2} dx$$

Optimal. Leaf size=51

$$\frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)$$

[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rubi [A] time = 0.0490906, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2021, 2008, 206}

$$\frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]

[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rule 2021

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^3}}{x^2} dx &= \frac{2\sqrt{ax^2 + bx^3}}{x} + a \int \frac{1}{\sqrt{ax^2 + bx^3}} dx \\ &= \frac{2\sqrt{ax^2 + bx^3}}{x} - (2a) \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}} \right) \\ &= \frac{2\sqrt{ax^2 + bx^3}}{x} - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a}x}{\sqrt{ax^2 + bx^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.0278593, size = 53, normalized size = 1.04

$$\frac{2x \left(-\sqrt{a}\sqrt{a+bx} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right) + a + bx \right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

Maple [A] time = 0.006, size = 52, normalized size = 1.

$$-2 \frac{\sqrt{bx^3 + ax^2}}{x\sqrt{bx + a}} \left(\sqrt{a} \operatorname{Artanh} \left(\frac{\sqrt{bx + a}}{\sqrt{a}} \right) - \sqrt{bx + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] -2*(b*x^3+a*x^2)^(1/2)*(a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-
(b*x+a)^(1/2))/x/(b*x+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)

Fricas [A] time = 0.850497, size = 247, normalized size = 4.84

$$\left[\frac{\sqrt{ax} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2\sqrt{bx^3+ax^2}}{x}, \frac{2\left(\sqrt{-a}x \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}\right)}{x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")

[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2))/x]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)

[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)

Giac [A] time = 1.21336, size = 88, normalized size = 1.73

$$2 \left(\frac{a \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx+a} \right) \operatorname{sgn}(x) - \frac{2 \left(a \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a} \right) \operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")

[Out] 2*(a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a))*sgn(x) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)

$$3.237 \quad \int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$$

Optimal. Leaf size=52

$$-\frac{\sqrt{ax^2+bx^3}}{x^2} - \frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3]/x^2) - (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/\text{Sqrt}[a]$

Rubi [A] time = 0.0519346, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{ax^2+bx^3}}{x^2} - \frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^2 + b*x^3]/x^3, x]$

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3]/x^2) - (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/\text{Sqrt}[a]$

Rule 2020

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:\> \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m+j*p+1, 0]$

Rule 2008

$\text{Int}[1/\text{Sqrt}[(a_*)(x_)^2 + (b_*)(x_)^{(n_*)}], x_Symbol] :\> \text{Dist}[2/(2-n), \text{Subst}[\text{Int}[1/(1-a*x^2)], x], x, x/\text{Sqrt}[a*x^2 + b*x^n], x] /;$ $\text{FreeQ}\{a, b, n\}, x\} \&\& \text{NeQ}[n, 2]$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^3}}{x^3} dx &= -\frac{\sqrt{ax^2 + bx^3}}{x^2} + \frac{1}{2}b \int \frac{1}{\sqrt{ax^2 + bx^3}} dx \\ &= -\frac{\sqrt{ax^2 + bx^3}}{x^2} - b \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}} \right) \\ &= -\frac{\sqrt{ax^2 + bx^3}}{x^2} - \frac{b \tanh^{-1} \left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}} \right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0360123, size = 48, normalized size = 0.92

$$\frac{bx\sqrt{\frac{bx}{a} + 1} \tanh^{-1} \left(\sqrt{\frac{bx}{a} + 1} \right) + a + bx}{\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3, x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[x^2*(a
+ b*x)])
```

Maple [A] time = 0.01, size = 56, normalized size = 1.1

$$-\frac{1}{x^2} \sqrt{bx^3 + ax^2} \left(\operatorname{Artanh} \left(\sqrt{bx + a} \frac{1}{\sqrt{a}} \right) bx + \sqrt{bx + a} \sqrt{a} \right) \frac{1}{\sqrt{bx + a}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3, x)
```

```
[Out] -(b*x^3+a*x^2)^(1/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+(b*x+a)^(1/2)*a^(1
/2))/x^2/(b*x+a)^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)

Fricas [A] time = 0.869586, size = 282, normalized size = 5.42

$$\left[\frac{\sqrt{abx^2} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\sqrt{bx^3+ax^2}a}{2ax^2}, \frac{\sqrt{-abx^2} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) - \sqrt{bx^3+ax^2}a}{ax^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)

[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)

Giac [A] time = 1.20562, size = 58, normalized size = 1.12

$$\frac{\left(\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+ab}}{x} \right) \operatorname{sgn}(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")

[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)*sgn(x)/b

$$3.238 \quad \int \frac{\sqrt{ax^2+bx^3}}{x^4} dx$$

Optimal. Leaf size=84

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{\sqrt{ax^2+bx^3}}{2x^3}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(2*x^3) - (b*\text{Sqrt}[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*a^{(3/2)})$

Rubi [A] time = 0.0915473, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^2 + b*x^3]/x^4, x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(2*x^3) - (b*\text{Sqrt}[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*a^{(3/2)})$

Rule 2020

$\text{Int}[(c*(x_))^{(m_)}*((a_)*(x_)^{(j_.)} + (b_)*(x_)^{(n_.)})^{(p_)}, x_Symbol]$
 $\rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p / (c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j)) / (c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \ || \ \text{GtQ}[c, 0]) \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m+j*p+1, 0]$

Rule 2025

$\text{Int}[(c*(x_))^{(m_.)}*((a_)*(x_)^{(j_.)} + (b_)*(x_)^{(n_.)})^{(p_)}, x_Symbol]$
 $\rightarrow \text{Simp}[c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)} / (a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1)) / (a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, m, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{LtQ}[0, j, n] \ \&\& \ (\text{IntegersQ}[j, n] \ || \ \text{GtQ}[c, 0]) \ \&\& \ \text{LtQ}[m+j*p+1, 0]$

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3}}{x^4} dx &= -\frac{\sqrt{ax^2 + bx^3}}{2x^3} + \frac{1}{4}b \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3}}{4ax^2} - \frac{b^2 \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{8a} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3}}{4ax^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{4a} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3}}{4ax^2} + \frac{b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0118765, size = 42, normalized size = 0.5

$$\frac{2b^2 \left(x^2(a + bx)\right)^{3/2} {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{bx}{a} + 1\right)}{3a^3 x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4, x]
```

```
[Out] (-2*b^2*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/
(3*a^3*x^3)
```

Maple [A] time = 0.01, size = 73, normalized size = 0.9

$$-\frac{1}{4x^3}\sqrt{bx^3+ax^2}\left(a^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}-\operatorname{Artanh}\left(\sqrt{bx+a}\frac{1}{\sqrt{a}}\right)ab^2x^2+a^{\frac{5}{2}}\sqrt{bx+a}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(1/2)/x^4,x)

[Out] $-1/4*(b*x^3+a*x^2)^{(1/2)}*(a^{(3/2)}*(b*x+a)^{(3/2)}-\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)}))*a*b^2*x^2+a^{(5/2)}*(b*x+a)^{(1/2)}/x^3/(b*x+a)^{(1/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)

Fricas [A] time = 0.865725, size = 338, normalized size = 4.02

$$\left[\frac{\sqrt{ab^2x^3} \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\sqrt{bx^3+ax^2}(abx+2a^2)}{8a^2x^3}, -\frac{\sqrt{-ab^2x^3} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}(abx+2a^2)}{4a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] $[1/8*(\operatorname{sqrt}(a)*b^2*x^3*\log((b*x^2+2*a*x+2*\operatorname{sqrt}(b*x^3+a*x^2))*\operatorname{sqrt}(a))/x^2)-2*\operatorname{sqrt}(b*x^3+a*x^2)*(a*b*x+2*a^2))/(a^2*x^3), -1/4*(\operatorname{sqrt}(-a)*b^2*x^3*\operatorname{arctan}(\operatorname{sqrt}(b*x^3+a*x^2)*\operatorname{sqrt}(-a)/(a*x))+\operatorname{sqrt}(b*x^3+a*x^2)*(a*b*x+2*a^2))/(a^2*x^3)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a+bx)}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)

[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)

Giac [A] time = 1.21381, size = 92, normalized size = 1.1

$$\frac{\left(\frac{b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{(bx+a)^{\frac{3}{2}} b^3 + \sqrt{bx+aa} b^3}{ab^2 x^2} \right) \operatorname{sgn}(x)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))*sgn(x)/b

$$3.239 \quad \int \frac{\sqrt{ax^2+bx^3}}{x^5} dx$$

Optimal. Leaf size=112

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}} - \frac{b\sqrt{ax^2+bx^3}}{12ax^3} - \frac{\sqrt{ax^2+bx^3}}{3x^4}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(3*x^4) - (b*\text{Sqrt}[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*\text{Sqrt}[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(8*a^{(5/2)})$

Rubi [A] time = 0.137877, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}} - \frac{b\sqrt{ax^2+bx^3}}{12ax^3} - \frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^2 + b*x^3]/x^5, x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(3*x^4) - (b*\text{Sqrt}[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*\text{Sqrt}[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(8*a^{(5/2)})$

Rule 2020

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*(m+j*p+1)), x] - \text{Dist}[(b*p*(n-j))/(c^n*(m+j*p+1)), \text{Int}[(c*x)^{(m+n)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m+j*p+1, 0]

Rule 2025

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Simp}[c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)}/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, p}, x]

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3}}{x^5} dx &= -\frac{\sqrt{ax^2 + bx^3}}{3x^4} + \frac{1}{6}b \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3}}{12ax^3} - \frac{b^2 \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{8a} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3}}{12ax^3} + \frac{b^2\sqrt{ax^2 + bx^3}}{8a^2x^2} + \frac{b^3 \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{16a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3}}{12ax^3} + \frac{b^2\sqrt{ax^2 + bx^3}}{8a^2x^2} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{8a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3}}{12ax^3} + \frac{b^2\sqrt{ax^2 + bx^3}}{8a^2x^2} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0124094, size = 42, normalized size = 0.38

$$\frac{2b^3 (x^2(a + bx))^{3/2} {}_2F_1\left(\frac{3}{2}, 4; \frac{5}{2}; \frac{bx}{a} + 1\right)}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5, x]
```


[Out] $(2*b^3*(x^2*(a + b*x))^{(3/2)}*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/ (3*a^4*x^3)$

Maple [A] time = 0.012, size = 89, normalized size = 0.8

$$-\frac{1}{24x^4}\sqrt{bx^3+ax^2}\left(3a^{9/2}\sqrt{bx+a}+8a^{7/2}(bx+a)^{3/2}-3a^{5/2}(bx+a)^{5/2}+3\operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3\right)a^{-\frac{9}{2}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a*x^2)^(1/2)/x^5,x)`

[Out] $-1/24*(b*x^3+a*x^2)^{(1/2)}*(3*a^{(9/2)}*(b*x+a)^{(1/2)}+8*a^{(7/2)}*(b*x+a)^{(3/2)}-3*a^{(5/2)}*(b*x+a)^{(5/2)}+3*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*a^2*b^3*x^3)/x^4/(b*x+a)^{(1/2)}/a^{(9/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^3 + a*x^2)/x^5, x)`

Fricas [A] time = 0.933463, size = 393, normalized size = 3.51

$$\left[\frac{3\sqrt{ab^3x^4}\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2(3ab^2x^2-2a^2bx-8a^3)\sqrt{bx^3+ax^2}}{48a^3x^4}, \frac{3\sqrt{-ab^3x^4}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(3ab^3x^4)}{24a^3x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")`

[Out] $[1/48*(3*\sqrt{a}*b^3*x^4*\log((b*x^2 + 2*a*x - 2*\sqrt{b*x^3 + a*x^2})*\sqrt{a})/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*\sqrt{b*x^3 + a*x^2})/(a^3*x^4)$
 $, 1/24*(3*\sqrt{-a}*b^3*x^4*\arctan(\sqrt{b*x^3 + a*x^2}*\sqrt{-a}/(a*x)) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*\sqrt{b*x^3 + a*x^2})/(a^3*x^4)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a+bx)}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)

[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)

Giac [A] time = 1.18835, size = 116, normalized size = 1.04

$$\frac{\left(\frac{3b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{3(bx+a)^5 b^4 - 8(bx+a)^3 ab^4 - 3\sqrt{bx+aa^2} b^4}{a^2 b^3 x^3} \right) \operatorname{sgn}(x)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")

[Out] $1/24*(3*b^4*\arctan(\sqrt{b*x + a}/\sqrt{-a})/(\sqrt{-a}*a^2) + (3*(b*x + a)^(5/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*\sqrt{b*x + a}*a^2*b^4)/(a^2*b^3*x^3))$
 $*\operatorname{sgn}(x)/b$

3.240 $\int x^2 (ax^2 + bx^3)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{512a^5 (ax^2 + bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4 (ax^2 + bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3 (ax^2 + bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2 (ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a (ax^2 + bx^3)^{5/2}}{39b^2x} + \frac{2(ax^2 + bx^3)^{3/2}}{15b}$$

[Out] $(2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 + b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)$

Rubi [A] time = 0.23122, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$\frac{512a^5 (ax^2 + bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4 (ax^2 + bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3 (ax^2 + bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2 (ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a (ax^2 + bx^3)^{5/2}}{39b^2x} + \frac{2(ax^2 + bx^3)^{3/2}}{15b}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3)^(3/2), x]

[Out] $(2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 + b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)$

Rule 2016

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*

```
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
 \int x^2 (ax^2 + bx^3)^{3/2} dx &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} - \frac{(2a) \int x (ax^2 + bx^3)^{3/2} dx}{3b} \\
 &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} - \frac{4a(ax^2 + bx^3)^{5/2}}{39b^2x} + \frac{(16a^2) \int (ax^2 + bx^3)^{3/2} dx}{39b^2} \\
 &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2 + bx^3)^{5/2}}{39b^2x} - \frac{(32a^3) \int \frac{(ax^2 + bx^3)^{3/2}}{x} dx}{143b^3} \\
 &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} - \frac{64a^3(ax^2 + bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2 + bx^3)^{5/2}}{39b^2x} + \frac{(128a^4) \int}{12} \\
 &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} + \frac{256a^4(ax^2 + bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2 + bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2 + bx^3)^{5/2}}{39b^2x} \\
 &= \frac{2(ax^2 + bx^3)^{5/2}}{15b} - \frac{512a^5(ax^2 + bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4(ax^2 + bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2 + bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2 + bx^3)^{5/2}}{39b^2x}
 \end{aligned}$$

Mathematica [A] time = 0.0414666, size = 80, normalized size = 0.5

$$\frac{2x(a + bx)^3(-1120a^3b^2x^2 + 1680a^2b^3x^3 + 640a^4bx - 256a^5 - 2310ab^4x^4 + 3003b^5x^5)}{45045b^6\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2), x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Maple [A] time = 0.005, size = 79, normalized size = 0.5

$$\frac{(2bx + 2a)(-3003x^5b^5 + 2310ax^4b^4 - 1680x^3a^2b^3 + 1120x^2a^3b^2 - 640a^4xb + 256a^5)}{45045b^6x^3} (bx^3 + ax^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^3+a*x^2)^(3/2),x)`

[Out] `-2/45045*(b*x+a)*(-3003*b^5*x^5+2310*a*b^4*x^4-1680*a^2*b^3*x^3+1120*a^3*b^2*x^2-640*a^4*b*x+256*a^5)*(b*x^3+a*x^2)^(3/2)/b^6/x^3`

Maxima [A] time = 1.00878, size = 116, normalized size = 0.72

$$\frac{2(3003b^7x^7 + 3696ab^6x^6 + 63a^2b^5x^5 - 70a^3b^4x^4 + 80a^4b^3x^3 - 96a^5b^2x^2 + 128a^6bx - 256a^7)\sqrt{bx+a}}{45045b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 + 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6`

Fricas [A] time = 0.868384, size = 217, normalized size = 1.35

$$\frac{2(3003b^7x^7 + 3696ab^6x^6 + 63a^2b^5x^5 - 70a^3b^4x^4 + 80a^4b^3x^3 - 96a^5b^2x^2 + 128a^6bx - 256a^7)\sqrt{bx^3 + ax^2}}{45045b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] `2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 + 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2)/(b^6*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (x^2 (a + bx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)

Giac [A] time = 1.13546, size = 242, normalized size = 1.5

$$\frac{512 a^{\frac{15}{2}} \operatorname{sgn}(x)}{45045 b^6} + \frac{2 \left(\frac{5 \left(693 (bx+a)^{\frac{13}{2}} - 4095 (bx+a)^{\frac{11}{2}} a + 10010 (bx+a)^{\frac{9}{2}} a^2 - 12870 (bx+a)^{\frac{7}{2}} a^3 + 9009 (bx+a)^{\frac{5}{2}} a^4 - 3003 (bx+a)^{\frac{3}{2}} a^5 \right) \operatorname{sgn}(x)}{b^5} + \frac{(3003 (bx+a)^{\frac{15}{2}} - 20790 (bx+a)^{\frac{13}{2}} a + 61425 (bx+a)^{\frac{11}{2}} a^2 - 100100 (bx+a)^{\frac{9}{2}} a^3 + 96525 (bx+a)^{\frac{7}{2}} a^4 - 54054 (bx+a)^{\frac{5}{2}} a^5 + 15015 (bx+a)^{\frac{3}{2}} a^6) \operatorname{sgn}(x)}{b^5} \right)}{45045 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(5*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*a*sgn(x)/b^5 + (3003*(b*x + a)^(15/2) - 20790*(b*x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 100100*(b*x + a)^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6)*sgn(x)/b^5/b

3.241 $\int x(ax^2 + bx^3)^{3/2} dx$

Optimal. Leaf size=136

$$\frac{256a^4(ax^2 + bx^3)^{5/2}}{15015b^5x^5} - \frac{128a^3(ax^2 + bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx}$$

[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)

Rubi [A] time = 0.171624, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2016, 2002, 2014}

$$\frac{256a^4(ax^2 + bx^3)^{5/2}}{15015b^5x^5} - \frac{128a^3(ax^2 + bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]

[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -

j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
 \int x(ax^2 + bx^3)^{3/2} dx &= \frac{2(ax^2 + bx^3)^{5/2}}{13bx} - \frac{(8a) \int (ax^2 + bx^3)^{3/2} dx}{13b} \\
 &= -\frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx} + \frac{(48a^2) \int \frac{(ax^2 + bx^3)^{3/2}}{x} dx}{143b^2} \\
 &= \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx} - \frac{(64a^3) \int \frac{(ax^2 + bx^3)^{3/2}}{x^2} dx}{429b^3} \\
 &= -\frac{128a^3(ax^2 + bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx} + \frac{(128a^4)}{15015b^5x^5} \\
 &= \frac{256a^4(ax^2 + bx^3)^{5/2}}{15015b^5x^5} - \frac{128a^3(ax^2 + bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2 + bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2 + bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2 + bx^3)^{5/2}}{13bx}
 \end{aligned}$$

Mathematica [A] time = 0.0332815, size = 69, normalized size = 0.51

$$\frac{2x(a + bx)^3 (560a^2b^2x^2 - 320a^3bx + 128a^4 - 840ab^3x^3 + 1155b^4x^4)}{15015b^5\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 + 1155*b^4*x^4))/(15015*b^5*sqrt[x^2*(a + b*x)])

Maple [A] time = 0.004, size = 68, normalized size = 0.5

$$\frac{(2bx + 2a)(1155x^4b^4 - 840ab^3x^3 + 560a^2x^2b^2 - 320xa^3b + 128a^4)}{15015b^5x^3} (bx^3 + ax^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^3+a*x^2)^(3/2),x)`

[Out] `2/15015*(b*x+a)*(1155*b^4*x^4-840*a*b^3*x^3+560*a^2*b^2*x^2-320*a^3*b*x+128*a^4)*(b*x^3+a*x^2)^(3/2)/b^5/x^3`

Maxima [A] time = 1.05686, size = 101, normalized size = 0.74

$$\frac{2(1155b^6x^6 + 1470ab^5x^5 + 35a^2b^4x^4 - 40a^3b^3x^3 + 48a^4b^2x^2 - 64a^5bx + 128a^6)\sqrt{bx+a}}{15015b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 + 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5`

Fricas [A] time = 0.808423, size = 193, normalized size = 1.42

$$\frac{2(1155b^6x^6 + 1470ab^5x^5 + 35a^2b^4x^4 - 40a^3b^3x^3 + 48a^4b^2x^2 - 64a^5bx + 128a^6)\sqrt{bx^3 + ax^2}}{15015b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] `2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 + 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(x^2 (a + bx) \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)

Giac [A] time = 1.17089, size = 211, normalized size = 1.55

$$-\frac{256 a^{\frac{13}{2}} \operatorname{sgn}(x)}{15015 b^5} + \frac{2 \left(\frac{13 \left(315 (bx+a)^{\frac{11}{2}} - 1540 (bx+a)^{\frac{9}{2}} a + 2970 (bx+a)^{\frac{7}{2}} a^2 - 2772 (bx+a)^{\frac{5}{2}} a^3 + 1155 (bx+a)^{\frac{3}{2}} a^4 \right) \operatorname{sgn}(x)}{b^4} + \frac{5 \left(693 (bx+a)^{\frac{13}{2}} - 4095 (bx+a)^{\frac{11}{2}} a + 10010 (bx+a)^{\frac{9}{2}} a^2 - 12870 (bx+a)^{\frac{7}{2}} a^3 + 9009 (bx+a)^{\frac{5}{2}} a^4 - 3003 (bx+a)^{\frac{3}{2}} a^5 \right) \operatorname{sgn}(x)}{45045 b} \right)}{45045 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(13*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*a*sgn(x)/b^4 + 5*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*sgn(x)/b^4)/b

3.242 $\int (ax^2 + bx^3)^{3/2} dx$

Optimal. Leaf size=108

$$-\frac{32a^3(ax^2 + bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2 + bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2 + bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2}$$

[Out] $(-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2)$

Rubi [A] time = 0.141219, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2002, 2016, 2014}

$$-\frac{32a^3(ax^2 + bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2 + bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2 + bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2), x]

[Out] $(-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2)$

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p+1)/(a*(j*p+1)*x^(j-1)), x] - Dist[(b*(n*p+n-j+1))/(a*(j*p+1)), Int[x^(n-j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p+n-j+1)/(n-j)], 0] && NeQ[j*p+1, 0]

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(m+j*p+1)), x] - Dist[(b*(m+n*p+n-j+1))/(a*c^(n-j)*(m+j*p+1)), Int[(c*x)^(m+n-j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m+n*p+n-j+1)/

$(n - j)], 0] \&\& \text{NeQ}[m + j*p + 1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2014

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \text{:> } -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[m + n*p + n - j + 1, 0] \&\& (\text{IntegerQ}[j] \parallel \text{GtQ}[c, 0])$

Rubi steps

$$\begin{aligned} \int (ax^2 + bx^3)^{3/2} dx &= \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2} - \frac{(6a) \int \frac{(ax^2 + bx^3)^{3/2}}{x} dx}{11b} \\ &= -\frac{4a(ax^2 + bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2} + \frac{(8a^2) \int \frac{(ax^2 + bx^3)^{3/2}}{x^2} dx}{33b^2} \\ &= \frac{16a^2(ax^2 + bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2 + bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2} - \frac{(16a^3) \int \frac{(ax^2 + bx^3)^{3/2}}{x^3} dx}{231b^3} \\ &= -\frac{32a^3(ax^2 + bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2 + bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2 + bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2 + bx^3)^{5/2}}{11bx^2} \end{aligned}$$

Mathematica [A] time = 0.0270002, size = 58, normalized size = 0.54

$$\frac{2x(a + bx)^3 (40a^2bx - 16a^3 - 70ab^2x^2 + 105b^3x^3)}{1155b^4\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155*b^4*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.003, size = 57, normalized size = 0.5

$$-\frac{(2bx + 2a)(-105x^3b^3 + 70ab^2x^2 - 40a^2xb + 16a^3)}{1155b^4x^3} (bx^3 + ax^2)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a*x^2)^(3/2),x)`

[Out] $-2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^{(3/2)}/b^4/x^3$

Maxima [A] time = 0.987819, size = 86, normalized size = 0.8

$$\frac{2(105b^5x^5 + 140ab^4x^4 + 5a^2b^3x^3 - 6a^3b^2x^2 + 8a^4bx - 16a^5)\sqrt{bx+a}}{1155b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] $2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4*b*x - 16*a^5)*\text{sqrt}(b*x + a)/b^4$

Fricas [A] time = 0.740348, size = 161, normalized size = 1.49

$$\frac{2(105b^5x^5 + 140ab^4x^4 + 5a^2b^3x^3 - 6a^3b^2x^2 + 8a^4bx - 16a^5)\sqrt{bx^3 + ax^2}}{1155b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] $2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4*b*x - 16*a^5)*\text{sqrt}(b*x^3 + a*x^2)/(b^4*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^2 + bx^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2),x)

[Out] Integral((a*x**2 + b*x**3)**(3/2), x)

Giac [A] time = 1.26395, size = 177, normalized size = 1.64

$$\frac{32 a^{\frac{11}{2}} \operatorname{sgn}(x)}{1155 b^4} + \frac{2 \left(\frac{11 \left(35 (bx+a)^{\frac{9}{2}} - 135 (bx+a)^{\frac{7}{2}} a + 189 (bx+a)^{\frac{5}{2}} a^2 - 105 (bx+a)^{\frac{3}{2}} a^3 \right) a \operatorname{sgn}(x)}{b^3} + \frac{\left(315 (bx+a)^{\frac{11}{2}} - 1540 (bx+a)^{\frac{9}{2}} a + 2970 (bx+a)^{\frac{7}{2}} a^2 - 2772 (bx+a)^{\frac{5}{2}} a^3 + 1155 (bx+a)^{\frac{3}{2}} a^4 \right) \operatorname{sgn}(x)}{b^3}}{3465 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(11*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a*sgn(x)/b^3 + (315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*sgn(x)/b^3/b

$$3.243 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x} dx$$

Optimal. Leaf size=80

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)

Rubi [A] time = 0.132562, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x, x]

[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x} dx &= \frac{2(ax^2 + bx^3)^{5/2}}{9bx^3} - \frac{(4a) \int \frac{(ax^2 + bx^3)^{3/2}}{x^2} dx}{9b} \\
&= -\frac{8a(ax^2 + bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2 + bx^3)^{5/2}}{9bx^3} + \frac{(8a^2) \int \frac{(ax^2 + bx^3)^{3/2}}{x^3} dx}{63b^2} \\
&= \frac{16a^2(ax^2 + bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2 + bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2 + bx^3)^{5/2}}{9bx^3}
\end{aligned}$$

Mathematica [A] time = 0.0224445, size = 47, normalized size = 0.59

$$\frac{2x(a + bx)^3(8a^2 - 20abx + 35b^2x^2)}{315b^3\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]

[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.004, size = 46, normalized size = 0.6

$$\frac{(2bx + 2a)(35b^2x^2 - 20abx + 8a^2)}{315b^3x^3} (bx^3 + ax^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x,x)

[Out] 2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(b*x^3+a*x^2)^(3/2)/b^3/x^3

Maxima [A] time = 1.00761, size = 72, normalized size = 0.9

$$\frac{2(35b^4x^4 + 50ab^3x^3 + 3a^2b^2x^2 - 4a^3bx + 8a^4)\sqrt{bx + a}}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")

[Out] $2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*\sqrt{bx^3 + ax^2}$
 $b*x + a)/b^3$

Fricas [A] time = 0.829455, size = 134, normalized size = 1.68

$$\frac{2(35b^4x^4 + 50ab^3x^3 + 3a^2b^2x^2 - 4a^3bx + 8a^4)\sqrt{bx^3 + ax^2}}{315b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")

[Out] $2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*\sqrt{bx^3 + ax^2}$
 $b*x^3 + a*x^2)/(b^3*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)

Giac [A] time = 1.28919, size = 144, normalized size = 1.8

$$-\frac{16a^{\frac{9}{2}}\operatorname{sgn}(x)}{315b^3} + \frac{2\left(\frac{3\left(15(bx+a)^{\frac{7}{2}}-42(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2\right)\operatorname{sgn}(x)}{b^2} + \frac{\left(35(bx+a)^{\frac{9}{2}}-135(bx+a)^{\frac{7}{2}}a+189(bx+a)^{\frac{5}{2}}a^2-105(bx+a)^{\frac{3}{2}}a^3\right)\operatorname{sgn}(x)}{b^2}\right)}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(3*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)*a*sgn(x)/b^2 + (35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*sgn(x)/b^2)/b
```

$$3.244 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^2} dx$$

Optimal. Leaf size=52

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

[Out] $(-4*a*(a*x^2 + b*x^3)^{(5/2)})/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^{(5/2)})/(7*b*x^4)$

Rubi [A] time = 0.0826054, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]

[Out] $(-4*a*(a*x^2 + b*x^3)^{(5/2)})/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^{(5/2)})/(7*b*x^4)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x^2} dx = \frac{2(ax^2 + bx^3)^{5/2}}{7bx^4} - \frac{(2a) \int \frac{(ax^2 + bx^3)^{3/2}}{x^3} dx}{7b}$$

$$= -\frac{4a(ax^2 + bx^3)^{5/2}}{35b^2x^5} + \frac{2(ax^2 + bx^3)^{5/2}}{7bx^4}$$

Mathematica [A] time = 0.01721, size = 36, normalized size = 0.69

$$\frac{2x(a + bx)^3(5bx - 2a)}{35b^2\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]

[Out] (2*x*(a + b*x)^3*(-2*a + 5*b*x))/(35*b^2*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.003, size = 35, normalized size = 0.7

$$-\frac{(2bx + 2a)(-5bx + 2a)}{35b^2x^3} (bx^3 + ax^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x^2,x)

[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3

Maxima [A] time = 1.02657, size = 55, normalized size = 1.06

$$\frac{2(5b^3x^3 + 8ab^2x^2 + a^2bx - 2a^3)\sqrt{bx + a}}{35b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2

Fricas [A] time = 0.817854, size = 105, normalized size = 2.02

$$\frac{2(5b^3x^3 + 8ab^2x^2 + a^2bx - 2a^3)\sqrt{bx^3 + ax^2}}{35b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)

Giac [A] time = 1.27493, size = 112, normalized size = 2.15

$$\frac{4a^{\frac{7}{2}}\operatorname{sgn}(x)}{35b^2} + \frac{2\left(\frac{7\left(3(bx+a)^{\frac{5}{2}}-5(bx+a)^{\frac{3}{2}}a\right)\operatorname{sgn}(x)}{b} + \frac{\left(15(bx+a)^{\frac{7}{2}}-42(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2\right)\operatorname{sgn}(x)}{b}\right)}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(7*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a
)*a*sgn(x)/b + (15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2)*sgn(x)/b)/b
```

$$3.245 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx$$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)

Rubi [A] time = 0.0406793, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2014}

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]

[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :-> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Mathematica [A] time = 0.0128377, size = 23, normalized size = 0.92

$$\frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]

[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)

Maple [A] time = 0.002, size = 27, normalized size = 1.1

$$\frac{2bx + 2a}{5bx^3} (bx^3 + ax^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x^3,x)

[Out] 2/5*(b*x+a)*(b*x^3+a*x^2)^(3/2)/b/x^3

Maxima [A] time = 0.988703, size = 38, normalized size = 1.52

$$\frac{2(b^2x^2 + 2abx + a^2)\sqrt{bx + a}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")

[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b

Fricas [A] time = 0.811251, size = 77, normalized size = 3.08

$$\frac{2(b^2x^2 + 2abx + a^2)\sqrt{bx^3 + ax^2}}{5bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] $2/5*(b^2*x^2 + 2*a*b*x + a^2)*\text{sqrt}(b*x^3 + a*x^2)/(b*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**(3/2)/x**3,x)`

[Out] `Integral((x**2*(a + b*x))**(3/2)/x**3, x)`

Giac [B] time = 1.24223, size = 70, normalized size = 2.8

$$-\frac{2a^{\frac{5}{2}}\text{sgn}(x)}{5b} + \frac{2\left(5(bx+a)^{\frac{3}{2}}a\text{sgn}(x) + \left(3(bx+a)^{\frac{5}{2}} - 5(bx+a)^{\frac{3}{2}}a\right)\text{sgn}(x)\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")`

[Out] $-2/5*a^{(5/2)}*\text{sgn}(x)/b + 2/15*(5*(b*x + a)^{(3/2)}*a*\text{sgn}(x) + (3*(b*x + a)^{(5/2)} - 5*(b*x + a)^{(3/2)}*a)*\text{sgn}(x))/b$

$$3.246 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx$$

Optimal. Leaf size=74

$$-2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right) + \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rubi [A] time = 0.0958373, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2021, 2008, 206}

$$-2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right) + \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4, x]

[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x]
/; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol]
:> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x]
/; FreeQ[{a, b, n}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3)^{3/2}}{x^4} dx &= \frac{2(ax^2 + bx^3)^{3/2}}{3x^3} + a \int \frac{\sqrt{ax^2 + bx^3}}{x^2} dx \\ &= \frac{2a\sqrt{ax^2 + bx^3}}{x} + \frac{2(ax^2 + bx^3)^{3/2}}{3x^3} + a^2 \int \frac{1}{\sqrt{ax^2 + bx^3}} dx \\ &= \frac{2a\sqrt{ax^2 + bx^3}}{x} + \frac{2(ax^2 + bx^3)^{3/2}}{3x^3} - (2a^2) \text{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}} \right) \\ &= \frac{2a\sqrt{ax^2 + bx^3}}{x} + \frac{2(ax^2 + bx^3)^{3/2}}{3x^3} - 2a^{3/2} \tanh^{-1} \left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.0426705, size = 68, normalized size = 0.92

$$\frac{2x\sqrt{a+bx} \left(\sqrt{a+bx}(4a+bx) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right) \right)}{3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4, x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Maple [A] time = 0.007, size = 63, normalized size = 0.9

$$-\frac{2}{3x^3} (bx^3 + ax^2)^{\frac{3}{2}} \left(3a^{3/2} \text{Artanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right) - (bx+a)^{\frac{3}{2}} - 3a\sqrt{bx+a} \right) (bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4, x)
```

[Out] $-2/3*(b*x^3+a*x^2)^{(3/2)}*(3*a^{(3/2)}*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})-(b*x+a)^{(3/2)}-3*a*(b*x+a)^{(1/2)})/x^3/(b*x+a)^{(3/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)`

Fricas [A] time = 0.810448, size = 296, normalized size = 4.

$$\left[\frac{3 a^{\frac{3}{2}} x \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2\sqrt{bx^3+ax^2}(bx+4a)}{3x}, \frac{2\left(3\sqrt{-a}ax \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}(bx+4a)\right)}{3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")`

[Out] `[1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx))^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)

Giac [A] time = 1.19226, size = 115, normalized size = 1.55

$$\frac{2a^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2}{3} (bx+a)^{\frac{3}{2}} \operatorname{sgn}(x) + 2\sqrt{bx+aa} \operatorname{sgn}(x) - \frac{2\left(3a^2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 4\sqrt{-aa^{\frac{3}{2}}}\right) \operatorname{sgn}(x)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")

[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)

$$3.247 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx$$

Optimal. Leaf size=73

$$-\frac{(ax^2+bx^3)^{3/2}}{x^4} + \frac{3b\sqrt{ax^2+bx^3}}{x} - 3\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)$$

[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rubi [A] time = 0.0931158, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2021, 2008, 206}

$$-\frac{(ax^2+bx^3)^{3/2}}{x^4} + \frac{3b\sqrt{ax^2+bx^3}}{x} - 3\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5, x]

[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]

Rule 2020

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x^5} dx &= -\frac{(ax^2 + bx^3)^{3/2}}{x^4} + \frac{1}{2}(3b) \int \frac{\sqrt{ax^2 + bx^3}}{x^2} dx \\
&= \frac{3b\sqrt{ax^2 + bx^3}}{x} - \frac{(ax^2 + bx^3)^{3/2}}{x^4} + \frac{1}{2}(3ab) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx \\
&= \frac{3b\sqrt{ax^2 + bx^3}}{x} - \frac{(ax^2 + bx^3)^{3/2}}{x^4} - (3ab) \operatorname{Subst}\left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right) \\
&= \frac{3b\sqrt{ax^2 + bx^3}}{x} - \frac{(ax^2 + bx^3)^{3/2}}{x^4} - 3\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0134288, size = 40, normalized size = 0.55

$$\frac{2b(x^2(a + bx))^{5/2} {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; \frac{bx}{a} + 1\right)}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (2*b*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*
a^2*x^5)
```

Maple [A] time = 0.011, size = 72, normalized size = 1.

$$-\frac{1}{x^4} (bx^3 + ax^2)^{\frac{3}{2}} \left(a^{\frac{3}{2}} \sqrt{bx+a} - 2\sqrt{bx+a}bx\sqrt{a} + 3 \operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)xab \right) (bx+a)^{-\frac{3}{2}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x^5,x)

[Out] -(b*x^3+a*x^2)^(3/2)*(a^(3/2)*(b*x+a)^(1/2)-2*(b*x+a)^(1/2)*b*x*a^(1/2)+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)/x^4/(b*x+a)^(3/2)/a^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)

Fricas [A] time = 0.898792, size = 304, normalized size = 4.16

$$\left[\frac{3\sqrt{a}bx^2 \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2\sqrt{bx^3+ax^2}(2bx-a)}{2x^2}, \frac{3\sqrt{-a}bx^2 \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}(2bx-a)}{x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")

[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx))^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)

Giac [A] time = 1.45642, size = 84, normalized size = 1.15

$$\frac{\frac{3ab^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + 2\sqrt{bx+a}ab^2 \operatorname{sgn}(x) - \frac{\sqrt{bx+a}b \operatorname{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")

[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b

$$3.248 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx$$

Optimal. Leaf size=81

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}} - \frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

[Out] $(-3*b*\text{Sqrt}[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^{(3/2)}/(2*x^5) - (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*\text{Sqrt}[a])$

Rubi [A] time = 0.0921833, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2020, 2008, 206}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}} - \frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3)^{(3/2)}/x^6, x]$

[Out] $(-3*b*\text{Sqrt}[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^{(3/2)}/(2*x^5) - (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*\text{Sqrt}[a])$

Rule 2020

```
Int[((c_.)*(x_)^(m_))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x^6} dx &= -\frac{(ax^2 + bx^3)^{3/2}}{2x^5} + \frac{1}{4}(3b) \int \frac{\sqrt{ax^2 + bx^3}}{x^3} dx \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{4x^2} - \frac{(ax^2 + bx^3)^{3/2}}{2x^5} + \frac{1}{8}(3b^2) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{4x^2} - \frac{(ax^2 + bx^3)^{3/2}}{2x^5} - \frac{1}{4}(3b^2) \text{Subst}\left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right) \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{4x^2} - \frac{(ax^2 + bx^3)^{3/2}}{2x^5} - \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0463637, size = 72, normalized size = 0.89

$$-\frac{2a^2 + 3b^2x^2\sqrt{\frac{bx}{a} + 1} \tanh^{-1}\left(\sqrt{\frac{bx}{a} + 1}\right) + 7abx + 5b^2x^2}{4x\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6, x]
```

```
[Out] -(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1
+ (b*x)/a]])/(4*x*Sqrt[x^2*(a + b*x)])
```

Maple [A] time = 0.01, size = 74, normalized size = 0.9

$$\frac{1}{4x^5} (bx^3 + ax^2)^{\frac{3}{2}} \left(-3 \operatorname{Arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) x^2 b^2 + 3a^{3/2} \sqrt{bx+a} - 5\sqrt{a} (bx+a)^{3/2} \right) \frac{1}{\sqrt{a}} (bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6, x)
```

[Out] $\frac{1}{4} \cdot (bx^3 + ax^2)^{3/2} \cdot (-3 \cdot \operatorname{arctanh}((bx+a)^{1/2}/a^{1/2})) \cdot x^2 \cdot b^2 + 3 \cdot a^{3/2} \cdot (bx+a)^{1/2} - 5 \cdot a^{1/2} \cdot (bx+a)^{3/2} / x^5 / (bx+a)^{3/2} / a^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{3/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)`

Fricas [A] time = 0.928418, size = 342, normalized size = 4.22

$$\left[\frac{3 \sqrt{ab^2x^3} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\sqrt{bx^3+ax^2}(5abx+2a^2)}{8ax^3}, \frac{3\sqrt{-ab^2x^3} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) - \sqrt{bx^3+ax^2}(5abx+2a^2)}{4ax^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")`

[Out] `[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{3/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)

Giac [A] time = 1.27062, size = 95, normalized size = 1.17

$$\frac{\frac{3b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} - \frac{5(bx+a)^{\frac{3}{2}} b^3 \operatorname{sgn}(x) - 3\sqrt{bx+a} ab^3 \operatorname{sgn}(x)}{b^2 x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")

[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b

$$3.249 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^7} dx$$

Optimal. Leaf size=109

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}} - \frac{b^2 \sqrt{ax^2+bx^3}}{8ax^2} - \frac{b \sqrt{ax^2+bx^3}}{4x^3} - \frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

[Out] $-(b\sqrt{ax^2+bx^3})/(4x^3) - (b^2\sqrt{ax^2+bx^3})/(8ax^2) - (ax^2+bx^3)^{(3/2)}/(3x^6) + (b^3\text{ArcTanh}[(\sqrt{a}x)/\sqrt{ax^2+bx^3}])/(8a^{(3/2)})$

Rubi [A] time = 0.134152, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}} - \frac{b^2 \sqrt{ax^2+bx^3}}{8ax^2} - \frac{b \sqrt{ax^2+bx^3}}{4x^3} - \frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7, x]

[Out] $-(b\sqrt{ax^2+bx^3})/(4x^3) - (b^2\sqrt{ax^2+bx^3})/(8ax^2) - (ax^2+bx^3)^{(3/2)}/(3x^6) + (b^3\text{ArcTanh}[(\sqrt{a}x)/\sqrt{ax^2+bx^3}])/(8a^{(3/2)})$

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x^7} dx &= -\frac{(ax^2 + bx^3)^{3/2}}{3x^6} + \frac{1}{2}b \int \frac{\sqrt{ax^2 + bx^3}}{x^4} dx \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{4x^3} - \frac{(ax^2 + bx^3)^{3/2}}{3x^6} + \frac{1}{8}b^2 \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2 + bx^3}}{8ax^2} - \frac{(ax^2 + bx^3)^{3/2}}{3x^6} - \frac{b^3 \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{16a} \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2 + bx^3}}{8ax^2} - \frac{(ax^2 + bx^3)^{3/2}}{3x^6} + \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{8a} \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2 + bx^3}}{8ax^2} - \frac{(ax^2 + bx^3)^{3/2}}{3x^6} + \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0157798, size = 42, normalized size = 0.39

$$\frac{2b^3 (x^2(a + bx))^{5/2} {}_2F_1\left(\frac{5}{2}, 4; \frac{7}{2}; \frac{bx}{a} + 1\right)}{5a^4 x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7, x]
```

[Out] $(2*b^3*(x^2*(a + b*x))^{5/2}*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/ (5*a^4*x^5)$

Maple [A] time = 0.012, size = 87, normalized size = 0.8

$$\frac{1}{24x^6} (bx^3 + ax^2)^{\frac{3}{2}} \left(3a^{7/2}\sqrt{bx+a} - 8a^{5/2}(bx+a)^{3/2} - 3a^{3/2}(bx+a)^{5/2} + 3 \operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) x^3 ab^3 \right) a^{-\frac{5}{2}} (bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((b*x^3+a*x^2)^{(3/2)}/x^7, x)$

[Out] $1/24*(b*x^3+a*x^2)^{(3/2)}*(3*a^{(7/2)}*(b*x+a)^{(1/2)}-8*a^{(5/2)}*(b*x+a)^{(3/2)}-3*a^{(3/2)}*(b*x+a)^{(5/2)}+3*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*x^3*a*b^3)/x^6/(b*x+a)^{(3/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((b*x^3+a*x^2)^{(3/2)}/x^7, x, \operatorname{algorithm}="maxima")$

[Out] $\operatorname{integrate}((b*x^3 + a*x^2)^{(3/2)}/x^7, x)$

Fricas [A] time = 0.785699, size = 397, normalized size = 3.64

$$\left[\frac{3\sqrt{a}b^3x^4 \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2(3ab^2x^2 + 14a^2bx + 8a^3)\sqrt{bx^3+ax^2}}{48a^2x^4}, -\frac{3\sqrt{-a}b^3x^4 \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + (3a}{24a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2))*sqrt(a))/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(a^2*x^4), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)

Giac [A] time = 1.21501, size = 124, normalized size = 1.14

$$\frac{3b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-aa}} + \frac{3(bx+a)^{\frac{5}{2}} b^4 \operatorname{sgn}(x) + 8(bx+a)^{\frac{3}{2}} ab^4 \operatorname{sgn}(x) - 3\sqrt{bx+aa^2} b^4 \operatorname{sgn}(x)}{ab^3 x^3}$$

24 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")

[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x + a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2*b^4*sgn(x))/(a*b^3*x^3))/b

$$3.250 \quad \int \frac{(ax^2+bx^3)^{3/2}}{x^8} dx$$

Optimal. Leaf size=137

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} - \frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{(ax^2+bx^3)^{3/2}}{4x^7}$$

[Out] $-(b\sqrt{ax^2+bx^3})/(8x^4) - (b^2\sqrt{ax^2+bx^3})/(32ax^3) + (3b^3\sqrt{ax^2+bx^3})/(64a^2x^2) - (ax^2+bx^3)^{(3/2)}/(4x^7) - (3b^4\text{ArcTanh}[(\sqrt{a}x)/\sqrt{ax^2+bx^3}])/(64a^{(5/2)})$

Rubi [A] time = 0.18419, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2008, 206}

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} - \frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{(ax^2+bx^3)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8, x]

[Out] $-(b\sqrt{ax^2+bx^3})/(8x^4) - (b^2\sqrt{ax^2+bx^3})/(32ax^3) + (3b^3\sqrt{ax^2+bx^3})/(64a^2x^2) - (ax^2+bx^3)^{(3/2)}/(4x^7) - (3b^4\text{ArcTanh}[(\sqrt{a}x)/\sqrt{ax^2+bx^3}])/(64a^{(5/2)})$

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x^8} dx &= -\frac{(ax^2 + bx^3)^{3/2}}{4x^7} + \frac{1}{8}(3b) \int \frac{\sqrt{ax^2 + bx^3}}{x^5} dx \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{8x^4} - \frac{(ax^2 + bx^3)^{3/2}}{4x^7} + \frac{1}{16}b^2 \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2 + bx^3}}{32ax^3} - \frac{(ax^2 + bx^3)^{3/2}}{4x^7} - \frac{(3b^3) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{64a} \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2 + bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2 + bx^3}}{64a^2x^2} - \frac{(ax^2 + bx^3)^{3/2}}{4x^7} + \frac{(3b^4) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{128a^2} \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2 + bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2 + bx^3}}{64a^2x^2} - \frac{(ax^2 + bx^3)^{3/2}}{4x^7} - \frac{(3b^4) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{ax^2 + bx^3}}{a}\right)}{64a^2} \\
&= -\frac{b\sqrt{ax^2 + bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2 + bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2 + bx^3}}{64a^2x^2} - \frac{(ax^2 + bx^3)^{3/2}}{4x^7} - \frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{ax^2 + bx^3}}{\sqrt{ax^2 + bx^3}}\right)}{64a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0145932, size = 42, normalized size = 0.31

$$-\frac{2b^4 \left(x^2(a + bx)\right)^{5/2} {}_2F_1\left(\frac{5}{2}, 5; \frac{7}{2}; \frac{bx}{a} + 1\right)}{5a^5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]

[Out] $(-2*b^4*(x^2*(a + b*x))^{5/2}*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x)/a])/(5*a^5*x^5)$

Maple [A] time = 0.01, size = 101, normalized size = 0.7

$$\frac{1}{64x^7} (bx^3 + ax^2)^{\frac{3}{2}} \left(3a^{5/2} (bx + a)^{7/2} - 11a^{7/2} (bx + a)^{5/2} - 3 \operatorname{Artanh} \left(\frac{\sqrt{bx + a}}{\sqrt{a}} \right) a^2 b^4 x^4 - 11a^{9/2} (bx + a)^{3/2} + 3a^{11/2} \sqrt{bx} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x^8,x)

[Out] $\frac{1}{64}*(b*x^3+a*x^2)^{(3/2)}*(3*a^{(5/2)}*(b*x+a)^{(7/2)}-11*a^{(7/2)}*(b*x+a)^{(5/2)}-3*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*a^2*b^4*x^4-11*a^{(9/2)}*(b*x+a)^{(3/2)}+3*a^{(11/2)}*(b*x+a)^{(1/2)})/x^7/(b*x+a)^{(3/2)}/a^{(9/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)

Fricas [A] time = 0.893233, size = 443, normalized size = 3.23

$$\left[\frac{3\sqrt{ab^4x^5} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2(3ab^3x^3 - 2a^2b^2x^2 - 24a^3bx - 16a^4)\sqrt{bx^3+ax^2} - 3\sqrt{-ab^4x^5} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{a}}{ax}\right)}{128a^3x^5}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")

[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2))*sqrt(a))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3 + a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3 + a*x^2))/(a^3*x^5)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)

Giac [A] time = 1.23712, size = 147, normalized size = 1.07

$$\frac{3b^5 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-aa^2}} + \frac{3(bx+a)^{\frac{7}{2}} b^5 \operatorname{sgn}(x) - 11(bx+a)^{\frac{5}{2}} ab^5 \operatorname{sgn}(x) - 11(bx+a)^{\frac{3}{2}} a^2 b^5 \operatorname{sgn}(x) + 3\sqrt{bx+aa^3} b^5 \operatorname{sgn}(x)}{a^2 b^4 x^4}$$

$64b$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")

[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x + a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b

3.251

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9} dx$$

Optimal. Leaf size=165

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} + \frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} - \frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

[Out] $(-3*b*\text{Sqrt}[a*x^2 + b*x^3])/(40*x^5) - (b^2*\text{Sqrt}[a*x^2 + b*x^3])/(80*a*x^4) + (b^3*\text{Sqrt}[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*\text{Sqrt}[a*x^2 + b*x^3])/(128*a^3*x^2) - (a*x^2 + b*x^3)^{(3/2)}/(5*x^8) + (3*b^5*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(128*a^{(7/2)})$

Rubi [A] time = 0.236105, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} + \frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} - \frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3)^{(3/2)}/x^9, x]$

[Out] $(-3*b*\text{Sqrt}[a*x^2 + b*x^3])/(40*x^5) - (b^2*\text{Sqrt}[a*x^2 + b*x^3])/(80*a*x^4) + (b^3*\text{Sqrt}[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*\text{Sqrt}[a*x^2 + b*x^3])/(128*a^3*x^2) - (a*x^2 + b*x^3)^{(3/2)}/(5*x^8) + (3*b^5*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(128*a^{(7/2)})$

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b^p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2008

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]

```

Rule 206

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3)^{3/2}}{x^9} dx &= -\frac{(ax^2 + bx^3)^{3/2}}{5x^8} + \frac{1}{10}(3b) \int \frac{\sqrt{ax^2 + bx^3}}{x^6} dx \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} + \frac{1}{80}(3b^2) \int \frac{1}{x^3\sqrt{ax^2 + bx^3}} dx \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2 + bx^3}}{80ax^4} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} - \frac{b^3 \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx}{32a} \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2 + bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2 + bx^3}}{64a^2x^3} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} + \frac{(3b^4) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{128a^2} \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2 + bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2 + bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2 + bx^3}}{128a^3x^2} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} - \frac{(3b^5) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{2} \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2 + bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2 + bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2 + bx^3}}{128a^3x^2} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} + \frac{(3b^5) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{2} \\
&= -\frac{3b\sqrt{ax^2 + bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2 + bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2 + bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2 + bx^3}}{128a^3x^2} - \frac{(ax^2 + bx^3)^{3/2}}{5x^8} + \frac{3b^5 \tan^{-1}\left(\frac{\sqrt{ax^2 + bx^3}}{x}\right)}{2}
\end{aligned}$$

Mathematica [C] time = 0.014527, size = 42, normalized size = 0.25

$$\frac{2b^5 (x^2(a + bx))^{5/2} {}_2F_1\left(\frac{5}{2}, 6; \frac{7}{2}; \frac{bx}{a} + 1\right)}{5a^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]

[Out] (2*b^5*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (b*x)/a])/ (5*a^6*x^5)

Maple [A] time = 0.013, size = 113, normalized size = 0.7

$$\frac{1}{640x^8} (bx^3 + ax^2)^{\frac{3}{2}} \left(15a^{15/2} \sqrt{bx+a} - 70a^{13/2} (bx+a)^{3/2} - 128a^{11/2} (bx+a)^{5/2} + 70a^{9/2} (bx+a)^{7/2} - 15a^{7/2} (bx+a)^{9/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)^(3/2)/x^9,x)

[Out] 1/640*(b*x^3+a*x^2)^(3/2)*(15*a^(15/2)*(b*x+a)^(1/2)-70*a^(13/2)*(b*x+a)^(3/2)-128*a^(11/2)*(b*x+a)^(5/2)+70*a^(9/2)*(b*x+a)^(7/2)-15*a^(7/2)*(b*x+a)^(9/2)+15*arctanh((b*x+a)^(1/2)/a^(1/2))*a^3*b^5*x^5)/x^8/(b*x+a)^(3/2)/a^(13/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)

Fricas [A] time = 0.909493, size = 504, normalized size = 3.05

$$\left[\frac{15 \sqrt{ab^5} x^6 \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2(15ab^4x^4 - 10a^2b^3x^3 + 8a^3b^2x^2 + 176a^4bx + 128a^5)\sqrt{bx^3+ax^2} - 15\sqrt{-ab^5}x^6}{1280a^4x^6}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2))*sqrt(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2)/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)

[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)

Giac [A] time = 1.32584, size = 170, normalized size = 1.03

$$\frac{15b^6 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-aa^3}} + \frac{15(bx+a)^{\frac{9}{2}}b^6 \operatorname{sgn}(x) - 70(bx+a)^{\frac{7}{2}}ab^6 \operatorname{sgn}(x) + 128(bx+a)^{\frac{5}{2}}a^2b^6 \operatorname{sgn}(x) + 70(bx+a)^{\frac{3}{2}}a^3b^6 \operatorname{sgn}(x) - 15\sqrt{bx+aa^4}b^6 \operatorname{sgn}(x)}{a^3b^5x^5}$$

$640b$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")

[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b

$$3.252 \quad \int \frac{x^4}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=103

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} + \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/(7*b)

Rubi [A] time = 0.148466, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 1588}

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} + \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]

[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/(7*b)

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

Q[m, x] && PolyQ[pp, x] && PolyQ[qq, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\sqrt{ax^2 + bx^3}} dx &= \frac{2x^2\sqrt{ax^2 + bx^3}}{7b} - \frac{(6a) \int \frac{x^3}{\sqrt{ax^2 + bx^3}} dx}{7b} \\
 &= -\frac{12ax\sqrt{ax^2 + bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2 + bx^3}}{7b} + \frac{(24a^2) \int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx}{35b^2} \\
 &= \frac{16a^2\sqrt{ax^2 + bx^3}}{35b^3} - \frac{12ax\sqrt{ax^2 + bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2 + bx^3}}{7b} - \frac{(16a^3) \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{35b^3} \\
 &= \frac{16a^2\sqrt{ax^2 + bx^3}}{35b^3} - \frac{32a^3\sqrt{ax^2 + bx^3}}{35b^4x} - \frac{12ax\sqrt{ax^2 + bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2 + bx^3}}{7b}
 \end{aligned}$$

Mathematica [A] time = 0.0324024, size = 53, normalized size = 0.51

$$\frac{2\sqrt{x^2(a + bx)}(8a^2bx - 16a^3 - 6ab^2x^2 + 5b^3x^3)}{35b^4x}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3], x]

[Out] (2*sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4*x)

Maple [A] time = 0.004, size = 55, normalized size = 0.5

$$-\frac{(2bx + 2a)(-5x^3b^3 + 6ab^2x^2 - 8a^2xb + 16a^3)x}{35b^4} \frac{1}{\sqrt{bx^3 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x^2)^(1/2), x)

[Out] -2/35*(b*x+a)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)*x/b^4/(b*x^3+a*x^2)^(1/2)

Maxima [A] time = 1.06037, size = 72, normalized size = 0.7

$$\frac{2(5b^4x^4 - ab^3x^3 + 2a^2b^2x^2 - 8a^3bx - 16a^4)}{35\sqrt{bx + ab^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x + a)*b^4)

Fricas [A] time = 0.805416, size = 109, normalized size = 1.06

$$\frac{2(5b^3x^3 - 6ab^2x^2 + 8a^2bx - 16a^3)\sqrt{bx^3 + ax^2}}{35b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^4*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x^2), x)
```

3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}} dx$

Optimal. Leaf size=75

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x} - \frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{2x\sqrt{ax^2+bx^3}}{5b}$$

[Out] $(-8*a*\text{Sqrt}[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*\text{Sqrt}[a*x^2 + b*x^3])/(15*b^3*x) + (2*x*\text{Sqrt}[a*x^2 + b*x^3])/(5*b)$

Rubi [A] time = 0.0999528, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x} - \frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/\text{Sqrt}[a*x^2 + b*x^3], x]$

[Out] $(-8*a*\text{Sqrt}[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*\text{Sqrt}[a*x^2 + b*x^3])/(15*b^3*x) + (2*x*\text{Sqrt}[a*x^2 + b*x^3])/(5*b)$

Rule 2016

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2 + bx^3}} dx &= \frac{2x\sqrt{ax^2 + bx^3}}{5b} - \frac{(4a) \int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx}{5b} \\
&= -\frac{8a\sqrt{ax^2 + bx^3}}{15b^2} + \frac{2x\sqrt{ax^2 + bx^3}}{5b} + \frac{(8a^2) \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{15b^2} \\
&= -\frac{8a\sqrt{ax^2 + bx^3}}{15b^2} + \frac{16a^2\sqrt{ax^2 + bx^3}}{15b^3x} + \frac{2x\sqrt{ax^2 + bx^3}}{5b}
\end{aligned}$$

Mathematica [A] time = 0.0196869, size = 42, normalized size = 0.56

$$\frac{2\sqrt{x^2(a + bx)}(8a^2 - 4abx + 3b^2x^2)}{15b^3x}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3], x]

[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)

Maple [A] time = 0.004, size = 44, normalized size = 0.6

$$\frac{(2bx + 2a)(3b^2x^2 - 4abx + 8a^2)x}{15b^3} \frac{1}{\sqrt{bx^3 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x^2)^(1/2), x)

[Out] 2/15*(b*x+a)*(3*b^2*x^2-4*a*b*x+8*a^2)*x/b^3/(b*x^3+a*x^2)^(1/2)

Maxima [A] time = 1.15133, size = 57, normalized size = 0.76

$$\frac{2(3b^3x^3 - ab^2x^2 + 4a^2bx + 8a^3)}{15\sqrt{bx + ab^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)

Fricas [A] time = 0.792813, size = 86, normalized size = 1.15

$$\frac{2(3b^2x^2 - 4abx + 8a^2)\sqrt{bx^3 + ax^2}}{15b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(b*x^3 + a*x^2), x)

$$3.254 \quad \int \frac{x^2}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=49

$$\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)

Rubi [A] time = 0.0545702, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 1588}

$$\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
    + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
    t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
    }, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
    (n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x
  ]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
    , x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
    , Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; Free
    Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx = \frac{2\sqrt{ax^2 + bx^3}}{3b} - \frac{(2a) \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{3b}$$

$$= \frac{2\sqrt{ax^2 + bx^3}}{3b} - \frac{4a\sqrt{ax^2 + bx^3}}{3b^2x}$$

Mathematica [A] time = 0.0148682, size = 30, normalized size = 0.61

$$\frac{2(bx - 2a)\sqrt{x^2(a + bx)}}{3b^2x}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)

Maple [A] time = 0.003, size = 33, normalized size = 0.7

$$-\frac{(2bx + 2a)(-bx + 2a)x}{3b^2} \frac{1}{\sqrt{bx^3 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x^2)^(1/2),x)

[Out] -2/3*(b*x+a)*(-b*x+2*a)*x/b^2/(b*x^3+a*x^2)^(1/2)

Maxima [A] time = 1.03395, size = 41, normalized size = 0.84

$$\frac{2(b^2x^2 - abx - 2a^2)}{3\sqrt{bx + ab^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] $2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)$

Fricas [A] time = 0.858497, size = 61, normalized size = 1.24

$$\frac{2\sqrt{bx^3 + ax^2}(bx - 2a)}{3b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(x**2/sqrt(x**2*(a + b*x)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^2/sqrt(b*x^3 + a*x^2), x)`

$$3.255 \quad \int \frac{x}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=23

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)

Rubi [A] time = 0.0101149, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3}} dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

Mathematica [A] time = 0.0073949, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x^2(a+bx)}}{bx}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)

Maple [A] time = 0.001, size = 25, normalized size = 1.1

$$2 \frac{x(bx + a)}{b\sqrt{bx^3 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x^2)^(1/2),x)

[Out] 2*x*(b*x+a)/b/(b*x^3+a*x^2)^(1/2)

Maxima [A] time = 1.16918, size = 16, normalized size = 0.7

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(b*x + a)/b

Fricas [A] time = 0.797656, size = 39, normalized size = 1.7

$$\frac{2\sqrt{bx^3 + ax^2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $2\sqrt{bx^3 + ax^2}/(bx)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(x/sqrt(x**2*(a + b*x)), x)`

Giac [A] time = 1.18953, size = 35, normalized size = 1.52

$$\frac{2}{\sqrt{\frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

[Out] `2/(sqrt(b/x + a/x^2) - sqrt(a)/x)`

$$3.256 \quad \int \frac{1}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=30

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

[Out] $(-2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/\text{Sqrt}[a]$

Rubi [A] time = 0.0105473, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2008, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[a*x^2 + b*x^3], x]$

[Out] $(-2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/\text{Sqrt}[a]$

Rule 2008

$\text{Int}[1/\text{Sqrt}[(a_*)(x_)^2 + (b_*)(x_)^{(n_)}], x_Symbol] \rightarrow \text{Dist}[2/(2 - n), \text{Ssubst}[\text{Int}[1/(1 - a*x^2), x], x, x/\text{Sqrt}[a*x^2 + b*x^n]], x] /;$ FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

$\text{Int}[(a_*) + (b_*)(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{ax^2 + bx^3}} dx = -\left(2 \operatorname{Subst}\left(\int \frac{1}{1 - ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)\right)$$

$$= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0089751, size = 46, normalized size = 1.53

$$-\frac{2x\sqrt{a+bx} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^3], x]

[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.004, size = 39, normalized size = 1.3

$$-2 \frac{x\sqrt{bx+a}}{\sqrt{bx^3+ax^2}\sqrt{a}} \operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a*x^2)^(1/2), x)

[Out] -2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)^(1/2)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)

Fricas [A] time = 0.803761, size = 171, normalized size = 5.7

$$\left[\frac{\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)

Giac [A] time = 1.16818, size = 61, normalized size = 2.03

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*sgn(x))
```

$$3.257 \quad \int \frac{1}{x\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=54

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}} - \frac{\sqrt{ax^2+bx^3}}{ax^2}$$

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3]/(a*x^2)) + (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/a^{(3/2)}$

Rubi [A] time = 0.0489975, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2008, 206}

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}} - \frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[a*x^2 + b*x^3]), x]$

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3]/(a*x^2)) + (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/a^{(3/2)}$

Rule 2025

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[m+j*p+1, 0]$

Rule 2008

$\text{Int}[1/\text{Sqrt}[(a_*)(x_*)^2 + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[2/(2-n), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x/\text{Sqrt}[a*x^2 + b*x^n]], x] /; \text{FreeQ}\{a, b, n\}, x] \&\& \text{NeQ}[n, 2]$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{ax^2+bx^3}} dx &= -\frac{\sqrt{ax^2+bx^3}}{ax^2} - \frac{b \int \frac{1}{\sqrt{ax^2+bx^3}} dx}{2a} \\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a} \\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2} + \frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0445731, size = 66, normalized size = 1.22

$$\frac{2bx(a+bx) \left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{2\sqrt{\frac{bx}{a}+1}} - \frac{a}{2bx} \right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b*x*(a + b*x)*(-a/(2*b*x) + ArcTanh[Sqrt[1 + (b*x)/a]]/(2*Sqrt[1 + (b*x)
/a]]))/(a^2*Sqrt[x^2*(a + b*x)])
```

Maple [A] time = 0.007, size = 55, normalized size = 1.

$$-\sqrt{bx+a} \left(a^{\frac{3}{2}} \sqrt{bx+a} - \operatorname{Artanh}\left(\sqrt{bx+a} \frac{1}{\sqrt{a}}\right) xab \right) \frac{1}{\sqrt{bx^3+ax^2}} a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

[Out] $-(b*x+a)^{(1/2)}*(a^{(3/2)}*(b*x+a)^{(1/2)}-\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)}))*x*a*b$
 $/(b*x^3+a*x^2)^{(1/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)`

Fricas [A] time = 0.828951, size = 289, normalized size = 5.35

$$\left[\frac{\sqrt{a}bx^2 \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\sqrt{bx^3+ax^2}a}{2a^2x^2}, -\frac{\sqrt{-a}bx^2 \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}a}{a^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**3+a*x**2)**(1/2),x)`

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.258 \quad \int \frac{1}{x^2 \sqrt{ax^2 + bx^3}} dx$$

Optimal. Leaf size=87

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*\text{Sqrt}[a*x^2 + b*x^3])/(4*a^2*x^2) - (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*a^{(5/2)})$

Rubi [A] time = 0.0911053, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2008, 206}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[a*x^2 + b*x^3]),x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*\text{Sqrt}[a*x^2 + b*x^3])/(4*a^2*x^2) - (3*b^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(4*a^{(5/2)})$

Rule 2025

$\text{Int}[\left((c_.)*(x_)\right)^{(m_)}*((a_.)*(x_)\right)^{(j_)} + (b_.)*(x_)\right)^{(n_)}\right)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{LtQ}[m+j*p+1, 0]$

Rule 2008

$\text{Int}[1/\text{Sqrt}[(a_.)*(x_)\right)^2 + (b_.)*(x_)\right)^{(n_)}], x_Symbol] \rightarrow \text{Dist}[2/(2-n), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x/\text{Sqrt}[a*x^2 + b*x^n]], x] /; \text{FreeQ}\{a, b, n\}, x] \&\& \text{NeQ}[n, 2]$

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{ax^2 + bx^3}} dx &= -\frac{\sqrt{ax^2 + bx^3}}{2ax^3} - \frac{(3b) \int \frac{1}{x \sqrt{ax^2 + bx^3}} dx}{4a} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3}}{4a^2x^2} + \frac{(3b^2) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{8a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3}}{4a^2x^2} - \frac{(3b^2) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{4a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3}}{4a^2x^2} - \frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0103942, size = 40, normalized size = 0.46

$$-\frac{2b^2 \sqrt{x^2(a+bx)} {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{bx}{a} + 1\right)}{a^3 x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*b^2*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/(a^3*x)
```

Maple [A] time = 0.007, size = 77, normalized size = 0.9

$$-\frac{1}{4x} \sqrt{bx+a} \left(2a^{5/2} \sqrt{bx+a} - 3a^{3/2} \sqrt{bx+axb} + 3 \operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) ab^2x^2 \right) \frac{1}{\sqrt{bx^3+ax^2}} a^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```


[Out] $-1/4*(b*x+a)^{(1/2)}*(2*a^{(5/2)}*(b*x+a)^{(1/2)}-3*a^{(3/2)}*(b*x+a)^{(1/2)}*x*b+3*a$
 $rctanh((b*x+a)^{(1/2)}/a^{(1/2)})*a*b^2*x^2)/x/(b*x^3+a*x^2)^{(1/2)}/a^{(7/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)`

Fricas [A] time = 0.901449, size = 347, normalized size = 3.99

$$\left[\frac{3\sqrt{ab^2x^3} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2\sqrt{bx^3+ax^2}(3abx-2a^2)}{8a^3x^3}, \frac{3\sqrt{-ab^2x^3} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}(3abx-2a^2)}{4a^3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $[1/8*(3*\sqrt{a}*b^2*x^3*\log((b*x^2 + 2*a*x - 2*\sqrt{b*x^3 + a*x^2})*\sqrt{a}))/x^2 + 2*\sqrt{b*x^3 + a*x^2}*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*\sqrt{-a})*b^2*x^3*\arctan(\sqrt{b*x^3 + a*x^2}*\sqrt{-a}/(a*x)) + \sqrt{b*x^3 + a*x^2}*(3*a*b*x - 2*a^2))/(a^3*x^3)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.259 \quad \int \frac{1}{x^3 \sqrt{ax^2 + bx^3}} dx$$

Optimal. Leaf size=115

$$-\frac{5b^2 \sqrt{ax^2 + bx^3}}{8a^3 x^2} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{7/2}} + \frac{5b \sqrt{ax^2 + bx^3}}{12a^2 x^3} - \frac{\sqrt{ax^2 + bx^3}}{3ax^4}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*\text{Sqrt}[a*x^2 + b*x^3])/(12*a^2*x^3) - (5*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(8*a^{(7/2)})$

Rubi [A] time = 0.135388, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2008, 206}

$$-\frac{5b^2 \sqrt{ax^2 + bx^3}}{8a^3 x^2} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{7/2}} + \frac{5b \sqrt{ax^2 + bx^3}}{12a^2 x^3} - \frac{\sqrt{ax^2 + bx^3}}{3ax^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^3*\text{Sqrt}[a*x^2 + b*x^3]),x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*\text{Sqrt}[a*x^2 + b*x^3])/(12*a^2*x^3) - (5*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^3]])/(8*a^{(7/2)})$

Rule 2025

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \|\| \text{GtQ}[c, 0]) \&\& \text{LtQ}[m+j*p+1, 0]$

Rule 2008

$\text{Int}[1/\text{Sqrt}[(a_*)(x_*)^2 + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[2/(2-n), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x/\text{Sqrt}[a*x^2 + b*x^n]], x] /; \text{FreeQ}\{a, b, n\}, x] \&\& \text{NeQ}[n, 2]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \sqrt{ax^2 + bx^3}} dx &= -\frac{\sqrt{ax^2 + bx^3}}{3ax^4} - \frac{(5b) \int \frac{1}{x^2 \sqrt{ax^2 + bx^3}} dx}{6a} \\
 &= -\frac{\sqrt{ax^2 + bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2 + bx^3}}{12a^2x^3} + \frac{(5b^2) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{8a^2} \\
 &= -\frac{\sqrt{ax^2 + bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2 + bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2 + bx^3}}{8a^3x^2} - \frac{(5b^3) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{16a^3} \\
 &= -\frac{\sqrt{ax^2 + bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2 + bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2 + bx^3}}{8a^3x^2} + \frac{(5b^3) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{8a^3} \\
 &= -\frac{\sqrt{ax^2 + bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2 + bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2 + bx^3}}{8a^3x^2} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{7/2}}
 \end{aligned}$$

Mathematica [C] time = 0.0102403, size = 40, normalized size = 0.35

$$\frac{2b^3 \sqrt{x^2(a + bx)} {}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; \frac{bx}{a} + 1\right)}{a^4 x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]

[Out] (2*b^3*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/(a^4*x)

Maple [A] time = 0.006, size = 95, normalized size = 0.8

$$-\frac{1}{24x^2} \sqrt{bx + a} \left(15 a^{3/2} \sqrt{bx + ax^2b^2} - 15 \text{Artanh}\left(\frac{\sqrt{bx + a}}{\sqrt{a}}\right) x^3 ab^3 - 10 a^{5/2} \sqrt{bx + axb} + 8 a^{7/2} \sqrt{bx + a} \right) \frac{1}{\sqrt{bx^3 + ax^2}} a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^3+a*x^2)^(1/2),x)`

[Out]
$$-1/24/x^2*(b*x+a)^{(1/2)}*(15*a^{(3/2)}*(b*x+a)^{(1/2)}*x^2*b^2-15*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*x^3*a*b^3-10*a^{(5/2)}*(b*x+a)^{(1/2)}*x*b+8*a^{(7/2)}*(b*x+a)^{(1/2)})/(b*x^3+a*x^2)^{(1/2)}/a^{(9/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)`

Fricas [A] time = 0.809012, size = 402, normalized size = 3.5

$$\left[\frac{15\sqrt{a}b^3x^4 \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2(15ab^2x^2 - 10a^2bx + 8a^3)\sqrt{bx^3+ax^2}}{48a^4x^4}, -\frac{15\sqrt{-ab^3x^4} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{24a^4x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out]
$$\left[\frac{1}{48}*(15*\sqrt{a}*b^3*x^4*\log((b*x^2 + 2*a*x + 2*\sqrt{b*x^3 + a*x^2})*\sqrt{a}))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*\sqrt{b*x^3 + a*x^2})/(a^4*x^4), -1/24*(15*\sqrt{-a}*b^3*x^4*\arctan(\sqrt{b*x^3 + a*x^2}*\sqrt{-a}/(a*x)) + (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*\sqrt{b*x^3 + a*x^2})/(a^4*x^4) \right]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.260 \quad \int \frac{x^6}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=98

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} - \frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

[Out] $(-2*x^4)/(b*\text{Sqrt}[a*x^2 + b*x^3]) - (16*a*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^3) + (32*a^2*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^2)$

Rubi [A] time = 0.150882, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 1588}

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} - \frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6/(a*x^2 + b*x^3)^{(3/2)}, x]$

[Out] $(-2*x^4)/(b*\text{Sqrt}[a*x^2 + b*x^3]) - (16*a*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^3) + (32*a^2*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*\text{Sqrt}[a*x^2 + b*x^3])/(5*b^2)$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)
*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

$(n - j)]], 0] \&\& \text{NeQ}[m + j*p + 1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 1588

$\text{Int}[(\text{Pp}_*)(\text{Qq}_*)^{(m_.)}, x_Symbol] \text{ :> With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p]*x^{(p - q + 1)}*\text{Qq}^{(m + 1)})/((p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]), x] /; \text{NeQ}[p + m*q + 1, 0] \&\& \text{EqQ}[(p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]*\text{Pp}, \text{Coeff}[\text{Pp}, x, p]*x^{(p - q)}*((p - q + 1)*\text{Qq} + (m + 1)*x*\text{D}[\text{Qq}, x])]] /; \text{FreeQ}[m, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(ax^2 + bx^3)^{3/2}} dx &= -\frac{2x^4}{b\sqrt{ax^2 + bx^3}} + \frac{6 \int \frac{x^3}{\sqrt{ax^2 + bx^3}} dx}{b} \\ &= -\frac{2x^4}{b\sqrt{ax^2 + bx^3}} + \frac{12x\sqrt{ax^2 + bx^3}}{5b^2} - \frac{(24a) \int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx}{5b^2} \\ &= -\frac{2x^4}{b\sqrt{ax^2 + bx^3}} - \frac{16a\sqrt{ax^2 + bx^3}}{5b^3} + \frac{12x\sqrt{ax^2 + bx^3}}{5b^2} + \frac{(16a^2) \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{5b^3} \\ &= -\frac{2x^4}{b\sqrt{ax^2 + bx^3}} - \frac{16a\sqrt{ax^2 + bx^3}}{5b^3} + \frac{32a^2\sqrt{ax^2 + bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2 + bx^3}}{5b^2} \end{aligned}$$

Mathematica [A] time = 0.0242158, size = 50, normalized size = 0.51

$$\frac{2x(8a^2bx + 16a^3 - 2ab^2x^2 + b^3x^3)}{5b^4\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]

[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.005, size = 56, normalized size = 0.6

$$\frac{(2bx + 2a)(x^3b^3 - 2ab^2x^2 + 8a^2xb + 16a^3)x^3}{5b^4} (bx^3 + ax^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^3+a*x^2)^(3/2),x)`

[Out] $2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3/2)$

Maxima [A] time = 1.15932, size = 55, normalized size = 0.56

$$\frac{2(b^3x^3 - 2ab^2x^2 + 8a^2bx + 16a^3)}{5\sqrt{bx + ab^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] $2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)$

Fricas [A] time = 0.905986, size = 122, normalized size = 1.24

$$\frac{2(b^3x^3 - 2ab^2x^2 + 8a^2bx + 16a^3)\sqrt{bx^3 + ax^2}}{5(b^5x^2 + ab^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] $2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x^2 + a*b^4*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x^2(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x^2)^(3/2), x)
```

$$3.261 \quad \int \frac{x^5}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x} - \frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

[Out] $(-2*x^3)/(b*\text{Sqrt}[a*x^2 + b*x^3]) + (8*\text{Sqrt}[a*x^2 + b*x^3])/(3*b^2) - (16*a*\text{Sqrt}[a*x^2 + b*x^3])/(3*b^3*x)$

Rubi [A] time = 0.105215, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2015, 2016, 1588}

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x} - \frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]

[Out] $(-2*x^3)/(b*\text{Sqrt}[a*x^2 + b*x^3]) + (8*\text{Sqrt}[a*x^2 + b*x^3])/(3*b^2) - (16*a*\text{Sqrt}[a*x^2 + b*x^3])/(3*b^3*x)$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)
*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

$(n - j)]], 0] \&\& \text{NeQ}[m + j*p + 1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 1588

$\text{Int}[(\text{Pp}_*) * (\text{Qq}_*)^{(m_.)}, x_Symbol] \text{ :> With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p] * x^{(p - q + 1)} * \text{Qq}^{(m + 1)}) / ((p + m*q + 1) * \text{Coeff}[\text{Qq}, x, q]), x] /; \text{NeQ}[p + m*q + 1, 0] \&\& \text{EqQ}[(p + m*q + 1) * \text{Coeff}[\text{Qq}, x, q] * \text{Pp}, \text{Coeff}[\text{Pp}, x, p] * x^{(p - q)} * ((p - q + 1) * \text{Qq} + (m + 1) * x * \text{D}[\text{Qq}, x])]] /; \text{FreeQ}[m, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(ax^2 + bx^3)^{3/2}} dx &= -\frac{2x^3}{b\sqrt{ax^2 + bx^3}} + \frac{4 \int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx}{b} \\ &= -\frac{2x^3}{b\sqrt{ax^2 + bx^3}} + \frac{8\sqrt{ax^2 + bx^3}}{3b^2} - \frac{(8a) \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{3b^2} \\ &= -\frac{2x^3}{b\sqrt{ax^2 + bx^3}} + \frac{8\sqrt{ax^2 + bx^3}}{3b^2} - \frac{16a\sqrt{ax^2 + bx^3}}{3b^3x} \end{aligned}$$

Mathematica [A] time = 0.0172749, size = 39, normalized size = 0.54

$$\frac{2x(-8a^2 - 4abx + b^2x^2)}{3b^3\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.006, size = 46, normalized size = 0.6

$$-\frac{(2bx + 2a)(-b^2x^2 + 4abx + 8a^2)x^3}{3b^3} (bx^3 + ax^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^3+a*x^2)^(3/2),x)`

[Out] `-2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)`

Maxima [A] time = 1.14535, size = 41, normalized size = 0.57

$$\frac{2(b^2x^2 - 4abx - 8a^2)}{3\sqrt{bx + ab^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)`

Fricas [A] time = 0.829334, size = 99, normalized size = 1.38

$$\frac{2(b^2x^2 - 4abx - 8a^2)\sqrt{bx^3 + ax^2}}{3(b^4x^2 + ab^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] `2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(x^2(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**3+a*x**2)**(3/2),x)`

[Out] Integral($x^{5/(x^{2}(a + bx))^{3/2}}$, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^5/(b*x^3+a*x^2)^{(3/2)}$,x, algorithm="giac")

[Out] integrate($x^5/(b*x^3 + a*x^2)^{(3/2)}$, x)

$$3.262 \quad \int \frac{x^4}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=47

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x} - \frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

[Out] $(-2*x^2)/(b*\text{Sqrt}[a*x^2 + b*x^3]) + (4*\text{Sqrt}[a*x^2 + b*x^3])/(b^2*x)$

Rubi [A] time = 0.0567734, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 1588}

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x} - \frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(a*x^2 + b*x^3)^{(3/2)}, x]$

[Out] $(-2*x^2)/(b*\text{Sqrt}[a*x^2 + b*x^3]) + (4*\text{Sqrt}[a*x^2 + b*x^3])/(b^2*x)$

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
&& LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x]
/; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)
*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{(ax^2 + bx^3)^{3/2}} dx = -\frac{2x^2}{b\sqrt{ax^2 + bx^3}} + \frac{2 \int \frac{x}{\sqrt{ax^2 + bx^3}} dx}{b}$$

$$= -\frac{2x^2}{b\sqrt{ax^2 + bx^3}} + \frac{4\sqrt{ax^2 + bx^3}}{b^2x}$$

Mathematica [A] time = 0.0124749, size = 26, normalized size = 0.55

$$\frac{2x(2a + bx)}{b^2\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.003, size = 34, normalized size = 0.7

$$2 \frac{(bx + a)(bx + 2a)x^3}{b^2 (bx^3 + ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^3+a*x^2)^(3/2), x)

[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)

Maxima [A] time = 1.11266, size = 26, normalized size = 0.55

$$\frac{2(bx + 2a)}{\sqrt{bx + ab^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)

Fricas [A] time = 0.82862, size = 74, normalized size = 1.57

$$\frac{2\sqrt{bx^3 + ax^2}(bx + 2a)}{b^3x^2 + ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)

Giac [A] time = 1.13415, size = 38, normalized size = 0.81

$$\frac{2\left(\frac{1}{b} + \frac{2a}{b^2x}\right)}{\sqrt{\frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 2*(1/b + 2*a/(b^2*x))/sqrt(b/x + a/x^2)

$$3.263 \quad \int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=21

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

[Out] $(-2*x)/(b*\text{Sqrt}[a*x^2 + b*x^3])$

Rubi [A] time = 0.0175054, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1588}

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/(a*x^2 + b*x^3)^{(3/2)}, x]$

[Out] $(-2*x)/(b*\text{Sqrt}[a*x^2 + b*x^3])$

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Mathematica [A] time = 0.0059717, size = 19, normalized size = 0.9

$$-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]

[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.002, size = 27, normalized size = 1.3

$$-2 \frac{(bx + a)x^3}{b(bx^3 + ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x^2)^(3/2),x)

[Out] -2*(b*x+a)*x^3/b/(b*x^3+a*x^2)^(3/2)

Maxima [A] time = 1.07839, size = 16, normalized size = 0.76

$$-\frac{2}{\sqrt{bx + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] -2/(sqrt(b*x + a)*b)

Fricas [A] time = 0.83839, size = 57, normalized size = 2.71

$$-\frac{2\sqrt{bx^3 + ax^2}}{b^2x^2 + abx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] $-2\sqrt{bx^3 + ax^2}/(b^2x^2 + abx)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2(a+bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x**3/(x**2*(a + b*x))**(3/2), x)`

Giac [A] time = 1.18193, size = 50, normalized size = 2.38

$$\frac{2}{\left(\sqrt{a}\left(\sqrt{\frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right) - b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")`

[Out] `2/((sqrt(a)*(sqrt(b/x + a/x^2) - sqrt(a)/x) - b)*sqrt(a))`

$$3.264 \quad \int \frac{x^2}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=52

$$\frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)

Rubi [A] time = 0.0583374, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2023, 2008, 206}

$$\frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)

Rule 2023

```
Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax^2 + bx^3)^{3/2}} dx &= \frac{2x}{a\sqrt{ax^2 + bx^3}} + \frac{\int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{a} \\ &= \frac{2x}{a\sqrt{ax^2 + bx^3}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{a} \\ &= \frac{2x}{a\sqrt{ax^2 + bx^3}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{a^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.0085071, size = 35, normalized size = 0.67

$$\frac{2x {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx}{a} + 1\right)}{a\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2), x]

[Out] (2*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.007, size = 53, normalized size = 1.

$$2 \frac{x^3 (bx + a)}{(bx^3 + ax^2)^{3/2} a^{5/2}} \left(a^{3/2} - \operatorname{Artanh}\left(\frac{\sqrt{bx + a}}{\sqrt{a}}\right) a\sqrt{bx + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x^2)^(3/2), x)

[Out] $2*x^3*(b*x+a)*(a^{(3/2)}-\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)}))*a*(b*x+a)^{(1/2)}/(b*x^3+a*x^2)^{(3/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)`

Fricas [A] time = 0.801286, size = 333, normalized size = 6.4

$$\left[\frac{(bx^2 + ax)\sqrt{a} \log\left(\frac{bx^2 + 2ax - 2\sqrt{bx^3 + ax^2}\sqrt{a}}{x^2}\right) + 2\sqrt{bx^3 + ax^2}a}{a^2bx^2 + a^3x}, \frac{2\left((bx^2 + ax)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3 + ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^3 + ax^2}a\right)}{a^2bx^2 + a^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] `[((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.265 \quad \int \frac{x}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=75

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}} + \frac{2}{a\sqrt{ax^2+bx^3}}$$

[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)

Rubi [A] time = 0.0857002, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2023, 2025, 2008, 206}

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}} + \frac{2}{a\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3)^(3/2), x]

[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(ax^2 + bx^3)^{3/2}} dx &= \frac{2}{a\sqrt{ax^2 + bx^3}} + \frac{3 \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{a} \\ &= \frac{2}{a\sqrt{ax^2 + bx^3}} - \frac{3\sqrt{ax^2 + bx^3}}{a^2x^2} - \frac{(3b) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{2a^2} \\ &= \frac{2}{a\sqrt{ax^2 + bx^3}} - \frac{3\sqrt{ax^2 + bx^3}}{a^2x^2} + \frac{(3b) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{a^2} \\ &= \frac{2}{a\sqrt{ax^2 + bx^3}} - \frac{3\sqrt{ax^2 + bx^3}}{a^2x^2} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{a^{5/2}} \end{aligned}$$

Mathematica [C] time = 0.0087374, size = 36, normalized size = 0.48

$$\frac{2bx {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{bx}{a} + 1\right)}{a^2\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2), x]
```

```
[Out] (-2*b*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[x^2*(a + b*
x)])
```

Maple [A] time = 0.012, size = 62, normalized size = 0.8

$$x^2 (bx + a) \left(3 \operatorname{Artanh} \left(\frac{\sqrt{bx + a}}{\sqrt{a}} \right) \sqrt{bx + a} - 3bx\sqrt{a} - a^{\frac{3}{2}} \right) (bx^3 + ax^2)^{-\frac{3}{2}} a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a*x^2)^(3/2),x)

[Out] $x^2*(b*x+a)*(3*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*(b*x+a)^{(1/2)}*x*b-3*b*x*a^{(1/2)}-a^{(3/2)})/(b*x^3+a*x^2)^{(3/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)

Fricas [A] time = 0.804222, size = 402, normalized size = 5.36

$$\left[\frac{3(b^2x^3 + abx^2)\sqrt{a} \log\left(\frac{bx^2+2ax+2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\sqrt{bx^3+ax^2}(3abx+a^2)}{2(a^3bx^3+a^4x^2)}, -\frac{3(b^2x^3+abx^2)\sqrt{-a} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{a^3bx^3+a^4x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] $[1/2*(3*(b^2*x^3 + a*b*x^2)*\operatorname{sqrt}(a)*\log((b*x^2 + 2*a*x + 2*\operatorname{sqrt}(b*x^3 + a*x^2)*\operatorname{sqrt}(a))/x^2) - 2*\operatorname{sqrt}(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2), -(3*(b^2*x^3 + a*b*x^2)*\operatorname{sqrt}(-a)*\operatorname{arctan}(\operatorname{sqrt}(b*x^3 + a*x^2)*\operatorname{sqrt}(-a))$

$/(a*x)) + \text{sqrt}(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2(a+bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.266 \quad \int \frac{1}{(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=110

$$-\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{2}{ax\sqrt{ax^2+bx^3}}$$

[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*Sqrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(7/2))

Rubi [A] time = 0.105175, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2006, 2025, 2008, 206}

$$-\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{2}{ax\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^(-3/2), x]

[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*Sqrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(7/2))

Rule 2006

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j + b*x^n)^(p+1)/(a*(n-j)*(p+1)*x^(j-1)), x] + Dist[(n*p + n - j + 1)/(a*(n-j)*(p+1)), Int[(a*x^j + b*x^n)^(p+1)/x^j, x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]

Rule 2025

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(m+j*p+1)), x] - Dist[(b*(m+n*p+n-j+1))/(a*c^(n-j)*(m+j*p+1)), Int[(c*x)^(m+n-j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2 + bx^3)^{3/2}} dx &= \frac{2}{ax\sqrt{ax^2 + bx^3}} + \frac{5 \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx}{a} \\
&= \frac{2}{ax\sqrt{ax^2 + bx^3}} - \frac{5\sqrt{ax^2 + bx^3}}{2a^2x^3} - \frac{(15b) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{4a^2} \\
&= \frac{2}{ax\sqrt{ax^2 + bx^3}} - \frac{5\sqrt{ax^2 + bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2 + bx^3}}{4a^3x^2} + \frac{(15b^2) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{8a^3} \\
&= \frac{2}{ax\sqrt{ax^2 + bx^3}} - \frac{5\sqrt{ax^2 + bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2 + bx^3}}{4a^3x^2} - \frac{(15b^2) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^3}}\right)}{4a^3} \\
&= \frac{2}{ax\sqrt{ax^2 + bx^3}} - \frac{5\sqrt{ax^2 + bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2 + bx^3}}{4a^3x^2} - \frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0072385, size = 38, normalized size = 0.35

$$\frac{2b^2x {}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; \frac{bx}{a} + 1\right)}{a^3\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2), x]
```

[Out] $(2*b^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*sqrt[x^2*(a + b*x)])$

Maple [A] time = 0.012, size = 76, normalized size = 0.7

$$-\frac{x(bx+a)}{4} \left(15 \operatorname{Arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right) \sqrt{bx+ax^2b^2-5a^{3/2}xb-15x^2b^2\sqrt{a}+2a^{5/2}} \right) (bx^3+ax^2)^{-\frac{3}{2}} a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^3+a*x^2)^(3/2),x)`

[Out] $-1/4*x*(b*x+a)*(15*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*(b*x+a)^{(1/2)}*x^2*b^2-5*a^{(3/2)}*x*b-15*x^2*b^2*a^{(1/2)}+2*a^{(5/2)})/(b*x^3+a*x^2)^{(3/2)}/a^{(7/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x^2)^(-3/2), x)`

Fricas [A] time = 0.901782, size = 466, normalized size = 4.24

$$\left[\frac{15(b^3x^4 + ab^2x^3)\sqrt{a} \log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) + 2(15ab^2x^2 + 5a^2bx - 2a^3)\sqrt{bx^3+ax^2} - 15(b^3x^4 + ab^2x^3)\sqrt{-a} \operatorname{arctan}\left(\frac{\sqrt{bx^3+ax^2}}{\sqrt{-a}}\right)}{8(a^4bx^4 + a^5x^3)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

```
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2))*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^2 + bx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```


$$3.267 \quad \int \frac{1}{x(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=138

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))

Rubi [A] time = 0.186272, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2008, 206}

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]

[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)
*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```

+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2008

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3)^{3/2}} dx &= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} + \frac{7 \int \frac{1}{x^3\sqrt{ax^2 + bx^3}} dx}{a} \\
&= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} - \frac{7\sqrt{ax^2 + bx^3}}{3a^2x^4} - \frac{(35b) \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx}{6a^2} \\
&= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} - \frac{7\sqrt{ax^2 + bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2 + bx^3}}{12a^3x^3} + \frac{(35b^2) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{8a^3} \\
&= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} - \frac{7\sqrt{ax^2 + bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2 + bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2 + bx^3}}{8a^4x^2} - \frac{(35b^3) \int \frac{1}{\sqrt{ax^2 + bx^3}} dx}{16a^4} \\
&= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} - \frac{7\sqrt{ax^2 + bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2 + bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2 + bx^3}}{8a^4x^2} + \frac{(35b^3) \text{Subst}\left(\int \frac{1}{1-ax^2} dx\right)}{8a^4} \\
&= \frac{2}{ax^2\sqrt{ax^2 + bx^3}} - \frac{7\sqrt{ax^2 + bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2 + bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2 + bx^3}}{8a^4x^2} + \frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^3}}\right)}{8a^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.0087413, size = 38, normalized size = 0.28

$$-\frac{2b^3x {}_2F_1\left(-\frac{1}{2}, 4; \frac{1}{2}; \frac{bx}{a} + 1\right)}{a^4\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]

[Out] (-2*b^3*x*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (b*x)/a])/(a^4*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.013, size = 86, normalized size = 0.6

$$-\frac{bx+a}{24} \left(-105 \operatorname{Arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right) \sqrt{bx+ax^3} b^3 - 14 a^{5/2} x b + 35 a^{3/2} x^2 b^2 + 105 b^3 x^3 \sqrt{a} + 8 a^{7/2} \right) (bx^3 + ax^2)^{-\frac{3}{2}} a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^3+a*x^2)^(3/2),x)

[Out] -1/24*(b*x+a)*(-105*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^3*b^3-14*a^(5/2)*x*b+35*a^(3/2)*x^2*b^2+105*b^3*x^3*a^(1/2)+8*a^(7/2))/(b*x^3+a*x^2)^(3/2)/a^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)

Fricas [A] time = 0.816872, size = 524, normalized size = 3.8

$$\left[\frac{105 (b^4 x^5 + ab^3 x^4) \sqrt{a} \log \left(\frac{bx^2 + 2ax + 2\sqrt{bx^3 + ax^2} \sqrt{a}}{x^2} \right) - 2 (105 ab^3 x^3 + 35 a^2 b^2 x^2 - 14 a^3 bx + 8 a^4) \sqrt{bx^3 + ax^2} - 105 (b^4 x^5 - 105 ab^3 x^3 + 35 a^2 b^2 x^2 - 14 a^3 bx + 8 a^4) \sqrt{bx^3 + ax^2}}{48 (a^5 bx^5 + a^6 x^4)}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2))*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left(x^2 (a + bx)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)

$$3.268 \quad \int \frac{1}{x^2(ax^2+bx^3)^{3/2}} dx$$

Optimal. Leaf size=166

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{315b^4 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))

Rubi [A] time = 0.232448, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2023, 2025, 2008, 206}

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{315b^4 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]

[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1]
]
```

Rule 2025

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2008

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax^2 + bx^3)^{3/2}} dx &= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} + \frac{9 \int \frac{1}{x^4\sqrt{ax^2 + bx^3}} dx}{a} \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} - \frac{(63b) \int \frac{1}{x^3\sqrt{ax^2 + bx^3}} dx}{8a^2} \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2 + bx^3}}{8a^3x^4} + \frac{(105b^2) \int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx}{16a^3} \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2 + bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2 + bx^3}}{32a^4x^3} - \frac{(315b^3) \int \frac{1}{x\sqrt{ax^2 + bx^3}} dx}{64a^4} \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2 + bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2 + bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2 + bx^3}}{64a^5x^2} + \dots \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2 + bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2 + bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2 + bx^3}}{64a^5x^2} - \dots \\
&= \frac{2}{ax^3\sqrt{ax^2 + bx^3}} - \frac{9\sqrt{ax^2 + bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2 + bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2 + bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2 + bx^3}}{64a^5x^2} - \dots
\end{aligned}$$

Mathematica [C] time = 0.0099285, size = 38, normalized size = 0.23

$$\frac{2b^4x {}_2F_1\left(-\frac{1}{2}, 5; \frac{1}{2}; \frac{bx}{a} + 1\right)}{a^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)), x]

[Out] (2*b^4*x*Hypergeometric2F1[-1/2, 5, 1/2, 1 + (b*x)/a])/(a^5*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.014, size = 100, normalized size = 0.6

$$-\frac{bx+a}{64x} \left(315\sqrt{bx+a} \operatorname{Arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) x^4 b^4 - 24 a^{7/2} x b + 42 a^{5/2} x^2 b^2 - 105 a^{3/2} x^3 b^3 - 315 b^4 x^4 \sqrt{a} + 16 a^{9/2} \right) (bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a*x^2)^(3/2), x)

[Out] -1/64*(b*x+a)*(315*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*x^4*b^4-24*a^(7/2)*x*b+42*a^(5/2)*x^2*b^2-105*a^(3/2)*x^3*b^3-315*b^4*x^4*a^(1/2)+16*a^(9/2))/x/(b*x^3+a*x^2)^(3/2)/a^(11/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)

Fricas [A] time = 0.821706, size = 575, normalized size = 3.46

$$\frac{315 (b^5 x^6 + ab^4 x^5) \sqrt{a} \log\left(\frac{bx^2 + 2ax - 2\sqrt{bx^3 + ax^2}\sqrt{a}}{x^2}\right) + 2 (315 ab^4 x^4 + 105 a^2 b^3 x^3 - 42 a^3 b^2 x^2 + 24 a^4 bx - 16 a^5) \sqrt{bx^3 + ax^2}}{128 (a^6 bx^6 + a^7 x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (x^2 (a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)

[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)

$$3.269 \quad \int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=125

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))

Rubi [A] time = 0.169449, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]

[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],

$x] /; \text{FreeQ}[\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x^{7/2}}{\sqrt{ax^2 + bx^3}} dx &= \frac{x^{3/2} \sqrt{ax^2 + bx^3}}{3b} - \frac{(5a) \int \frac{x^{5/2}}{\sqrt{ax^2 + bx^3}} dx}{6b} \\ &= -\frac{5a \sqrt{x} \sqrt{ax^2 + bx^3}}{12b^2} + \frac{x^{3/2} \sqrt{ax^2 + bx^3}}{3b} + \frac{(5a^2) \int \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}} dx}{8b^2} \\ &= \frac{5a^2 \sqrt{ax^2 + bx^3}}{8b^3 \sqrt{x}} - \frac{5a \sqrt{x} \sqrt{ax^2 + bx^3}}{12b^2} + \frac{x^{3/2} \sqrt{ax^2 + bx^3}}{3b} - \frac{(5a^3) \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^3}} dx}{16b^3} \\ &= \frac{5a^2 \sqrt{ax^2 + bx^3}}{8b^3 \sqrt{x}} - \frac{5a \sqrt{x} \sqrt{ax^2 + bx^3}}{12b^2} + \frac{x^{3/2} \sqrt{ax^2 + bx^3}}{3b} - \frac{(5a^3) \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}} \right)}{8b^3} \\ &= \frac{5a^2 \sqrt{ax^2 + bx^3}}{8b^3 \sqrt{x}} - \frac{5a \sqrt{x} \sqrt{ax^2 + bx^3}}{12b^2} + \frac{x^{3/2} \sqrt{ax^2 + bx^3}}{3b} - \frac{5a^3 \tanh^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2 + bx^3}} \right)}{8b^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.108556, size = 104, normalized size = 0.83

$$\frac{\sqrt{x^2(a+bx)} \left(\sqrt{b} \sqrt{x} \sqrt{\frac{bx}{a} + 1} (15a^2 - 10abx + 8b^2x^2) - 15a^{5/2} \sinh^{-1} \left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right) \right)}{24b^{7/2} x \sqrt{\frac{bx}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3], x]

[Out] (Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(24*b^(7/2)*x*Sqrt[1 + (b*x)/a])

Maple [A] time = 0.006, size = 103, normalized size = 0.8

$$-\frac{1}{48}\sqrt{x}\left(-16b^{9/2}x^4 + 4b^{7/2}x^3a - 10b^{5/2}x^2a^2 - 30b^{3/2}xa^3 + 15\sqrt{x(bx+a)}\ln\left(1/2\frac{2\sqrt{bx^2+ax}\sqrt{b}+2bx+a}{\sqrt{b}}\right)a^3b\right)\frac{1}{\sqrt{bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x)

[Out]
$$-1/48*x^{(1/2)}*(-16*b^{(9/2)}*x^4+4*b^{(7/2)}*x^3*a-10*b^{(5/2)}*x^2*a^2-30*b^{(3/2)}*x*a^3+15*(x*(b*x+a))^{(1/2)}*\ln(1/2*(2*(b*x^2+a*x)^{(1/2)}*b^{(1/2)}+2*b*x+a)/b^{(1/2)})*a^3*b)/(b*x^3+a*x^2)^{(1/2)}/b^{(9/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)

Fricas [A] time = 0.792716, size = 428, normalized size = 3.42

$$\left[\frac{15a^3\sqrt{bx}\log\left(\frac{2bx^2+ax-2\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right) + 2(8b^3x^2 - 10ab^2x + 15a^2b)\sqrt{bx^3+ax^2}\sqrt{x}}{48b^4x}, \frac{15a^3\sqrt{-bx}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{48b^4x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out]
$$[1/48*(15*a^3*\sqrt{b}*x*\log((2*b*x^2 + a*x - 2*\sqrt{b*x^3 + a*x^2})*\sqrt{b}*\sqrt{x}))/x + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*\sqrt{b*x^3 + a*x^2}*\sqrt{b*x^3 + a*x^2}*\arctan(x)/(b^4*x), 1/24*(15*a^3*\sqrt{-b})*x*\arctan(\sqrt{b*x^3 + a*x^2}*\sqrt{-b})/$$

$(b*x^{(3/2)}) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*\text{sqrt}(b*x^3 + a*x^2)*\text{sqrt}(x)/(b^4*x]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)

Giac [A] time = 1.30755, size = 86, normalized size = 0.69

$$\frac{1}{24} \sqrt{bx+a} \left(2x \left(\frac{4x}{b} - \frac{5a}{b^2} \right) + \frac{15a^2}{b^3} \right) \sqrt{x} + \frac{5a^3 \log \left(\left| -\sqrt{b}\sqrt{x} + \sqrt{bx+a} \right| \right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)

$$3.270 \quad \int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=95

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}$$

[Out] $(-3*a*\text{Sqrt}[a*x^2 + b*x^3])/(4*b^2*\text{Sqrt}[x]) + (\text{Sqrt}[x]*\text{Sqrt}[a*x^2 + b*x^3])/(2*b) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x^2 + b*x^3]])/(4*b^{(5/2)})$

Rubi [A] time = 0.125441, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2024, 2029, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(5/2)}/\text{Sqrt}[a*x^2 + b*x^3], x]$

[Out] $(-3*a*\text{Sqrt}[a*x^2 + b*x^3])/(4*b^2*\text{Sqrt}[x]) + (\text{Sqrt}[x]*\text{Sqrt}[a*x^2 + b*x^3])/(2*b) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x^{(3/2)})/\text{Sqrt}[a*x^2 + b*x^3]])/(4*b^{(5/2)})$

Rule 2024

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a*x^j + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n-j)}*(m+j*p-n+j+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-(n-j))}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \|\| \text{GtQ}[c, 0]) \&\& \text{GtQ}[m+j*p+1-n+j, 0] \&\& \text{NeQ}[m+n*p+1, 0]$

Rule 2029

$\text{Int}[(x_*)^{(m_*)}/\text{Sqrt}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[-2/(n-j), \text{Subst}[\text{Int}[1/(1-a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /; \text{FreeQ}\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2-1] \&\& \text{NeQ}[n, j]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^{5/2}}{\sqrt{ax^2 + bx^3}} dx &= \frac{\sqrt{x}\sqrt{ax^2 + bx^3}}{2b} - \frac{(3a) \int \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}} dx}{4b} \\ &= -\frac{3a\sqrt{ax^2 + bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2 + bx^3}}{2b} + \frac{(3a^2) \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^3}} dx}{8b^2} \\ &= -\frac{3a\sqrt{ax^2 + bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2 + bx^3}}{2b} + \frac{(3a^2) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}}\right)}{4b^2} \\ &= -\frac{3a\sqrt{ax^2 + bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2 + bx^3}}{2b} + \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2 + bx^3}}\right)}{4b^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.0519435, size = 90, normalized size = 0.95

$$\frac{\sqrt{bx^{3/2}}(-3a^2 - abx + 2b^2x^2) + 3a^{5/2}x\sqrt{\frac{bx}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3], x]

[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.005, size = 92, normalized size = 1.

$$\frac{1}{8}\sqrt{x}\left(4b^{7/2}x^3 - 2b^{5/2}x^2a - 6b^{3/2}xa^2 + 3\sqrt{x(bx + a)}\ln\left(\frac{1}{2}\frac{2\sqrt{bx^2 + ax}\sqrt{b} + 2bx + a}{\sqrt{b}}\right)a^2b\right)\frac{1}{\sqrt{bx^3 + ax^2}}b^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x)`

[Out] $\frac{1}{8}x^{1/2}(4b^{7/2}x^3-2b^{5/2}x^2a-6b^{3/2}xa^2+3(x(bx+a))^{1/2})\ln\left(\frac{1}{2}(2(bx^2+ax)^{1/2}b^{1/2}+2bx+a)/b^{1/2}\right)a^2b/(bx^3+ax^2)^{1/2}/b^{7/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)`

Fricas [A] time = 0.830067, size = 375, normalized size = 3.95

$$\left[\frac{3a^2\sqrt{bx} \log\left(\frac{2bx^2+ax+2\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right) + 2\sqrt{bx^3+ax^2}(2b^2x-3ab)\sqrt{x}}{8b^3x}, \frac{3a^2\sqrt{-bx} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right) - \sqrt{bx^3+ax^2}}{4b^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{8}(3a^2\sqrt{b}x\log((2bx^2+ax+2\sqrt{bx^3+ax^2})\sqrt{b}\sqrt{x}))/x + 2\sqrt{bx^3+ax^2}(2b^2x-3ab)\sqrt{x}}{(b^3x)}, \frac{-1}{4}(3a^2\sqrt{-b}x\arctan(\sqrt{bx^3+ax^2}\sqrt{-b}/(bx^{3/2}))) - \sqrt{bx^3+ax^2}(2b^2x-3ab)\sqrt{x}}{(b^3x)} \right]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)

Giac [A] time = 1.29259, size = 70, normalized size = 0.74

$$\frac{1}{4} \sqrt{bx+a} \sqrt{x} \left(\frac{2x}{b} - \frac{3a}{b^2} \right) - \frac{3a^2 \log \left(\left| -\sqrt{b} \sqrt{x} + \sqrt{bx+a} \right| \right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)

$$3.271 \quad \int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=60

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)

Rubi [A] time = 0.0833389, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3], x]

[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x]
  - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*
  (a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] &&
  (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[-2/(n - j),
  Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}} dx &= \frac{\sqrt{ax^2 + bx^3}}{b\sqrt{x}} - \frac{a \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^3}} dx}{2b} \\ &= \frac{\sqrt{ax^2 + bx^3}}{b\sqrt{x}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}}\right)}{b} \\ &= \frac{\sqrt{ax^2 + bx^3}}{b\sqrt{x}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2 + bx^3}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0392275, size = 73, normalized size = 1.22

$$\frac{\sqrt{bx^{3/2}}(a + bx) - a^{3/2}x\sqrt{\frac{bx}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3], x]

[Out] (Sqrt[b]*x^(3/2)*(a + b*x) - a^(3/2)*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^2*(a + b*x)])

Maple [A] time = 0.005, size = 79, normalized size = 1.3

$$\frac{1}{2}\sqrt{x}\left(2b^{5/2}x^2 + 2b^{3/2}xa - a\sqrt{x(bx + a)}\ln\left(\frac{1}{2}\left(2\sqrt{bx^2 + ax}\sqrt{b} + 2bx + a\right)\frac{1}{\sqrt{b}}\right)b\right)\frac{1}{\sqrt{bx^3 + ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2), x)

[Out] $\frac{1}{2}x^{1/2}(2b^{5/2}x^2+2b^{3/2}xa-a(x(bx+a))^{1/2})\ln(1/2(2(bx^2+ax)^{1/2}b^{1/2}+2bx+a)/b^{1/2})/b^{5/2}+(bx^3+ax^2)^{1/2}/b^{5/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)`

Fricas [A] time = 0.885123, size = 315, normalized size = 5.25

$$\left[\frac{a\sqrt{bx} \log\left(\frac{2bx^2+ax-2\sqrt{bx^3+ax^2}\sqrt{bx}}{x}\right) + 2\sqrt{bx^3+ax^2}b\sqrt{x}}{2b^2x}, \frac{a\sqrt{-bx} \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right) + \sqrt{bx^3+ax^2}b\sqrt{x}}{b^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

Giac [A] time = 1.35451, size = 51, normalized size = 0.85

$$\frac{a \log\left(\left|-\sqrt{b}\sqrt{x} + \sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x)/b
```

$$3.272 \quad \int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=34

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]

Rubi [A] time = 0.0424737, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3], x]

[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^3}} dx = 2 \operatorname{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^2 + bx^3}} \right)$$

$$= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{ax^2 + bx^3}} \right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0161086, size = 55, normalized size = 1.62

$$\frac{2\sqrt{ax}\sqrt{\frac{bx}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}} \right)}{\sqrt{b}\sqrt{x^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[a]*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x^2*(a + b*x)])

Maple [B] time = 0.005, size = 58, normalized size = 1.7

$$\sqrt{x}\sqrt{x(bx+a)} \ln \left(\frac{1}{2} \left(2\sqrt{bx^2+ax}\sqrt{b} + 2bx + a \right) \frac{1}{\sqrt{b}} \right) \frac{1}{\sqrt{bx^3+ax^2}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x)

[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)

Fricas [A] time = 0.840399, size = 189, normalized size = 5.56

$$\left[\frac{\log\left(\frac{2bx^2+ax+2\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)

Giac [A] time = 1.39691, size = 31, normalized size = 0.91

$$\frac{2 \log\left(\left|-\sqrt{b}\sqrt{x} + \sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```


$$3.273 \quad \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=25

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(a*x^{(3/2)})$

Rubi [A] time = 0.0377593, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[x]*\text{Sqrt}[a*x^2 + b*x^3]), x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(a*x^{(3/2)})$

Rule 2014

$\text{Int}[\left((c_)*(x_)\right)^{(m_)}*((a_)*(x_)\right)^{(j_)} + (b_)*(x_)\right)^{(n_)}\right)^{(p_)}, x_Symbol]$
 $:\> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /;$
 $\text{FreeQ}\{a, b, c, j, m, n, p\}, x\} \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[m + n*p + n - j + 1, 0] \ \&\& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Mathematica [A] time = 0.0082031, size = 23, normalized size = 0.92

$$-\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]

[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))

Maple [A] time = 0.003, size = 27, normalized size = 1.1

$$-2 \frac{\sqrt{x}(bx+a)}{a\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x)

[Out] -2*x^(1/2)*(b*x+a)/a/(b*x^3+a*x^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)

Fricas [A] time = 0.713405, size = 49, normalized size = 1.96

$$-\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $-2\sqrt{bx^3 + ax^2}/(ax^{3/2})$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)`

Giac [A] time = 1.26793, size = 41, normalized size = 1.64

$$\frac{4\sqrt{b}}{(\sqrt{b}\sqrt{x} - \sqrt{bx+a})^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

[Out] `4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)`

$$3.274 \quad \int \frac{1}{x^{3/2} \sqrt{ax^2 + bx^3}} dx$$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2 + bx^3}}{3a^2x^{3/2}} - \frac{2\sqrt{ax^2 + bx^3}}{3ax^{5/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(3*a*x^{(5/2)}) + (4*b*\text{Sqrt}[a*x^2 + b*x^3])/(3*a^2*x^{(3/2)})$

Rubi [A] time = 0.0766247, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2 + bx^3}}{3a^2x^{3/2}} - \frac{2\sqrt{ax^2 + bx^3}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(3/2)}*\text{Sqrt}[a*x^2 + b*x^3]),x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(3*a*x^{(5/2)}) + (4*b*\text{Sqrt}[a*x^2 + b*x^3])/(3*a^2*x^{(3/2)})$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x]
- Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x]
/; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}} dx = -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}} - \frac{(2b) \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx}{3a}$$

$$= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}} + \frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}$$

Mathematica [A] time = 0.0150395, size = 31, normalized size = 0.55

$$-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]

[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))

Maple [A] time = 0.003, size = 33, normalized size = 0.6

$$-\frac{(2bx+2a)(-2bx+a)}{3a^2} \frac{1}{\sqrt{x}} \frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x)

[Out] -2/3*(b*x+a)*(-2*b*x+a)/x^(1/2)/a^2/(b*x^3+a*x^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2x^{\frac{3}{2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)

Fricas [A] time = 0.890541, size = 69, normalized size = 1.23

$$\frac{2\sqrt{bx^3 + ax^2}(2bx - a)}{3a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)

Giac [A] time = 1.20807, size = 74, normalized size = 1.32

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a}\right)^2 - a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3

$$3.275 \quad \int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}} dx$$

Optimal. Leaf size=86

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} - \frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(5*a*x^{(7/2)}) + (8*b*\text{Sqrt}[a*x^2 + b*x^3])/(15*a^2*x^{(5/2)}) - (16*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(15*a^3*x^{(3/2)})$

Rubi [A] time = 0.1178, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} - \frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^3]), x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(5*a*x^{(7/2)}) + (8*b*\text{Sqrt}[a*x^2 + b*x^3])/(15*a^2*x^{(5/2)}) - (16*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(15*a^3*x^{(3/2)})$

Rule 2016

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m+n*p+n-j+1)/(n-j)], 0] && NeQ[m+j*p+1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m+n*p+n-j+1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}} dx &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} - \frac{(4b) \int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}} dx}{5a} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} + \frac{(8b^2) \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx}{15a^2} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0152781, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{x^2(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]

[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))

Maple [A] time = 0.004, size = 46, normalized size = 0.5

$$-\frac{(2bx+2a)(8b^2x^2-4abx+3a^2)}{15a^3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x)

[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(3/2)/a^3/(b*x^3+a*x^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2x^{\frac{5}{2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)

Fricas [A] time = 0.789147, size = 96, normalized size = 1.12

$$-\frac{2(8b^2x^2 - 4abx + 3a^2)\sqrt{bx^3 + ax^2}}{15a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)

Giac [A] time = 1.15967, size = 104, normalized size = 1.21

$$\frac{32\left(10(\sqrt{b}\sqrt{x} - \sqrt{bx+a})^4 - 5a(\sqrt{b}\sqrt{x} - \sqrt{bx+a})^2 + a^2\right)b^{\frac{5}{2}}}{15\left((\sqrt{b}\sqrt{x} - \sqrt{bx+a})^2 - a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

```
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

$$3.276 \quad \int \frac{1}{x^{7/2} \sqrt{ax^2 + bx^3}} dx$$

Optimal. Leaf size=116

$$\frac{32b^3 \sqrt{ax^2 + bx^3}}{35a^4 x^{3/2}} - \frac{16b^2 \sqrt{ax^2 + bx^3}}{35a^3 x^{5/2}} + \frac{12b \sqrt{ax^2 + bx^3}}{35a^2 x^{7/2}} - \frac{2\sqrt{ax^2 + bx^3}}{7ax^{9/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(7*a*x^{(9/2)}) + (12*b*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^2*x^{(7/2)}) - (16*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^3*x^{(5/2)}) + (32*b^3*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^4*x^{(3/2)})$

Rubi [A] time = 0.162766, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{32b^3 \sqrt{ax^2 + bx^3}}{35a^4 x^{3/2}} - \frac{16b^2 \sqrt{ax^2 + bx^3}}{35a^3 x^{5/2}} + \frac{12b \sqrt{ax^2 + bx^3}}{35a^2 x^{7/2}} - \frac{2\sqrt{ax^2 + bx^3}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(7/2)}*\text{Sqrt}[a*x^2 + b*x^3]), x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^3])/(7*a*x^{(9/2)}) + (12*b*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^2*x^{(7/2)}) - (16*b^2*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^3*x^{(5/2)}) + (32*b^3*\text{Sqrt}[a*x^2 + b*x^3])/(35*a^4*x^{(3/2)})$

Rule 2016

$\text{Int}[\text{((c_.)*(x_))}^{(m_.)} * \text{((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})}^{(p_.)}, x_Symbol]$
 $]:> \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m+n*p+n-j+1)/(n-j)], 0] && NeQ[m+j*p+1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

$\text{Int}[\text{((c_.)*(x_))}^{(m_.)} * \text{((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})}^{(p_.)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m+n*p+n-j+1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}} dx &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} - \frac{(6b) \int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}} dx}{7a} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} + \frac{(24b^2) \int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}} dx}{35a^2} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} - \frac{(16b^3) \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}} dx}{35a^3} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0177761, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2(a+bx)}(6a^2bx-5a^3-8ab^2x^2+16b^3x^3)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]

[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35*a^4*x^(9/2))

Maple [A] time = 0.004, size = 57, normalized size = 0.5

$$-\frac{(2bx+2a)(-16b^3x^3+8ab^2x^2-6bxa^2+5a^3)}{35a^4}x^{-\frac{5}{2}}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x)

[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(5/2)/a^4/(b*x^3+a*x^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2} x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)

Fricas [A] time = 0.756968, size = 117, normalized size = 1.01

$$\frac{2(16b^3x^3 - 8ab^2x^2 + 6a^2bx - 5a^3)\sqrt{bx^3 + ax^2}}{35a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^4*x^(9/2))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}} \sqrt{x^2(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)

[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)

Giac [A] time = 1.26918, size = 139, normalized size = 1.2

$$\frac{64 \left(35 \left(\sqrt{b}\sqrt{x} - \sqrt{bx+a} \right)^6 - 21 a \left(\sqrt{b}\sqrt{x} - \sqrt{bx+a} \right)^4 + 7 a^2 \left(\sqrt{b}\sqrt{x} - \sqrt{bx+a} \right)^2 - a^3 \right) b^{\frac{7}{2}}}{35 \left(\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a} \right)^2 - a \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7

$$3.277 \quad \int x^{1-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=61

$$\frac{x^{2-3n} \left(\frac{bx}{a} + 1\right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(2-n, -n; 3-n; -\frac{bx}{a}\right)}{2-n}$$

[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -(b*x)/a]))/((2 - n)*(1 + (b*x)/a)^n)

Rubi [A] time = 0.038824, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 66, 64}

$$\frac{x^{2-3n} \left(\frac{bx}{a} + 1\right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(2-n, -n; 3-n; -\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

[In] Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -(b*x)/a]))/((2 - n)*(1 + (b*x)/a)^n)

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 66

Int[((b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)]) && EqQ[c^2 - d^2, 0])) || !RationalQ[n]

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c^n*(b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)]/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

Rubi steps

$$\begin{aligned} \int x^{1-3n} (ax^2 + bx^3)^n dx &= \left(x^{-2n} (a + bx)^{-n} (ax^2 + bx^3)^n \right) \int x^{1-n} (a + bx)^n dx \\ &= \left(x^{-2n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n \right) \int x^{1-n} \left(1 + \frac{bx}{a} \right)^n dx \\ &= \frac{x^{2-3n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n {}_2F_1 \left(2 - n, -n; 3 - n; -\frac{bx}{a} \right)}{2 - n} \end{aligned}$$

Mathematica [A] time = 0.0175152, size = 59, normalized size = 0.97

$$\frac{x^{2-3n} (x^2(a + bx))^n \left(\frac{bx}{a} + 1 \right)^{-n} {}_2F_1 \left(2 - n, -n; 3 - n; -\frac{bx}{a} \right)}{2 - n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] (x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/a)])/((2 - n)*(1 + (b*x)/a)^n)

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int x^{1-3n} (bx^3 + ax^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)

[Out] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3 + ax^2\right)^n x^{-3n+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")

[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

$$3.278 \quad \int x^{-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=48

$$\frac{x^{-3n-1} (ax^2 + bx^3)^{n+1} {}_2F_1\left(1, 2; 2-n; -\frac{bx}{a}\right)}{a(1-n)}$$

[Out] (x^(-1 - 3*n)*(a*x^2 + b*x^3)^(1 + n)*Hypergeometric2F1[1, 2, 2 - n, -(b*x)/a])/(a*(1 - n))

Rubi [A] time = 0.0371099, antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2032, 66, 64}

$$\frac{x^{1-3n} \left(\frac{bx}{a} + 1\right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(1-n, -n; 2-n; -\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)^n/x^(3*n), x]

[Out] (x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -(b*x)/a])/(1 - n)*(1 + (b*x)/a)^n

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 66

Int[((b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)]) && EqQ[c^2 - d^2, 0])) || !RationalQ[n]

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c^n*(b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)]/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

Rubi steps

$$\begin{aligned} \int x^{-3n} (ax^2 + bx^3)^n dx &= \left(x^{-2n} (a + bx)^{-n} (ax^2 + bx^3)^n \right) \int x^{-n} (a + bx)^n dx \\ &= \left(x^{-2n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n \right) \int x^{-n} \left(1 + \frac{bx}{a} \right)^n dx \\ &= \frac{x^{1-3n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n {}_2F_1 \left(1 - n, -n; 2 - n; -\frac{bx}{a} \right)}{1 - n} \end{aligned}$$

Mathematica [A] time = 0.012427, size = 59, normalized size = 1.23

$$\frac{x^{1-3n} (x^2(a + bx))^n \left(\frac{bx}{a} + 1 \right)^{-n} {}_2F_1 \left(1 - n, -n; 2 - n; -\frac{bx}{a} \right)}{1 - n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^n/x^(3*n), x]
```

```
[Out] (x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^n}{x^{3n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^n/(x^(3*n)), x)
```

[Out] `int((b*x^3+a*x^2)^n/(x^(3*n)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^n}{x^{3n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a*x^2)^n/x^(3*n), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 + ax^2)^n}{x^{3n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")`

[Out] `integral((b*x^3 + a*x^2)^n/x^(3*n), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-3n} (x^2 (a + bx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)`

[Out] `Integral(x**(-3*n)*(x**2*(a + b*x))**n, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + ax^2)^n}{x^{3n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

$$3.279 \quad \int x^{-1-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=54

$$\frac{x^{-3n} \left(\frac{bx}{a} + 1\right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(-n, -n; 1 - n; -\frac{bx}{a}\right)}{n}$$

[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -(b*x)/a]))/(n*x^(3*n)*(1 + (b*x)/a)^n)

Rubi [A] time = 0.0382429, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 66, 64}

$$\frac{x^{-3n} \left(\frac{bx}{a} + 1\right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(-n, -n; 1 - n; -\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -(b*x)/a]))/(n*x^(3*n)*(1 + (b*x)/a)^n)

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 66

Int[((b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)]) && EqQ[c^2 - d^2, 0])) || !RationalQ[n]

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c^n*(b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)]/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

Rubi steps

$$\begin{aligned} \int x^{-1-3n} (ax^2 + bx^3)^n dx &= \left(x^{-2n} (a + bx)^{-n} (ax^2 + bx^3)^n \right) \int x^{-1-n} (a + bx)^n dx \\ &= \left(x^{-2n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n \right) \int x^{-1-n} \left(1 + \frac{bx}{a} \right)^n dx \\ &= -\frac{x^{-3n} \left(1 + \frac{bx}{a} \right)^{-n} (ax^2 + bx^3)^n {}_2F_1\left(-n, -n; 1 - n; -\frac{bx}{a}\right)}{n} \end{aligned}$$

Mathematica [A] time = 0.00813, size = 52, normalized size = 0.96

$$-\frac{x^{-3n} (x^2(a + bx))^n \left(\frac{bx}{a} + 1\right)^{-n} {}_2F_1\left(-n, -n; 1 - n; -\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] -(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*n)*(1 + (b*x)/a)^n))

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int x^{-1-3n} (bx^3 + ax^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)

[Out] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="maxima")

[Out] integrate((b*x³ + a*x²)ⁿ*x^(-3*n - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3 + ax^2\right)^n x^{-3n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="fricas")

[Out] integral((b*x³ + a*x²)ⁿ*x^(-3*n - 1), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-3n-1} \left(x^2 (a + bx)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1-3*n)}*(b*x^{**3}+a*x^{**2})^{**n},x)

[Out] Integral(x^{**(-3*n - 1)}*(x^{**2}*(a + b*x))^{**n}, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

$$3.280 \quad \int x^{-2-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=32

$$\frac{x^{-3(n+1)} (ax^2 + bx^3)^{n+1}}{a(n+1)}$$

[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))

Rubi [A] time = 0.0251077, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2014}

$$\frac{x^{-3(n+1)} (ax^2 + bx^3)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-2-3n} (ax^2 + bx^3)^n dx = -\frac{x^{-3(1+n)} (ax^2 + bx^3)^{1+n}}{a(1+n)}$$

Mathematica [A] time = 0.014193, size = 30, normalized size = 0.94

$$\frac{x^{-3(n+1)} (x^2(a + bx))^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 - 3*n)*(a*x² + b*x³)ⁿ,x]

[Out] -((x²*(a + b*x))^(1 + n)/(a*(1 + n)*x^{(3*(1 + n))}))

Maple [A] time = 0.003, size = 36, normalized size = 1.1

$$\frac{x^{-1-3n} (bx + a) (bx^3 + ax^2)^n}{a(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-2-3*n)*(b*x³+a*x²)ⁿ,x)

[Out] -x^(-1-3*n)*(b*x+a)/a/(1+n)*(b*x³+a*x²)ⁿ

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="maxima")

[Out] integrate((b*x³ + a*x²)ⁿ*x^(-3*n - 2), x)

Fricas [A] time = 0.946151, size = 77, normalized size = 2.41

$$\frac{(bx^2 + ax)(bx^3 + ax^2)^n x^{-3n-2}}{an + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="fricas")

[Out] $-(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^{(-3*n - 2)}/(a*n + a)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)`

$$3.281 \quad \int x^{-3-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=70

$$\frac{bx^{-3(n+1)}(ax^2 + bx^3)^{n+1}}{a^2(n+1)(n+2)} - \frac{x^{-3n-4}(ax^2 + bx^3)^{n+1}}{a(n+2)}$$

[Out] $-\left(\frac{x^{-4-3n}(ax^2 + bx^3)^{n+1}}{a^2(n+1)(n+2)} + \frac{b x^{-3n-4}(ax^2 + bx^3)^{n+1}}{a(n+2)}\right)$

Rubi [A] time = 0.0543414, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{bx^{-3(n+1)}(ax^2 + bx^3)^{n+1}}{a^2(n+1)(n+2)} - \frac{x^{-3n-4}(ax^2 + bx^3)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

[In] Int[x⁻³⁻³ⁿ(a*x² + b*x³)ⁿ,x]

[Out] $-\left(\frac{x^{-4-3n}(ax^2 + bx^3)^{n+1}}{a^2(n+1)(n+2)} + \frac{b x^{-3n-4}(ax^2 + bx^3)^{n+1}}{a(n+2)}\right)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(m+j*p+1)), x]
- Dist[(b*(m+n*p+n-j+1))/(a*c^(n-j)*(m+j*p+1)), Int[(c*x)^(m+n-j)*(a*x^j + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m+n*p+n-j+1)/(n-j)], 0] && NeQ[m+j*p+1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(n-j)*(p+1)), x]
/; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m+n*p+n-j+1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int x^{-3-3n} (ax^2 + bx^3)^n dx &= -\frac{x^{-4-3n} (ax^2 + bx^3)^{1+n}}{a(2+n)} - \frac{b \int x^{-2-3n} (ax^2 + bx^3)^n dx}{a(2+n)} \\ &= -\frac{x^{-4-3n} (ax^2 + bx^3)^{1+n}}{a(2+n)} + \frac{bx^{-3(1+n)} (ax^2 + bx^3)^{1+n}}{a^2(1+n)(2+n)} \end{aligned}$$

Mathematica [A] time = 0.0208736, size = 44, normalized size = 0.63

$$-\frac{x^{-3n-4}(an+a-bx)(x^2(a+bx))^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n, x]

[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 + n)))

Maple [A] time = 0.005, size = 50, normalized size = 0.7

$$\frac{x^{-2-3n} (bx^3 + ax^2)^n (an - bx + a)(bx + a)}{(2+n)(1+n)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n, x)

[Out] -(b*x^3+a*x^2)^n*x^(-2-3*n)*(a*n-b*x+a)*(b*x+a)/(2+n)/(1+n)/a^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="maxima")

[Out] integrate((b*x³ + a*x²)ⁿ*x^(-3*n - 3), x)

Fricas [A] time = 0.811785, size = 136, normalized size = 1.94

$$\frac{(abnx^2 - b^2x^3 + (a^2n + a^2)x)(bx^3 + ax^2)^n x^{-3n-3}}{a^2n^2 + 3a^2n + 2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="fricas")

[Out] -(a*b*n*x² - b²*x³ + (a²*n + a²)*x)*(b*x³ + a*x²)ⁿ*x^(-3*n - 3)/(a²*n² + 3*a²*n + 2*a²)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-3-3*n)}*(b*x^{**3}+a*x^{**2})^{**n},x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3-3*n)*(b*x³+a*x²)ⁿ,x, algorithm="giac")

[Out] integrate((b*x³ + a*x²)ⁿ*x^(-3*n - 3), x)

$$3.282 \quad \int x^{-4-3n} (ax^2 + bx^3)^n dx$$

Optimal. Leaf size=116

$$-\frac{2b^2x^{-3(n+1)}(ax^2 + bx^3)^{n+1}}{a^3(n+1)(n+2)(n+3)} + \frac{2bx^{-3n-4}(ax^2 + bx^3)^{n+1}}{a^2(n+2)(n+3)} - \frac{x^{-3n-5}(ax^2 + bx^3)^{n+1}}{a(n+3)}$$

[Out] $-\left(\frac{x^{-5-3n}(ax^2 + bx^3)^{n+1}}{a^3(n+1)(n+2)(n+3)} + \frac{2bx^{-3n-4}(ax^2 + bx^3)^{n+1}}{a^2(n+2)(n+3)} - \frac{x^{-3n-5}(ax^2 + bx^3)^{n+1}}{a(n+3)}\right)$

Rubi [A] time = 0.0914168, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$-\frac{2b^2x^{-3(n+1)}(ax^2 + bx^3)^{n+1}}{a^3(n+1)(n+2)(n+3)} + \frac{2bx^{-3n-4}(ax^2 + bx^3)^{n+1}}{a^2(n+2)(n+3)} - \frac{x^{-3n-5}(ax^2 + bx^3)^{n+1}}{a(n+3)}$$

Antiderivative was successfully verified.

[In] Int[x^{−4−3n}(ax² + bx³)ⁿ, x]

[Out] $-\left(\frac{x^{-5-3n}(ax^2 + bx^3)^{n+1}}{a^3(n+1)(n+2)(n+3)} + \frac{2bx^{-3n-4}(ax^2 + bx^3)^{n+1}}{a^2(n+2)(n+3)} - \frac{x^{-3n-5}(ax^2 + bx^3)^{n+1}}{a(n+3)}\right)$

Rule 2016

Int[((c_.)(x_))^(m_.)*((a_.)(x_)^(j_.) + (b_.)(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2014

Int[((c_.)(x_))^(m_.)*((a_.)(x_)^(j_.) + (b_.)(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,

j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int x^{-4-3n} (ax^2 + bx^3)^n dx &= -\frac{x^{-5-3n} (ax^2 + bx^3)^{1+n}}{a(3+n)} - \frac{(2b) \int x^{-3-3n} (ax^2 + bx^3)^n dx}{a(3+n)} \\ &= -\frac{x^{-5-3n} (ax^2 + bx^3)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n} (ax^2 + bx^3)^{1+n}}{a^2(2+n)(3+n)} + \frac{(2b^2) \int x^{-2-3n} (ax^2 + bx^3)^n dx}{a^2(2+n)(3+n)} \\ &= -\frac{x^{-5-3n} (ax^2 + bx^3)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n} (ax^2 + bx^3)^{1+n}}{a^2(2+n)(3+n)} - \frac{2b^2x^{-3(1+n)} (ax^2 + bx^3)^{1+n}}{a^3(1+n)(2+n)(3+n)} \end{aligned}$$

Mathematica [A] time = 0.0286695, size = 72, normalized size = 0.62

$$\frac{x^{-3(n+1)}(a+bx)(x^2(a+bx))^n(a^2(n^2+3n+2)-2ab(n+1)x+2b^2x^2)}{a^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]

[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))

Maple [A] time = 0.005, size = 84, normalized size = 0.7

$$\frac{(bx+a)x^{-3-3n}(a^2n^2-2abnx+2b^2x^2+3a^2n-2abx+2a^2)(bx^3+ax^2)^n}{(3+n)(2+n)(1+n)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x)

[Out] -(b*x+a)*x^(-3-3*n)*(a^2*n^2-2*a*b*n*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)*(b*x^3+a*x^2)^n/(3+n)/(2+n)/(1+n)/a^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)

Fricas [A] time = 0.91543, size = 217, normalized size = 1.87

$$\frac{(2ab^2nx^3 - 2b^3x^4 - (a^2bn^2 + a^2bn)x^2 - (a^3n^2 + 3a^3n + 2a^3)x)(bx^3 + ax^2)^n x^{-3n-4}}{a^3n^3 + 6a^3n^2 + 11a^3n + 6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")

[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n + 6*a^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + ax^2)^n x^{-3n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

$$3.283 \quad \int \frac{x^{11}}{(ax^2+bx^5)^3} dx$$

Optimal. Leaf size=19

$$\frac{x^6}{6a(a+bx^3)^2}$$

[Out] x^6/(6*a*(a + b*x^3)^2)

Rubi [A] time = 0.0100698, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 264}

$$\frac{x^6}{6a(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a*x^2 + b*x^5)^3,x]

[Out] x^6/(6*a*(a + b*x^3)^2)

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :-> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :-> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{11}}{(ax^2 + bx^5)^3} dx = \int \frac{x^5}{(a + bx^3)^3} dx$$

$$= \frac{x^6}{6a(a + bx^3)^2}$$

Mathematica [A] time = 0.00836, size = 24, normalized size = 1.26

$$-\frac{a + 2bx^3}{6b^2(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]

[Out] -(a + 2*b*x^3)/(6*b^2*(a + b*x^3)^2)

Maple [A] time = 0.007, size = 31, normalized size = 1.6

$$\frac{a}{6b^2(bx^3 + a)^2} - \frac{1}{3b^2(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(b*x^5+a*x^2)^3,x)

[Out] 1/6/b^2*a/(b*x^3+a)^2-1/3/b^2/(b*x^3+a)

Maxima [B] time = 1.17217, size = 49, normalized size = 2.58

$$-\frac{2bx^3 + a}{6(b^4x^6 + 2ab^3x^3 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁵+a*x²)³,x, algorithm="maxima")

[Out] $-\frac{1}{6} \frac{(2bx^3 + a)}{(b^4x^6 + 2ab^3x^3 + a^2b^2)}$

Fricas [B] time = 0.71859, size = 73, normalized size = 3.84

$$-\frac{2bx^3 + a}{6(b^4x^6 + 2ab^3x^3 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁵+a*x²)³,x, algorithm="fricas")

[Out] $-\frac{1}{6} \frac{(2bx^3 + a)}{(b^4x^6 + 2ab^3x^3 + a^2b^2)}$

Sympy [B] time = 1.50915, size = 36, normalized size = 1.89

$$-\frac{a + 2bx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(b*x**5+a*x**2)**3,x)

[Out] $-\frac{(a + 2bx^3)}{(6a^2b^2 + 12ab^3x^3 + 6b^4x^6)}$

Giac [A] time = 1.16428, size = 30, normalized size = 1.58

$$-\frac{2bx^3 + a}{6(bx^3 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(b*x⁵+a*x²)³,x, algorithm="giac")

[Out] $-\frac{1}{6} \frac{(2bx^3 + a)}{((bx^3 + a)^2b^2)}$

$$3.284 \quad \int \frac{x^9}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=80

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)

Rubi [A] time = 0.114813, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Antiderivative was successfully verified.

[In] Int[x^9/Sqrt[a*x^2 + b*x^5], x]

[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:= Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```


Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\sqrt{ax^2 + bx^5}} dx &= \frac{2x^5\sqrt{ax^2 + bx^5}}{15b} - \frac{(4a) \int \frac{x^6}{\sqrt{ax^2 + bx^5}} dx}{5b} \\
&= -\frac{8ax^2\sqrt{ax^2 + bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2 + bx^5}}{15b} + \frac{(8a^2) \int \frac{x^3}{\sqrt{ax^2 + bx^5}} dx}{15b^2} \\
&= \frac{16a^2\sqrt{ax^2 + bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2 + bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2 + bx^5}}{15b}
\end{aligned}$$

Mathematica [A] time = 0.0246324, size = 46, normalized size = 0.57

$$\frac{2\sqrt{x^2(a + bx^3)}(8a^2 - 4abx^3 + 3b^2x^6)}{45b^3x}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)

Maple [A] time = 0.006, size = 48, normalized size = 0.6

$$\frac{(2bx^3 + 2a)(3b^2x^6 - 4abx^3 + 8a^2)x}{45b^3} \frac{1}{\sqrt{bx^5 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(b*x^5+a*x^2)^(1/2), x)

[Out] 2/45*(b*x^3+a)*(3*b^2*x^6-4*a*b*x^3+8*a^2)*x/b^3/(b*x^5+a*x^2)^(1/2)

Maxima [A] time = 1.0347, size = 62, normalized size = 0.78

$$\frac{2(3b^3x^9 - ab^2x^6 + 4a^2bx^3 + 8a^3)}{45\sqrt{bx^3 + ab^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)

Fricas [A] time = 0.65072, size = 89, normalized size = 1.11

$$\frac{2(3b^2x^6 - 4abx^3 + 8a^2)\sqrt{bx^5 + ax^2}}{45b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

```
[Out] integrate(x^9/sqrt(b*x^5 + a*x^2), x)
```

$$3.285 \quad \int \frac{x^6}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=52

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b} - \frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

[Out] $(-4*a*\text{Sqrt}[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*\text{Sqrt}[a*x^2 + b*x^5])/(9*b)$

Rubi [A] time = 0.0649265, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 1588}

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b} - \frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6/\text{Sqrt}[a*x^2 + b*x^5], x]$

[Out] $(-4*a*\text{Sqrt}[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*\text{Sqrt}[a*x^2 + b*x^5])/(9*b)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x]
  - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x]
  /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]},
  Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x]
  /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]
  /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{ax^2 + bx^5}} dx = \frac{2x^2\sqrt{ax^2 + bx^5}}{9b} - \frac{(2a) \int \frac{x^3}{\sqrt{ax^2 + bx^5}} dx}{3b}$$

$$= -\frac{4a\sqrt{ax^2 + bx^5}}{9b^2x} + \frac{2x^2\sqrt{ax^2 + bx^5}}{9b}$$

Mathematica [A] time = 0.0171806, size = 34, normalized size = 0.65

$$\frac{2(bx^3 - 2a)\sqrt{x^2(a + bx^3)}}{9b^2x}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)

Maple [A] time = 0.005, size = 37, normalized size = 0.7

$$-\frac{(2bx^3 + 2a)(-bx^3 + 2a)x}{9b^2} \frac{1}{\sqrt{bx^5 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^5+a*x^2)^(1/2), x)

[Out] -2/9*(b*x^3+a)*(-b*x^3+2*a)*x/b^2/(b*x^5+a*x^2)^(1/2)

Maxima [A] time = 1.15932, size = 46, normalized size = 0.88

$$\frac{2(b^2x^6 - abx^3 - 2a^2)}{9\sqrt{bx^3 + ab^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)

Fricas [A] time = 0.809841, size = 63, normalized size = 1.21

$$\frac{2\sqrt{bx^5 + ax^2}(bx^3 - 2a)}{9b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^6/sqrt(b*x^5 + a*x^2), x)

$$3.286 \quad \int \frac{x^3}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=25

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)

Rubi [A] time = 0.016886, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]

[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx = \frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Mathematica [A] time = 0.0077292, size = 25, normalized size = 1.

$$\frac{2\sqrt{x^2(a+bx^3)}}{3bx}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]

[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)

Maple [A] time = 0.004, size = 27, normalized size = 1.1

$$\frac{(2bx^3 + 2a)x}{3b} \frac{1}{\sqrt{bx^5 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^5+a*x^2)^(1/2),x)

[Out] 2/3*(b*x^3+a)*x/b/(b*x^5+a*x^2)^(1/2)

Maxima [A] time = 1.1064, size = 19, normalized size = 0.76

$$\frac{2\sqrt{bx^3 + a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/3*sqrt(b*x^3 + a)/b

Fricas [A] time = 0.796274, size = 42, normalized size = 1.68

$$\frac{2\sqrt{bx^5 + ax^2}}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $2/3*\text{sqrt}(b*x^5 + a*x^2)/(b*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**5+a*x**2)**(1/2),x)`

[Out] `Integral(x**3/sqrt(x**2*(a + b*x**3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^3/sqrt(b*x^5 + a*x^2), x)`

$$3.287 \quad \int \frac{1}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])

Rubi [A] time = 0.0109513, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^2 + b*x^5], x]

[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{ax^2 + bx^5}} dx = -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^5}}\right)\right) \\ = -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2 + bx^5}}\right)}{3\sqrt{a}}$$

Mathematica [A] time = 0.0103286, size = 54, normalized size = 1.69

$$\frac{2x\sqrt{a + bx^3} \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]

[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.005, size = 43, normalized size = 1.3

$$-\frac{2x}{3}\sqrt{bx^3 + a}\text{Artanh}\left(\sqrt{bx^3 + a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{bx^5 + ax^2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^5+a*x^2)^(1/2),x)

[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)

Fricas [A] time = 0.875145, size = 180, normalized size = 5.62

$$\left[\frac{\log\left(\frac{bx^4+2ax-2\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)}{3\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right)}{3a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)

Giac [A] time = 1.16395, size = 63, normalized size = 1.97

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/  
sqrt(-a))/(sqrt(-a)*sgn(x))
```

$$3.288 \quad \int \frac{1}{x^3 \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=59

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^5}}\right)}{3a^{3/2}} - \frac{\sqrt{ax^2 + bx^5}}{3ax^4}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^5]/(3*a*x^4) + (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^5]])/(3*a^{(3/2)})$

Rubi [A] time = 0.0545933, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2008, 206}

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^5}}\right)}{3a^{3/2}} - \frac{\sqrt{ax^2 + bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^3*\text{Sqrt}[a*x^2 + b*x^5]), x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^5]/(3*a*x^4) + (b*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^5]])/(3*a^{(3/2)})$

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{ax^2 + bx^5}} dx &= -\frac{\sqrt{ax^2 + bx^5}}{3ax^4} - \frac{b \int \frac{1}{\sqrt{ax^2 + bx^5}} dx}{2a} \\ &= -\frac{\sqrt{ax^2 + bx^5}}{3ax^4} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^5}}\right)}{3a} \\ &= -\frac{\sqrt{ax^2 + bx^5}}{3ax^4} + \frac{b \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^5}}\right)}{3a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.061394, size = 71, normalized size = 1.2

$$\frac{2b\sqrt{x^2(a + bx^3)} \left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a} + 1}\right)}{2\sqrt{\frac{bx^3}{a} + 1}} - \frac{a}{2bx^3} \right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*b*Sqrt[x^2*(a + b*x^3)]*(-a/(2*b*x^3) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/(2*
Sqrt[1 + (b*x^3)/a])))/(3*a^2*x)
```

Maple [A] time = 0.006, size = 66, normalized size = 1.1

$$-\frac{1}{3x^2} \sqrt{bx^3 + a} \left(-b \operatorname{Artanh}\left(\sqrt{bx^3 + a} \frac{1}{\sqrt{a}}\right) ax^3 + \sqrt{bx^3 + a} a^{\frac{3}{2}} \right) \frac{1}{\sqrt{bx^5 + ax^2}} a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x)
```

[Out] $-1/3/x^2*(b*x^3+a)^{(1/2)}*(-b*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})+a*x^3+(b*x^3+a)^{(1/2)}*a^{(3/2)})/(b*x^5+a*x^2)^{(1/2)}/a^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)`

Fricas [A] time = 0.809663, size = 294, normalized size = 4.98

$$\left[\frac{\sqrt{ab}x^4 \log\left(\frac{bx^4+2ax+2\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right) - 2\sqrt{bx^5+ax^2}a}{6a^2x^4}, -\frac{\sqrt{-ab}x^4 \arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^5+ax^2}a}{3a^2x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2(a+bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)`

[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)

Giac [A] time = 1.16582, size = 77, normalized size = 1.31

$$-\frac{b \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3 \sqrt{-a} \operatorname{sgn}(x)} - \frac{\sqrt{\frac{b}{x} + \frac{a}{x^4}}}{3 a x \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] -1/3*b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) - 1/3*sqrt(b/x + a/x^4)/(a*x*sgn(x))

$$3.289 \quad \int \frac{x^4}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=238

$$\frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right), -7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.141568, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2024, 2032, 218}

$$\frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right) \middle| -7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rule 2024

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax^2 + bx^5}} dx &= \frac{2\sqrt{ax^2 + bx^5}}{5b} - \frac{(2a) \int \frac{x}{\sqrt{ax^2 + bx^5}} dx}{5b} \\
&= \frac{2\sqrt{ax^2 + bx^5}}{5b} - \frac{(2ax\sqrt{a + bx^3}) \int \frac{1}{\sqrt{a + bx^3}} dx}{5b\sqrt{ax^2 + bx^5}} \\
&= \frac{2\sqrt{ax^2 + bx^5}}{5b} - \frac{4\sqrt{2 + \sqrt{3}}ax(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{(1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right) \mid -7 - 4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0243369, size = 68, normalized size = 0.29

$$\frac{2x^2 \left(-a\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{5b\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a]))/(5*b*Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.297, size = 248, normalized size = 1.

$$\frac{2x}{15b^2} \left(ia\sqrt{3}\sqrt[3]{-b^2a} \sqrt{-i\sqrt{3} \left(i\sqrt{3}\sqrt[3]{-b^2a} - 2bx - \sqrt[3]{-b^2a} \right)} \frac{1}{\sqrt[3]{-b^2a}} \sqrt{-2 \frac{-bx + \sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a} (i\sqrt{3} - 3)}} \sqrt{-i\sqrt{3} \left(i\sqrt{3}\sqrt[3]{-b^2a} + 2bx + \sqrt[3]{-b^2a} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^5+a*x^2)^(1/2), x)

[Out] 2/15*x*(I*a*3^(1/2)*(-b^2*a)^(1/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2), 2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*x^4*b^2+3*a*b*x)/(b*x^5+a*x^2)^(1/2)/b^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^5+a*x^2)^(1/2), x, algorithm="maxima")

[Out] integrate(x⁴/sqrt(b*x⁵ + a*x²), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2x^2}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁴/(b*x⁵+a*x²)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x⁵ + a*x²)*x²/(b*x³ + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**4/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁴/(b*x⁵+a*x²)^(1/2),x, algorithm="giac")

[Out] integrate(x⁴/sqrt(b*x⁵ + a*x²), x)

$$3.290 \quad \int \frac{x}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=212

$$\frac{2\sqrt{2+\sqrt{3}}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.0667158, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2032, 218}

$$\frac{2\sqrt{2+\sqrt{3}}x(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p]]/(x^(

FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rubi steps

$$\int \frac{x}{\sqrt{ax^2 + bx^5}} dx = \frac{\left(x\sqrt{a + bx^3}\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{\sqrt{ax^2 + bx^5}}$$

$$= \frac{2\sqrt{2 + \sqrt{3}}x \left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)^2}} F\left(\sin^{-1}\left(\frac{(1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)^2}} \sqrt{ax^2 + bx^5}}$$

Mathematica [C] time = 0.0094612, size = 52, normalized size = 0.25

$$\frac{x^2 \sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x^2 + b*x^5],x]

[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a])/Sqrt[x^2*(a + b*x^3)]

Maple [A] time = 0.005, size = 231, normalized size = 1.1

$$\frac{-\frac{i}{3}x\sqrt{3}}{b}\sqrt[3]{-b^2a}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}-2bx-\sqrt[3]{-b^2a}\right)}\frac{1}{\sqrt[3]{-b^2a}}\sqrt{-2\frac{-bx+\sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}(i\sqrt{3}-3)}}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}+2bx+\sqrt[3]{-b^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^5+a*x^2)^(1/2),x)

[Out]
$$-1/3*I/(b*x^5+a*x^2)^{(1/2)}*x*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(-I*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})*3^{(1/2)}/(-b^2*a)^{(1/3)})^{(1/2)}*(-2*(-b*x+(-b^2*a)^{(1/3)})/(-b^2*a)^{(1/3)}/(I*3^{(1/2)}-3))^{(1/2)}*(-I*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})*3^{(1/2)}/(-b^2*a)^{(1/3)})^{(1/2)}*EllipticF(1/6*3^{(1/2)}*2^{(1/2)}*(-I*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})*3^{(1/2)}/(-b^2*a)^{(1/3)})^{(1/2)},2^{(1/2)}*(I*3^{(1/2)}/(I*3^{(1/2)}-3))^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^4 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^4 + a*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)

$$3.291 \quad \int \frac{1}{x^2 \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=243

$$\frac{\sqrt{2 + \sqrt{3}} b^{2/3} x (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right)}{2 \sqrt[4]{3} a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{\sqrt{ax^2 + bx^5}}{2ax^3}$$

[Out] $-\operatorname{Sqrt}[a*x^2 + b*x^5]/(2*a*x^3) - (\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^{(2/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x]/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)], -7 - 4*\operatorname{Sqrt}[3])/(2*3^{(1/4)}*a*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.128627, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2025, 2032, 218}

$$\frac{\sqrt{2 + \sqrt{3}} b^{2/3} x (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}}\right) \mid -7 - 4\sqrt{3}\right)}{2 \sqrt[4]{3} a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{\sqrt{ax^2 + bx^5}}{2ax^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x^2*\operatorname{Sqrt}[a*x^2 + b*x^5]), x]$

[Out] $-\operatorname{Sqrt}[a*x^2 + b*x^5]/(2*a*x^3) - (\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^{(2/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x]/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)], -7 - 4*\operatorname{Sqrt}[3])/(2*3^{(1/4)}*a*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

Rule 2025

```

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{ax^2 + bx^5}} dx &= -\frac{\sqrt{ax^2 + bx^5}}{2ax^3} - \frac{b \int \frac{x}{\sqrt{ax^2 + bx^5}} dx}{4a} \\
&= -\frac{\sqrt{ax^2 + bx^5}}{2ax^3} - \frac{(bx\sqrt{a + bx^3}) \int \frac{1}{\sqrt{a + bx^3}} dx}{4a\sqrt{ax^2 + bx^5}} \\
&= -\frac{\sqrt{ax^2 + bx^5}}{2ax^3} - \frac{\sqrt{2 + \sqrt{3}b^{2/3}x(\sqrt[3]{a} + \sqrt[3]{bx})} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{(1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right) \middle| -7 - 4\sqrt{3}\right)}{2\sqrt[4]{3}a \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0130886, size = 55, normalized size = 0.23

$$\frac{\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; -\frac{bx^3}{a}\right)}{2x\sqrt{x^2(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]

[Out] -(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(2*x*Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.005, size = 248, normalized size = 1.

$$\frac{1}{12ax} \left(i\sqrt{3}\sqrt[3]{-b^2a} \sqrt{-i\sqrt{3} \left(i\sqrt{3}\sqrt[3]{-b^2a} - 2bx - \sqrt[3]{-b^2a} \right)} \frac{1}{\sqrt[3]{-b^2a}} \sqrt{-2 \frac{-bx + \sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a} (i\sqrt{3} - 3)}} \sqrt{-i\sqrt{3} \left(i\sqrt{3}\sqrt[3]{-b^2a} + 2bx + \sqrt[3]{-b^2a} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^5+a*x^2)^(1/2),x)

[Out] 1/12*(I*3^(1/2)*(-b^2*a)^(1/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/x/(b*x^5+a*x^2)^(1/2)/a

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^7 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^7 + a*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{x^2 (a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)

$$3.292 \quad \int \frac{x^5}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=514

$$\frac{8\sqrt{2}a^{4/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right), -7-4\sqrt{3}\right) + 4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x(\sqrt[3]{a} + \sqrt[3]{bx})}{7\sqrt[4]{3}b^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} \sqrt{ax^2+bx^5}} + \frac{7b^{5/3}}{7b^{5/3}}$$

[Out] $(-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (2*x*\operatorname{Sqrt}[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(7*b^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) - (8*\operatorname{Sqrt}[2]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(7*3^(1/4)*b^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.308019, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2024, 2032, 303, 218, 1877}

$$\frac{8\sqrt{2}a^{4/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right) \middle| -7-4\sqrt{3}\right) + 4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}}{7\sqrt[4]{3}b^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} \sqrt{ax^2+bx^5}} + \frac{7b^{5/3}}{7b^{5/3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^5/\operatorname{Sqrt}[a*x^2 + b*x^5], x]$

[Out] $(-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (2*x*\operatorname{Sqrt}[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(7*b^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) - (8*\operatorname{Sqrt}[2]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(7*3^(1/4)*b^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

$$\frac{2}{3}x^2 / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}] / (7b^{5/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} * \sqrt{a^2x + b^2x^5}) - (8\sqrt{2}a^{4/3}x(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}] / (7 \cdot 3^{1/4} b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} * \sqrt{a^2x + b^2x^5})$$
Rule 2024

$$\text{Int}[(c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a \cdot x^j + b \cdot x^n)^{p+1}) / (b \cdot (m+n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^{n-j} \cdot (m+j \cdot p - n + j + 1)) / (b \cdot (m+n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-(n-j)} \cdot (a \cdot x^j + b \cdot x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[0, j, n] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0]) \&\& \text{GtQ}[m + j \cdot p + 1 - n + j, 0] \&\& \text{NeQ}[m + n \cdot p + 1, 0]$$
Rule 2032

$$\text{Int}[(c \cdot x)^m \cdot (a \cdot x^j + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]} \cdot (c \cdot x)^{\text{FracPart}[m]} \cdot (a \cdot x^j + b \cdot x^n)^{\text{FracPart}[p]}) / (x^{(\text{FracPart}[m] + j \cdot \text{FracPart}[p])} \cdot (a + b \cdot x^{n-j})^{\text{FracPart}[p]}), \text{Int}[x^{m+j \cdot p} \cdot (a + b \cdot x^{n-j})^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$$
Rule 303

$$\text{Int}[x / \sqrt{(a) + (b \cdot x)^3}, x_Symbol] \rightarrow \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(\sqrt{2} \cdot s) / (\sqrt{2 + \sqrt{3}} \cdot r), \text{Int}[1 / \sqrt{a + b \cdot x^3}, x], x] + \text{Dist}[1/r, \text{Int}[(1 - \sqrt{3}) \cdot s + r \cdot x] / \sqrt{a + b \cdot x^3}, x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a]$$
Rule 218

$$\text{Int}[1 / \sqrt{(a) + (b \cdot x)^3}, x_Symbol] \rightarrow \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2 \cdot \sqrt{2 + \sqrt{3}} \cdot (s + r \cdot x) \cdot \sqrt{(s^2 - r \cdot s \cdot x + r^2 \cdot x^2) / ((1 + \sqrt{3}) \cdot s + r \cdot x)^2} * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3}) \cdot s + r \cdot x}{(1 + \sqrt{3}) \cdot s + r \cdot x}], -7 - 4\sqrt{3}]) / (3^{1/4} \cdot r \cdot \sqrt{a + b \cdot x^3} \cdot \sqrt{(s \cdot (s + r \cdot x)) / ((1 + \sqrt{3}) \cdot s + r \cdot x)^2}), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a]$$
Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{ax^2 + bx^5}} dx &= \frac{2x\sqrt{ax^2 + bx^5}}{7b} - \frac{(4a) \int \frac{x^2}{\sqrt{ax^2 + bx^5}} dx}{7b} \\
&= \frac{2x\sqrt{ax^2 + bx^5}}{7b} - \frac{(4ax\sqrt{a + bx^3}) \int \frac{x}{\sqrt{a + bx^3}} dx}{7b\sqrt{ax^2 + bx^5}} \\
&= \frac{2x\sqrt{ax^2 + bx^5}}{7b} - \frac{(4ax\sqrt{a + bx^3}) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{7b^{4/3}\sqrt{ax^2 + bx^5}} - \frac{(4\sqrt{2(2-\sqrt{3})}a^{4/3}x\sqrt{a + bx^3}) \int \frac{1}{\sqrt{a + bx^3}} dx}{7b^{4/3}\sqrt{ax^2 + bx^5}} \\
&= -\frac{8ax(a + bx^3)}{7b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})\sqrt{ax^2 + bx^5}} + \frac{2x\sqrt{ax^2 + bx^5}}{7b} + \frac{4\sqrt[4]{3}\sqrt{2 - \sqrt{3}}a^{4/3}x(\sqrt[3]{a} + \sqrt[3]{bx})}{7b^{5/3}\sqrt{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0233591, size = 68, normalized size = 0.13

$$\frac{2x^3 \left(-a\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{7b\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a]))/(7*b*Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.008, size = 676, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^5+a*x^2)^(1/2),x)`

[Out]
$$\begin{aligned} & -2/21*x*(3*I*(-2*(-b*x+(-b^2*a)^(1/3)))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)* \\ & (-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3)) \\ & ^{(1/2)*\text{EllipticE}(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*a-2*I*(-2*(-b*x+(-b^2*a)^(1/3)))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*\text{EllipticF}(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*a-3*x^5*b^3+3*(-2*(-b*x+(-b^2*a)^(1/3)))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*\text{EllipticE}(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*a-3*a*b^2*x^2)/(b*x^5+a*x^2)^(1/2)/b^3 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^5/sqrt(b*x^5 + a*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2x^3}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*x^3/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**5/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)

$$3.293 \quad \int \frac{x^2}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=484

$$\frac{2\sqrt{2}\sqrt[3]{ax}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) \sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{ax}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[4]{3}b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \quad b^{2/3}$$

[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.189697, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {2032, 303, 218, 1877}

$$\frac{2\sqrt{2}\sqrt[3]{ax}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}}\right) \mid -7 - 4\sqrt{3}\right) \sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{ax}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}{\sqrt[4]{3}b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \quad b^{2/3} \sqrt{\frac{\sqrt[3]{a}}{(1+\sqrt{3})}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

```
*a^(1/3) + b^(1/3)*x]], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2032

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p_], x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax^2 + bx^5}} dx &= \frac{(x\sqrt{a + bx^3}) \int \frac{x}{\sqrt{a+bx^3}} dx}{\sqrt{ax^2 + bx^5}} \\
&= \frac{(x\sqrt{a + bx^3}) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}\sqrt{ax^2 + bx^5}} + \frac{(\sqrt{2(2-\sqrt{3})}\sqrt[3]{ax}\sqrt{a + bx^3}) \int \frac{1}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}\sqrt{ax^2 + bx^5}} \\
&= \frac{2x(a + bx^3)}{b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})\sqrt{ax^2 + bx^5}} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{ax}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0110842, size = 55, normalized size = 0.11

$$\frac{x^3 \sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{2\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]

[Out] (x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.005, size = 394, normalized size = 0.8

$$\frac{-\frac{i}{6}x\sqrt{3}}{b^2} (-b^2a)^{\frac{2}{3}} \sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a} - 2bx - \sqrt[3]{-b^2a}\right)} \frac{1}{\sqrt[3]{-b^2a}} \sqrt{-2\frac{-bx + \sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}(i\sqrt{3} - 3)}} \sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a} + 2bx + \sqrt[3]{-b^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^5+a*x^2)^(1/2),x)

[Out] -1/6*I*x*3^(1/2)*(-b^2*a)^(2/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3))

$$\frac{1}{3} / (I \cdot 3^{1/2} - 3)^{1/2} * (-I * (I \cdot 3^{1/2}) * (-b^2 a)^{1/3} + 2 * b * x + (-b^2 a)^{1/3}) * 3^{1/2} / (-b^2 a)^{1/3})^{1/2} * (I * \text{EllipticE}(1/6 * 3^{1/2} * 2^{1/2}) * (-I * (I \cdot 3^{1/2}) * (-b^2 a)^{1/3} - 2 * b * x - (-b^2 a)^{1/3}) * 3^{1/2} / (-b^2 a)^{1/3})^{1/2}, 2^{1/2} * (I \cdot 3^{1/2} / (I \cdot 3^{1/2} - 3))^{1/2} * 3^{1/2} - 3 * \text{EllipticE}(1/6 * 3^{1/2} * 2^{1/2}) * (-I * (I \cdot 3^{1/2}) * (-b^2 a)^{1/3} - 2 * b * x - (-b^2 a)^{1/3}) * 3^{1/2} / (-b^2 a)^{1/3})^{1/2}, 2^{1/2} * (I \cdot 3^{1/2} / (I \cdot 3^{1/2} - 3))^{1/2} + 2 * \text{EllipticF}(1/6 * 3^{1/2} * 2^{1/2}) * (-I * (I \cdot 3^{1/2}) * (-b^2 a)^{1/3} - 2 * b * x - (-b^2 a)^{1/3}) * 3^{1/2} / (-b^2 a)^{1/3})^{1/2}, 2^{1/2} * (I \cdot 3^{1/2} / (I \cdot 3^{1/2} - 3))^{1/2}) / (b * x^5 + a * x^2)^{1/2} / b^2$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

$$3.294 \quad \int \frac{1}{x\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=510

$$\frac{\sqrt{2}\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) \sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}{\sqrt[4]{3}a^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \quad 2a^{2/3} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}$$

[Out] $(b^{(1/3)}*x*(a + b*x^3))/(a*((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) - \operatorname{Sqrt}[a*x^2 + b*x^5]/(a*x^2) - (3^{(1/4)}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\frac{(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\operatorname{Sqrt}[3]])/(2*a^{(2/3)}*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (\operatorname{Sqrt}[2]*b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\frac{(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\operatorname{Sqrt}[3]])/(3^{(1/4)}*a^{(2/3)}*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.273408, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2025, 2032, 303, 218, 1877}

$$\frac{\sqrt{2}\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}}\right) | -7 - 4\sqrt{3}\right) \sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}{\sqrt[4]{3}a^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \quad 2a^{2/3} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x*\operatorname{Sqrt}[a*x^2 + b*x^5]), x]$

[Out] $(b^{(1/3)}*x*(a + b*x^3))/(a*((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) - \operatorname{Sqrt}[a*x^2 + b*x^5]/(a*x^2) - (3^{(1/4)}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\frac{(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\operatorname{Sqrt}[3]])/(2*a^{(2/3)}*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (\operatorname{Sqrt}[2]*b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\frac{(1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\operatorname{Sqrt}[3]])/(3^{(1/4)}*a^{(2/3)}*\operatorname{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$


```
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x]^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax^2 + bx^5}} dx &= -\frac{\sqrt{ax^2 + bx^5}}{ax^2} + \frac{b \int \frac{x^2}{\sqrt{ax^2 + bx^5}} dx}{2a} \\
&= -\frac{\sqrt{ax^2 + bx^5}}{ax^2} + \frac{(bx\sqrt{a + bx^3}) \int \frac{x}{\sqrt{a + bx^3}} dx}{2a\sqrt{ax^2 + bx^5}} \\
&= -\frac{\sqrt{ax^2 + bx^5}}{ax^2} + \frac{(b^{2/3}x\sqrt{a + bx^3}) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{2a\sqrt{ax^2 + bx^5}} + \frac{\left(\sqrt{\frac{1}{2}(2 - \sqrt{3})}b^{2/3}x\sqrt{a + bx^3}\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{a^{2/3}\sqrt{ax^2 + bx^5}} \\
&= \frac{\sqrt[3]{bx}(a + bx^3)}{a((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})\sqrt{ax^2 + bx^5}} - \frac{\sqrt{ax^2 + bx^5}}{ax^2} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{2a^{2/3}} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})}}
\end{aligned}$$

Mathematica [C] time = 0.0107877, size = 50, normalized size = 0.1

$$-\frac{\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, -\frac{bx^3}{a}\right)}{\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]

[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sqrt[x^2*(a + b*x^3)])

Maple [A] time = 0.009, size = 673, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^5+a*x^2)^(1/2),x)`

[Out]
$$\frac{1}{12} \cdot (3 \cdot I \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2}) / ((-b^2 a)^{1/3})^{1/2} \cdot (-2 \cdot (-b \cdot x + (-b^2 a)^{1/3})) / ((-b^2 a)^{1/3}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} + 2 \cdot b \cdot x + (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2} \cdot \text{EllipticE}(1/6 \cdot 3^{1/2} \cdot 2^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2}, 2^{1/2} \cdot (I \cdot 3^{1/2}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot 3^{1/2} \cdot (-b^2 a)^{2/3} \cdot x - 2 \cdot I \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2} \cdot (-2 \cdot (-b \cdot x + (-b^2 a)^{1/3})) / ((-b^2 a)^{1/3}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} + 2 \cdot b \cdot x + (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2} \cdot \text{EllipticF}(1/6 \cdot 3^{1/2} \cdot 2^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2}, 2^{1/2} \cdot (I \cdot 3^{1/2}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot 3^{1/2} \cdot (-b^2 a)^{2/3} \cdot x + 3 \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2} \cdot (-2 \cdot (-b \cdot x + (-b^2 a)^{1/3})) / ((-b^2 a)^{1/3}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} + 2 \cdot b \cdot x + (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2} \cdot \text{EllipticE}(1/6 \cdot 3^{1/2} \cdot 2^{1/2} \cdot (-I \cdot (I \cdot 3^{1/2}) \cdot (-b^2 a)^{1/3} - 2 \cdot b \cdot x - (-b^2 a)^{1/3}) \cdot 3^{1/2} / ((-b^2 a)^{1/3})^{1/2}, 2^{1/2} \cdot (I \cdot 3^{1/2}) / (I \cdot 3^{1/2} - 3))^{1/2} \cdot (-b^2 a)^{2/3} \cdot x - 12 \cdot b^2 \cdot x^3 - 12 \cdot a \cdot b) / (b \cdot x^5 + a \cdot x^2)^{1/2} / a / b$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^5 + ax^2}}{bx^6 + ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^6 + a*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2(a+bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)

$$3.295 \quad \int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=265

$$\frac{7a^{5/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{40\sqrt[4]{3}b^2 \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b}$$

[Out] $(-7*a*\text{Sqrt}[a*x^2 + b*x^5])/(20*b^2*\text{Sqrt}[x]) + (x^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^5])/(5*b) + (7*a^{(5/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(40*3^{(1/4)}*b^2*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.281867, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2024, 2032, 329, 225}

$$\frac{7a^{5/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{40\sqrt[4]{3}b^2 \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(13/2)}/\text{Sqrt}[a*x^2 + b*x^5], x]$

[Out] $(-7*a*\text{Sqrt}[a*x^2 + b*x^5])/(20*b^2*\text{Sqrt}[x]) + (x^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^5])/(5*b) + (7*a^{(5/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(40*3^{(1/4)}*b^2*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x^2 + b*x^5])$

Rule 2024

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x]
  - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))
  *(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n]
  && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])
  *(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x]
  && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]},
  Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x]
  && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]},
  Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}}{\sqrt{ax^2 + bx^5}} dx &= \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b} - \frac{(7a) \int \frac{x^{7/2}}{\sqrt{ax^2 + bx^5}} dx}{10b} \\
&= -\frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b} + \frac{(7a^2) \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}} dx}{40b^2} \\
&= -\frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b} + \frac{(7a^2x\sqrt{a + bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{40b^2\sqrt{ax^2 + bx^5}} \\
&= -\frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b} + \frac{(7a^2x\sqrt{a + bx^3}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{20b^2\sqrt{ax^2 + bx^5}} \\
&= -\frac{7a\sqrt{ax^2 + bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2 + bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right)\right)}{40\sqrt[4]{3}b^2 \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0323308, size = 86, normalized size = 0.32

$$\frac{x^{3/2} \left(7a^2 \sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{bx^3}{a}\right) - 7a^2 - 3abx^3 + 4b^2x^6 \right)}{20b^2 \sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.533, size = 2017, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)/(b*x^5+a*x^2)^(1/2), x)

```
[Out] -1/20/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3/(-b^2*a)^(1/3)*(14*I*(-I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*3^(1/2)*x^2*a^2*b^2-28*I*(-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I
*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b
^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x*a^2
*b+14*I*(-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2
)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*a^2-14*(-I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b
^2-4*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2
*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*((b*x^3+a
)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^3*b^2+28*(-I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/
3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ellipt
icF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x
*a^2*b-14*(-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((
I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*a^2+12*x^3*((b*x^3+a)*x)^(1/2)*b^2*(
-b^2*a)^(1/3)*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x
+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)+7*
I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(
1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*((b*x^3+a)*x)
^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*a*b-21*a*((b*x^3+a)*x)^(1/2)*b*(-b^2*a)^(1/3)*
(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/
3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/((b*x^3+a)*x)^(
1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3))
```


$+2*b*x+(-b^2*a)^{(1/3)}*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3}))^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}x^{\frac{9}{2}}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)

$$3.296 \quad \int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=525

$$\frac{5(1-\sqrt{3})a^{4/3}x^{3/2}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}} + \frac{5\sqrt[4]{3}a^{4/3}x^{3/2}(\sqrt[3]{a}+\sqrt[3]{bx})}{8b^{5/3}\sqrt{ax^2+bx^5}}$$

[Out] $(-5*(1 + \operatorname{Sqrt}[3])*a*x^{(3/2)}*(a + b*x^3))/(8*b^{(5/3)}*(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (x^{(3/2)}*\operatorname{Sqrt}[a*x^2 + b*x^5])/(4*b) + (5*3^{(1/4)}*a^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{EllipticE}[\operatorname{ArcCos}[(a^{(1/3)} + (1 - \operatorname{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)], (2 + \operatorname{Sqrt}[3])/4])/(8*b^{(5/3)}*\operatorname{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (5*(1 - \operatorname{Sqrt}[3])*a^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcCos}[(a^{(1/3)} + (1 - \operatorname{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)], (2 + \operatorname{Sqrt}[3])/4])/(16*3^{(1/4)}*b^{(5/3)}*\operatorname{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.500006, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5(1-\sqrt{3})a^{4/3}x^{3/2}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}} + \frac{5\sqrt[4]{3}a^{4/3}x^{3/2}(\sqrt[3]{a}+\sqrt[3]{bx})}{8b^{5/3}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^{(11/2)}/\operatorname{Sqrt}[a*x^2 + b*x^5], x]$

[Out] $(-5*(1 + \operatorname{Sqrt}[3])*a*x^{(3/2)}*(a + b*x^3))/(8*b^{(5/3)}*(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (x^{(3/2)}*\operatorname{Sqrt}[a*x^2 + b*x^5])/(4*b) + (5*3^{(1/4)}*a^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{EllipticE}[\operatorname{ArcCos}[(a^{(1/3)} + (1 - \operatorname{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)], (2 + \operatorname{Sqrt}[3])/4])/(8*b^{(5/3)}*\operatorname{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5]) + (5*(1 - \operatorname{Sqrt}[3])*a^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcCos}[(a^{(1/3)} + (1 - \operatorname{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)], (2 + \operatorname{Sqrt}[3])/4])/(16*3^{(1/4)}*b^{(5/3)}*\operatorname{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \operatorname{Sqrt}[3])*b^{(1/3)}*x)^2]*\operatorname{Sqrt}[a*x^2 + b*x^5])$

$$\frac{1}{3}x + b^{2/3}x^2 / (a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2 * \text{EllipticE}[\text{ArcCos}[\frac{a^{1/3} + (1 - \sqrt{3})b^{1/3}x}{a^{1/3} + (1 + \sqrt{3})b^{1/3}x}], \frac{(2 + \sqrt{3})/4}{(8b^{5/3}\sqrt{(b^{1/3}x)(a^{1/3} + b^{1/3}x)}) / (a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2} * \sqrt{ax^2 + bx^5}] + (5(1 - \sqrt{3})a^{4/3}x^{3/2}(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)}) / (a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2 * \text{EllipticF}[\text{ArcCos}[\frac{a^{1/3} + (1 - \sqrt{3})b^{1/3}x}{a^{1/3} + (1 + \sqrt{3})b^{1/3}x}], \frac{(2 + \sqrt{3})/4}{(16 \cdot 3^{1/4} b^{5/3} \sqrt{(b^{1/3}x)(a^{1/3} + b^{1/3}x)}) / (a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2} * \sqrt{ax^2 + bx^5}]$$

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqrt[a + b*x^6], x], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol]
:> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqrt[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11/2}}{\sqrt{ax^2 + bx^5}} dx &= \frac{x^{3/2}\sqrt{ax^2 + bx^5}}{4b} - \frac{(5a) \int \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}} dx}{8b} \\
 &= \frac{x^{3/2}\sqrt{ax^2 + bx^5}}{4b} - \frac{(5ax\sqrt{a + bx^3}) \int \frac{x^{3/2}}{\sqrt{a + bx^3}} dx}{8b\sqrt{ax^2 + bx^5}} \\
 &= \frac{x^{3/2}\sqrt{ax^2 + bx^5}}{4b} - \frac{(5ax\sqrt{a + bx^3}) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a + bx^6}} dx, x, \sqrt{x}\right)}{4b\sqrt{ax^2 + bx^5}} \\
 &= \frac{x^{3/2}\sqrt{ax^2 + bx^5}}{4b} + \frac{(5ax\sqrt{a + bx^3}) \operatorname{Subst}\left(\int \frac{(-1 + \sqrt{3})a^{2/3} - 2b^{2/3}x^4}{\sqrt{a + bx^6}} dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2 + bx^5}} + \frac{(5(1 - \sqrt{3})a^{5/3}x\sqrt{a + bx^3})}{8b^{5/3}\sqrt{ax^2 + bx^5}} \\
 &= -\frac{5(1 + \sqrt{3})ax^{3/2}(a + bx^3)}{8b^{5/3}(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})\sqrt{ax^2 + bx^5}} + \frac{x^{3/2}\sqrt{ax^2 + bx^5}}{4b} + \frac{5\sqrt[4]{3}a^{4/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx})}{8b^{5/3}\sqrt{ax^2 + bx^5}} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx}}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})}}
 \end{aligned}$$

Mathematica [C] time = 0.0257397, size = 70, normalized size = 0.13

$$\frac{x^{7/2} \left(-a\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{4b\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] time = 0.052, size = 2586, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x^3+a)*(-5*I*(-b^2*a)^(2/3)*3^(1/2)*x*a-5*I*3^(1/2)*x^3*a*b^2-10*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*(-b^2*a)^(1/3)*x^2*a*b+15*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*(-b^2*a)^(1/3)*x^2*a*b+5*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*3^(1/2)*a^2*b-5*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^2*a*b+20*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I
```

$$\begin{aligned}
& 3^{1/2})/(-b*x+(-b^2*a)^{1/3})^{1/2}, ((I*3^{1/2}+3)*(-1+I*3^{1/2})/(1+I*3^{1/2})) / (I*3^{1/2}-3)^{1/2}) * (-b^2*a)^{2/3} * x * a - 30 * (-I*3^{1/2}-3) * x * b / (-1 \\
& + I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) / (1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2 \\
& *a)^{1/3} - 2*b*x - (-b^2*a)^{1/3}) / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} \\
& * \text{EllipticE}((-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3}))^{1/2}, \\
& ((I*3^{1/2}+3) * (-1+I*3^{1/2}) / (1+I*3^{1/2}) / (I*3^{1/2}-3))^{1/2} * (-b^2*a)^{2/3} * x * a + 10 * I * (-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) / (1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3}) / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * \text{EllipticE}((-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3}))^{1/2}, ((I*3^{1/2}+3) * (-1+I*3^{1/2}) / (1+I*3^{1/2}) / (I*3^{1/2}-3))^{1/2} * (-b^2*a)^{2/3} * 3^{1/2} * x * a + 10 * (-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) / (1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3}) / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * \text{EllipticF}((-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3}))^{1/2}, ((I*3^{1/2}+3) * (-1+I*3^{1/2}) / (1+I*3^{1/2}) / (I*3^{1/2}-3))^{1/2} * a^2 * b - 15 * (-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) / (1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * ((I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3}) / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3})^{1/2} * \text{EllipticE}((-I*3^{1/2}-3) * x * b / (-1+I*3^{1/2}) / (-b*x+(-b^2*a)^{1/3}))^{1/2}, ((I*3^{1/2}+3) * (-1+I*3^{1/2}) / (1+I*3^{1/2}) / (I*3^{1/2}-3))^{1/2} * a^2 * b + I * ((b*x^3+a)*x)^{1/2} * (1/b^2*x*(-b*x+(-b^2*a)^{1/3})) * (I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) * (I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3})^{1/2} * 3^{1/2} * x^2 * b^2 - 3 * ((b*x^3+a)*x)^{1/2} * (1/b^2*x*(-b*x+(-b^2*a)^{1/3})) * (I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) * (I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3})^{1/2} * x^2 * b^2 - 5 * I * (-b^2*a)^{1/3} * 3^{1/2} * x^2 * a * b + 15 * a * x^3 * b^2 + 15 * (-b^2*a)^{1/3} * x^2 * a * b + 15 * (-b^2*a)^{2/3} * x * a) / (b*x^5+a*x^2)^{1/2} / b^3 / ((b*x^3+a)*x)^{1/2} / (I*3^{1/2}-3) / (1/b^2*x*(-b*x+(-b^2*a)^{1/3})) * (I*3^{1/2}) * (-b^2*a)^{1/3} + 2*b*x + (-b^2*a)^{1/3}) * (I*3^{1/2}) * (-b^2*a)^{1/3} - 2*b*x - (-b^2*a)^{1/3})^{1/2}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11/2}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}x^{\frac{7}{2}}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)

$$3.297 \quad \int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*b^(3/2))

Rubi [A] time = 0.0896446, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*b^(3/2))

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^{9/2}}{\sqrt{ax^2 + bx^5}} dx &= \frac{\sqrt{x}\sqrt{ax^2 + bx^5}}{3b} - \frac{a \int \frac{x^{3/2}}{\sqrt{ax^2 + bx^5}} dx}{2b} \\ &= \frac{\sqrt{x}\sqrt{ax^2 + bx^5}}{3b} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}}\right)}{3b} \\ &= \frac{\sqrt{x}\sqrt{ax^2 + bx^5}}{3b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2 + bx^5}}\right)}{3b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0373513, size = 81, normalized size = 1.25

$$\frac{\sqrt{bx^{5/2}}(a + bx^3) - ax\sqrt{a + bx^3} \tanh^{-1}\left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a + bx^3}}\right)}{3b^{3/2}\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5], x]
```

```
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2)
)/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] time = 0.051, size = 3347, normalized size = 51.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2), x)
```

```
[Out] 1/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(6*I*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-
```


$$\begin{aligned} & \left(\frac{1}{3} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} / \left((1 + I * 3^{\frac{1}{2}}) \right) \\ & \left(\frac{1}{3} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} / \left((-1 + I * 3^{\frac{1}{2}}) \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \text{EllipticF} \left(\left(-I * 3^{\frac{1}{2}} - 3 \right) \right. \\ & * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right), \left(\left(I * 3^{\frac{1}{2}} + 3 \right) * \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(1 + I * 3^{\frac{1}{2}} \right) \right) / \left(I * 3^{\frac{1}{2}} - 3 \right) \left(\frac{1}{2} \right) * (-b^2 a)^{\frac{1}{3}} * x * a * b - 12 * \left(-I * 3^{\frac{1}{2}} - 3 \right) \\ & * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} / \left(1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \text{EllipticPi} \left(\left(-I * 3^{\frac{1}{2}} - 3 \right) * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \right) \left(\frac{1}{2} \right), \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(I * 3^{\frac{1}{2}} - 3 \right), \left(\left(I * 3^{\frac{1}{2}} + 3 \right) * \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(1 + I * 3^{\frac{1}{2}} \right) \right) / \left(I * 3^{\frac{1}{2}} - 3 \right) \left(\frac{1}{2} \right) * (-b^2 a)^{\frac{1}{3}} * x * a * b + I * \left((b^2 x^3 + a) * x \right)^{\frac{1}{2}} \\ & * \left(1/b^2 * x * (-b^2 x + (-b^2 a)^{\frac{1}{3}}) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} \right) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} \right) \right)^{\frac{1}{2}} * 3^{\frac{1}{2}} * x * b^2 - 6 \\ & * \left(-I * 3^{\frac{1}{2}} - 3 \right) * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} / \left(1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \text{EllipticF} \left(\left(-I * 3^{\frac{1}{2}} - 3 \right) * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \right) \left(\frac{1}{2} \right), \left(\left(I * 3^{\frac{1}{2}} + 3 \right) * \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(1 + I * 3^{\frac{1}{2}} \right) \right) / \left(I * 3^{\frac{1}{2}} - 3 \right) \left(\frac{1}{2} \right) * (-b^2 a)^{\frac{1}{3}} * a + 6 * \left(-I * 3^{\frac{1}{2}} - 3 \right) * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} / \left(1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \left(\frac{1}{3} \right) * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) * \text{EllipticPi} \left(\left(-I * 3^{\frac{1}{2}} - 3 \right) * x * b / \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(-b^2 x + (-b^2 a)^{\frac{1}{3}} \right) \right) \left(\frac{1}{2} \right), \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(I * 3^{\frac{1}{2}} - 3 \right), \left(\left(I * 3^{\frac{1}{2}} + 3 \right) * \left(-1 + I * 3^{\frac{1}{2}} \right) / \left(1 + I * 3^{\frac{1}{2}} \right) \right) / \left(I * 3^{\frac{1}{2}} - 3 \right) \left(\frac{1}{2} \right) * (-b^2 a)^{\frac{1}{3}} * a - 3 * x * \left((b^2 x^3 + a) * x \right)^{\frac{1}{2}} * b^2 * \left(1/b^2 * x * (-b^2 x + (-b^2 a)^{\frac{1}{3}}) \right) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} \right) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} \right) \left(\frac{1}{2} \right) / \left((b^2 x^3 + a) * x \right)^{\frac{1}{2}} / \left(I * 3^{\frac{1}{2}} - 3 \right) / \left(1/b^2 * x * (-b^2 x + (-b^2 a)^{\frac{1}{3}}) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} + 2 b^2 x + (-b^2 a)^{\frac{1}{3}} \right) * \left(I * 3^{\frac{1}{2}} * (-b^2 a)^{\frac{1}{3}} - 2 b^2 x - (-b^2 a)^{\frac{1}{3}} \right) \right)^{\frac{1}{2}} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{b x^5 + a x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)

Fricas [A] time = 1.13319, size = 358, normalized size = 5.51

$$\left[\frac{a\sqrt{b} \log\left(-8b^2x^6 - 8abx^3 + 4\sqrt{bx^5 + ax^2}(2bx^3 + a)\sqrt{b}\sqrt{x} - a^2\right) + 4\sqrt{bx^5 + ax^2}b\sqrt{x}}{12b^2}, \frac{a\sqrt{-b} \arctan\left(\frac{2\sqrt{bx^5 + ax^2}\sqrt{-b}\sqrt{x}}{2bx^3 + a}\right)}{6b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.39752, size = 59, normalized size = 0.91

$$\frac{\sqrt{bx^3 + ax^2}^{\frac{3}{2}}}{3b} + \frac{a \log\left(\left|-\sqrt{bx^2} + \sqrt{bx^3 + a}\right|\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/b + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a)))/b^(3/2)

$$3.298 \quad \int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=237

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}$$

[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.225232, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2024, 2032, 329, 225}

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rule 2024

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
+ 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 225

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\sqrt{ax^2 + bx^5}} dx &= \frac{\sqrt{ax^2 + bx^5}}{2b\sqrt{x}} - \frac{a \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}} dx}{4b} \\
&= \frac{\sqrt{ax^2 + bx^5}}{2b\sqrt{x}} - \frac{(ax\sqrt{a + bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{4b\sqrt{ax^2 + bx^5}} \\
&= \frac{\sqrt{ax^2 + bx^5}}{2b\sqrt{x}} - \frac{(ax\sqrt{a + bx^3}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{2b\sqrt{ax^2 + bx^5}} \\
&= \frac{\sqrt{ax^2 + bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right) \middle| \frac{1}{4}(2 + \sqrt{3})\right)}{4\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0249058, size = 70, normalized size = 0.3

$$\frac{x^{3/2} \left(-a\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{bx^3}{a}\right) + a + bx^3 \right)}{2b\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -(b*x^3)/a]))/(2*b*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.032, size = 1793, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(b*x^5+a*x^2)^(1/2), x)

[Out] 1/2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2/(-b^2*a)^(1/3)*(2*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}x^{\frac{3}{2}}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)

$$3.299 \quad \int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=492

$$(1 - \sqrt{3}) \sqrt[3]{ax^{3/2}} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \text{EllipticF} \left(\cos^{-1} \left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}} \right), \frac{1}{4} (2 + \sqrt{3}) \right) \sqrt[4]{3} \sqrt[3]{ax^{3/2}} (\sqrt[3]{a} + \sqrt[3]{bx})$$

$$2 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}$$

[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.398378, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2032, 329, 308, 225, 1881}

$$(1 - \sqrt{3}) \sqrt[3]{ax^{3/2}} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F \left(\cos^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right) \sqrt[4]{3} \sqrt[3]{ax^{3/2}} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}}$$

$$2 \sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]

[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

$$\begin{aligned} & + (1 + \sqrt{3})b^{(1/3)*x}], (2 + \sqrt{3})/4)/(b^{(2/3)*\sqrt{3}}(b^{(1/3)*x}(a \\ & ^{(1/3)} + b^{(1/3)*x}))/((a^{(1/3)} + (1 + \sqrt{3})b^{(1/3)*x})^2)*\sqrt{a*x^2 + b* \\ & x^5}) - ((1 - \sqrt{3})a^{(1/3)*x^{(3/2)}}(a^{(1/3)} + b^{(1/3)*x})*\sqrt{(a^{(2/3)} \\ & - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}})/(a^{(1/3)} + (1 + \sqrt{3})b^{(1/3)*x})^2}*E \\ & llipticF[ArcCos[(a^{(1/3)} + (1 - \sqrt{3})b^{(1/3)*x})/(a^{(1/3)} + (1 + \sqrt{3}) \\ &)b^{(1/3)*x}], (2 + \sqrt{3})/4)]/(2*3^{(1/4)}*b^{(2/3)*\sqrt{3}}(b^{(1/3)*x}(a^{(1/3)} \\ &) + b^{(1/3)*x}))/((a^{(1/3)} + (1 + \sqrt{3})b^{(1/3)*x})^2)*\sqrt{a*x^2 + b*x^5}) \end{aligned}$$

Rule 2032

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
+ b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^{(1/4)}*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r =
Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^{(1/4)}*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*E
```

lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4)]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}} dx &= \frac{\left(x\sqrt{a + bx^3}\right) \int \frac{x^{3/2}}{\sqrt{a+bx^3}} dx}{\sqrt{ax^2 + bx^5}} \\ &= \frac{\left(2x\sqrt{a + bx^3}\right) \text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{\sqrt{ax^2 + bx^5}} \\ &= -\frac{\left(x\sqrt{a + bx^3}\right) \text{Subst}\left(\int \frac{(-1+\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2 + bx^5}} - \frac{\left((1 - \sqrt{3}) a^{2/3} x \sqrt{a + bx^3}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2 + bx^5}} \\ &= \frac{(1 + \sqrt{3}) x^{3/2} (a + bx^3)}{b^{2/3} (\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}) \sqrt{ax^2 + bx^5}} - \frac{\sqrt[4]{3} \sqrt[3]{ax^3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} E\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}}\right)\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \end{aligned}$$

Mathematica [C] time = 0.0131643, size = 57, normalized size = 0.12

$$\frac{2x^{7/2} \sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^3}{a}\right)}{5\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.034, size = 2374, normalized size = 4.8

result too large to display

$$\begin{aligned} & (-b^2a)^{1/3} + 2bx + (-b^2a)^{1/3} / (1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2} * ((I\sqrt{3})^{1/2} * (-b^2a)^{1/3} - 2bx - (-b^2a)^{1/3}) / (-1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2} * \text{EllipticF}((-I\sqrt{3} - 3) * x * b / (-1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2}, ((I\sqrt{3} + 3) * (-1 + I\sqrt{3}) / (1 + I\sqrt{3}) / (I\sqrt{3} - 3))^{1/2}) * a * b - 3 * (-I\sqrt{3} - 3) * x * b / (-1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2} * ((I\sqrt{3})^{1/2} * (-b^2a)^{1/3} + 2bx + (-b^2a)^{1/3}) / (1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2} * ((I\sqrt{3})^{1/2} * (-b^2a)^{1/3} - 2bx - (-b^2a)^{1/3}) / (-1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2} * \text{EllipticE}((-I\sqrt{3} - 3) * x * b / (-1 + I\sqrt{3}) / (-bx + (-b^2a)^{1/3})^{1/2}, ((I\sqrt{3} + 3) * (-1 + I\sqrt{3}) / (1 + I\sqrt{3}) / (I\sqrt{3} - 3))^{1/2}) * a * b - I * (-b^2a)^{1/3} * \sqrt{3} * x^2 * b - I * (-b^2a)^{2/3} * \sqrt{3} * x + 3 * b^2 * x^3 + 3 * (-b^2a)^{1/3} * x^2 * b + 3 * (-b^2a)^{2/3} * x / (b * x^5 + a * x^2)^{1/2} / b^2 / ((b * x^3 + a) * x)^{1/2} / (I\sqrt{3} - 3) / (1/b^2 * x * (-bx + (-b^2a)^{1/3})) * (I\sqrt{3})^{1/2} * (-b^2a)^{1/3} + 2bx + (-b^2a)^{1/3}) * (I\sqrt{3})^{1/2} * (-b^2a)^{1/3} - 2bx - (-b^2a)^{1/3})^{1/2} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)

$$3.300 \quad \int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=36

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])

Rubi [A] time = 0.0475052, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{ax^2 + bx^5}} dx = \frac{2}{3} \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}} \right)$$

$$= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx^{5/2}}}{\sqrt{ax^2 + bx^5}} \right)}{3\sqrt{b}}$$

Mathematica [A] time = 0.0120326, size = 59, normalized size = 1.64

$$\frac{2x\sqrt{a + bx^3} \tanh^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a + bx^3}} \right)}{3\sqrt{b}\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5], x]

[Out] (2*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.118, size = 480, normalized size = 13.3

$$-4 \frac{x^{3/2} (bx^3 + a) (-1 + i\sqrt{3}) (-bx + \sqrt[3]{-b^2a})^2}{\sqrt{bx^5 + ax^2b^2} \sqrt{(bx^3 + a)x(i\sqrt{3} - 3)}} \sqrt{\frac{(i\sqrt{3} - 3)xb}{(-1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a} + 2bx + \sqrt[3]{-b^2a}}{(1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a}}{(-1 + i\sqrt{3})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2), x)

[Out] -4*x^(3/2)*(b*x^3+a)*(-1+I*3^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2)*(-b*x+(-b^2*a)^(1/3))^2*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2), ((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-EllipticPi((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2),

$$\frac{(-1 + I\sqrt{3})/(I\sqrt{3} - 3), ((I\sqrt{3} + 3)(-1 + I\sqrt{3})/(1 + I\sqrt{3}))/(I\sqrt{3} - 3)^{(1/2)}}{(bx^5 + ax^2)^{(1/2)}/b^2 / ((bx^3 + a)x)^{(1/2)}/(I\sqrt{3} - 3) / (1/b^2 * x * (-bx + (-b^2a)^{(1/3)}) * (I\sqrt{3}) * (-b^2a)^{(1/3)} + 2 * bx + (-b^2a)^{(1/3)}) * (I\sqrt{3}) * (-b^2a)^{(1/3)} - 2 * bx - (-b^2a)^{(1/3)})^{(1/2)}}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)

Fricas [A] time = 1.33068, size = 247, normalized size = 6.86

$$\left[\frac{\log\left(-8b^2x^6 - 8abx^3 - 4\sqrt{bx^5 + ax^2}(2bx^3 + a)\sqrt{b}\sqrt{x} - a^2\right)}{6\sqrt{b}}, -\frac{\sqrt{-b} \arctan\left(\frac{2\sqrt{bx^5 + ax^2}\sqrt{-b}\sqrt{x}}{2bx^3 + a}\right)}{3b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a))/b]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)

Giac [A] time = 1.28855, size = 55, normalized size = 1.53

$$-\frac{2 \arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}} + \frac{2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/3*arctan(sqrt(b)/sqrt(-b))/sqrt(-b)

3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}} dx$

Optimal. Leaf size=203

$$\frac{x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx}}\right), \frac{1}{4} (2 + \sqrt{3})\right)}{\sqrt[4]{3} \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}$$

[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/((3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.165711, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 329, 225}

$$\frac{x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}\right) \middle| \frac{1}{4} (2 + \sqrt{3})\right)}{\sqrt[4]{3} \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5], x]

[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/((3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(

FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
 erQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
 n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
 ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 225

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
 s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
 (s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
 + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
 t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}} dx &= \frac{(x\sqrt{a + bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{\sqrt{ax^2 + bx^5}} \\ &= \frac{(2x\sqrt{a + bx^3}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{\sqrt{ax^2 + bx^5}} \\ &= \frac{x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}}\right) \middle| \frac{1}{4} (2 + \sqrt{3})\right)}{\sqrt[4]{3} \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} \end{aligned}$$

Mathematica [C] time = 0.0135442, size = 55, normalized size = 0.27

$$\frac{2x^{3/2} \sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{bx^3}{a}\right)}{\sqrt{x^2 (a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]

[Out] (2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/Sqrt[x^2*(a + b*x^3)]

Maple [C] time = 0.126, size = 437, normalized size = 2.2

$$-4 \frac{x^{3/2} (bx^3 + a) \left(i\sqrt{3}x^2b^2 - 2i\sqrt[3]{-b^2a}\sqrt{3}xb + i(-b^2a)^{2/3} \sqrt{3} - b^2x^2 + 2\sqrt[3]{-b^2axb} - (-b^2a)^{2/3} \right)}{\sqrt{bx^5 + ax^2}\sqrt[3]{-b^2ab}\sqrt{(bx^3 + a)}x(i\sqrt{3} - 3)} \sqrt{\frac{(i\sqrt{3} - 3)xb}{(-1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2axb})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x)

[Out]
$$-4/(b*x^5+a*x^2)^{(1/2)}*x^{(3/2)}*(b*x^3+a)/(-b^2*a)^{(1/3)}/b*(-(I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)}))^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)}))^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)}))^{(1/2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(I*3^{(1/2)}*x^2*b^2-2*I*(-b^2*a)^{(1/3)}*3^{(1/2)}*x*b+I*(-b^2*a)^{(2/3)}*3^{(1/2)}-b^2*x^2+2*(-b^2*a)^{(1/3)}*x*b-(-b^2*a)^{(2/3)})/((b*x^3+a)*x)^{(1/2)}/(I*3^{(1/2)}-3)/(1/b^2*x*(-b*x+(-b^2*a)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)}))*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)}))^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(x)/sqrt(b*x^5 + a*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)`

[Out] `Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{bx^5 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)`

$$3.302 \quad \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=519

$$\frac{(1-\sqrt{3})\sqrt[3]{bx^{3/2}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

Rubi [A] time = 0.486824, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$\frac{(1-\sqrt{3})\sqrt[3]{bx^{3/2}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}F\left(\cos^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{bx}+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a}+\sqrt[3]{bx})}{(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]

[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])

```
)x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4)]/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
+ b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqrt[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{x}\sqrt{ax^2 + bx^5}} dx &= -\frac{2\sqrt{ax^2 + bx^5}}{ax^{3/2}} + \frac{(2b) \int \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}} dx}{a} \\
 &= -\frac{2\sqrt{ax^2 + bx^5}}{ax^{3/2}} + \frac{(2bx\sqrt{a + bx^3}) \int \frac{x^{3/2}}{\sqrt{a + bx^3}} dx}{a\sqrt{ax^2 + bx^5}} \\
 &= -\frac{2\sqrt{ax^2 + bx^5}}{ax^{3/2}} + \frac{(4bx\sqrt{a + bx^3}) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a + bx^6}} dx, x, \sqrt{x}\right)}{a\sqrt{ax^2 + bx^5}} \\
 &= -\frac{2\sqrt{ax^2 + bx^5}}{ax^{3/2}} - \frac{(2\sqrt[3]{bx}\sqrt{a + bx^3}) \operatorname{Subst}\left(\int \frac{(-1 + \sqrt{3})a^{2/3} - 2b^{2/3}x^4}{\sqrt{a + bx^6}} dx, x, \sqrt{x}\right)}{a\sqrt{ax^2 + bx^5}} - \frac{(2(1 - \sqrt{3})\sqrt[3]{bx}\sqrt{a + bx^3})}{a\sqrt{ax^2 + bx^5}} \\
 &= \frac{2(1 + \sqrt{3})\sqrt[3]{bx^{3/2}}(a + bx^3)}{a(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})\sqrt{ax^2 + bx^5}} - \frac{2\sqrt{ax^2 + bx^5}}{ax^{3/2}} - \frac{2\sqrt[4]{3}\sqrt[3]{bx^{3/2}}(\sqrt[3]{a} + \sqrt[3]{bx})}{a^{2/3}\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}}} \\
 &\qquad\qquad\qquad - \frac{2(1 - \sqrt{3})\sqrt[3]{bx}\sqrt{a + bx^3}}{a^{2/3}\sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}}}
 \end{aligned}$$

Mathematica [C] time = 0.0152446, size = 55, normalized size = 0.11

$$\frac{2\sqrt{x}\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; -\frac{bx^3}{a}\right)}{\sqrt{x^2(a + bx^3)}}$$

$$\begin{aligned}
&)) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * \text{EllipticF}((-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)})) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)}, ((I*3^{(1/2)} + 3)*(-1 + I*3^{(1/2)}) / (1 + I*3^{(1/2)})) / (I*3^{(1/2)} - 3)^{(1/2)} * ((b*x^3 + a)*x)^{(1/2)} * (-b^2*a)^{(2/3)} * x - 12 * (-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) / (1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}) / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * \text{EllipticE}((-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)} + 3)*(-1 + I*3^{(1/2)}) / (1 + I*3^{(1/2)})) / (I*3^{(1/2)} - 3)^{(1/2)} * ((b*x^3 + a)*x)^{(1/2)} * (-b^2*a)^{(2/3)} * x + I*(1/b^2*x*(-b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}))^{(1/2)} * 3^{(1/2)} * a*b + I*(1/b^2*x*(-b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}))^{(1/2)} * 3^{(1/2)} * x^3*b^2 + 4 * (-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) / (1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}) / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * \text{EllipticF}((-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)} + 3)*(-1 + I*3^{(1/2)}) / (1 + I*3^{(1/2)})) / (I*3^{(1/2)} - 3)^{(1/2)} * ((b*x^3 + a)*x)^{(1/2)} * a*b - 6 * (-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) / (1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}) / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * \text{EllipticE}((-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)} + 3)*(-1 + I*3^{(1/2)}) / (1 + I*3^{(1/2)})) / (I*3^{(1/2)} - 3)^{(1/2)} * ((b*x^3 + a)*x)^{(1/2)} * a*b - 2*I*((b*x^3 + a)*x)^{(1/2)} * 3^{(1/2)} * x^3*b^2 - 3*(1/b^2*x*(-b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}))^{(1/2)} * x^3*b^2 + 2*I*(-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) / (1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * ((I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}) / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)})^{(1/2)} * \text{EllipticE}((-I*3^{(1/2)} - 3)*x*b / (-1 + I*3^{(1/2)}) / (-b*x + (-b^2*a)^{(1/3)}))^{(1/2)}, ((I*3^{(1/2)} + 3)*(-1 + I*3^{(1/2)}) / (1 + I*3^{(1/2)})) / (I*3^{(1/2)} - 3)^{(1/2)} * ((b*x^3 + a)*x)^{(1/2)} * 3^{(1/2)} * a*b + 6*((b*x^3 + a)*x)^{(1/2)} * x^3*b^2 - 2*I*((b*x^3 + a)*x)^{(1/2)} * (-b^2*a)^{(1/3)} * 3^{(1/2)} * x^2*b + 6*((b*x^3 + a)*x)^{(1/2)} * (-b^2*a)^{(2/3)} * x - 3*(1/b^2*x*(-b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}))^{(1/2)} * a*b) / (b*x^5 + a*x^2)^{(1/2)} / b/a / (I*3^{(1/2)} - 3) / (1/b^2*x*(-b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} + 2*b*x + (-b^2*a)^{(1/3)}) * (I*3^{(1/2)} * (-b^2*a)^{(1/3)} - 2*b*x - (-b^2*a)^{(1/3)}))^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^6 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^6 + a*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```


$$3.303 \quad \int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}} dx$$

Optimal. Leaf size=27

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(3*a*x^{(5/2)})$

Rubi [A] time = 0.0400189, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(3/2)}*\text{Sqrt}[a*x^2 + b*x^5]),x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(3*a*x^{(5/2)})$

Rule 2014

$\text{Int}[\frac{((c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)})}{x_Symbol}] :> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] /; \text{FreeQ}[\{a, b, c, j, m, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[m + n*p + n - j + 1, 0] \ \&\& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}} dx = -\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Mathematica [A] time = 0.0108333, size = 27, normalized size = 1.

$$-\frac{2\sqrt{x^2(a+bx^3)}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))

Maple [A] time = 0.003, size = 29, normalized size = 1.1

$$-\frac{2bx^3 + 2a}{3a} \frac{1}{\sqrt{x}} \frac{1}{\sqrt{bx^5 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x)

[Out] -2/3*(b*x^3+a)/x^(1/2)/a/(b*x^5+a*x^2)^(1/2)

Maxima [A] time = 1.11984, size = 35, normalized size = 1.3

$$-\frac{2(bx^4 + ax)}{3\sqrt{bx^3 + aax^2}^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))

Fricas [A] time = 0.920576, size = 51, normalized size = 1.89

$$-\frac{2\sqrt{bx^5 + ax^2}}{3ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $-2/3*\sqrt{b*x^5 + a*x^2}/(a*x^{(5/2)})$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} \sqrt{x^2 (a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)`

[Out] `Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)`

Giac [A] time = 1.4494, size = 31, normalized size = 1.15

$$-\frac{2\sqrt{b + \frac{a}{x^3}}}{3a} + \frac{2\sqrt{b}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

[Out] $-2/3*\sqrt{b + a/x^3}/a + 2/3*\sqrt{b}/a$

$$3.304 \quad \int \frac{1}{x^{5/2} \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=235

$$\frac{2bx^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \text{EllipticF} \left(\cos^{-1} \left(\frac{\sqrt[3]{a} + (1 - \sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx}} \right), \frac{1}{4} (2 + \sqrt{3}) \right)}{5 \sqrt[4]{3} a^{4/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{2\sqrt{ax^2 + bx^5}}{5ax^{7/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(5*a*x^{(7/2)}) - (2*b*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x) * \text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)^2] * \text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) * b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4]) / (5*3^{(1/4)} * a^{(4/3)} * \text{Sqrt}[(b^{(1/3)}*x * (a^{(1/3)} + b^{(1/3)}*x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)^2] * \text{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.221022, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2025, 2032, 329, 225}

$$\frac{2bx^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} F \left(\cos^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}} \right), \frac{1}{4} (2 + \sqrt{3}) \right)}{5 \sqrt[4]{3} a^{4/3} \sqrt{\frac{\sqrt[3]{bx} (\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{2\sqrt{ax^2 + bx^5}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^{(5/2)} * \text{Sqrt}[a*x^2 + b*x^5]), x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(5*a*x^{(7/2)}) - (2*b*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x) * \text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)^2] * \text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) * b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4]) / (5*3^{(1/4)} * a^{(4/3)} * \text{Sqrt}[(b^{(1/3)}*x * (a^{(1/3)} + b^{(1/3)}*x)) / (a^{(1/3)} + (1 + \text{Sqrt}[3]) * b^{(1/3)}*x)^2] * \text{Sqrt}[a*x^2 + b*x^5])$

Rule 2025

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 225

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}} dx &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{(2b) \int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}} dx}{5a} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{(2bx\sqrt{a+bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{5a\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{(4bx\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{5a\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{2bx^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right) \middle| \frac{1}{4}(2+\sqrt{3})\right)}{5\sqrt[4]{3}a^{4/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2+bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0134694, size = 57, normalized size = 0.24

$$\frac{2\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; -\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -(b*x^3)/a])/(5*x^(3/2)*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.037, size = 1795, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x)

[Out] -2/5*(b*x^3+a)*(-4*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-

$$\begin{aligned}
& b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)}) \\
& /(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/ \\
& (-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(\\
& 1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*3^{(1/2)}*x^5*b^2+8*I*(-I*3^{(1/2)}-3)*x*b/ \\
& (-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b* \\
& x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(- \\
& b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1 \\
& /2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/ \\
& 2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-b^2*a \\
&)^{(1/3)}*3^{(1/2)}*x^4*b-4*I*(-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a \\
&)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/ \\
& 2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^ \\
& (1/3))/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*EllipticF((-I*3^{(1/2)}-3 \\
&)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1 \\
& /2)})/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-b^2*a)^{(2/3)}*3^{(1/2)}*x^3+4*(-I* \\
& 3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^ \\
& 2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)} \\
& *((I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})/(-1+I*3^{(1/2)})/(-b*x+(-b^ \\
& 2*a)^{(1/3)})^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2 \\
& *a)^{(1/3)})^{(1/2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3) \\
&)^{(1/2)}*x^5*b^2-8*(-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)}) \\
&)^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/2)})/(-b* \\
& x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})/(\\
& -1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(- \\
& 1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(1+ \\
& I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-b^2*a)^{(1/3)}*x^4*b+4*(-I*3^{(1/2)}-3)*x*b \\
& /(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b \\
& *x+(-b^2*a)^{(1/3)})/(1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1/2)}*((I*3^{(1/2)}*(- \\
& b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(\\
& 1/2)}*EllipticF((-I*3^{(1/2)}-3)*x*b/(-1+I*3^{(1/2)})/(-b*x+(-b^2*a)^{(1/3)})^{(1 \\
& /2)},((I*3^{(1/2)}+3)*(-1+I*3^{(1/2)})/(1+I*3^{(1/2)})/(I*3^{(1/2)}-3))^{(1/2)}*(-b^2 \\
& *a)^{(2/3)}*x^3+I*(1/b^2*x*(-b*x+(-b^2*a)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2* \\
& b*x+(-b^2*a)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})^{(1/2)}* \\
& (-b^2*a)^{(1/3)}*3^{(1/2)}*((b*x^3+a)*x)^{(1/2)}-3*((b*x^3+a)*x)^{(1/2)}*(-b^2*a)^{(\\
& 1/3)}*(1/b^2*x*(-b*x+(-b^2*a)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a \\
&)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^5+a*x \\
& ^2)^{(1/2)}/x^{(3/2)}/(-b^2*a)^{(1/3)}/a/((b*x^3+a)*x)^{(1/2)}/(I*3^{(1/2)}-3)/(1/b^2 \\
& *x*(-b*x+(-b^2*a)^{(1/3)})*(I*3^{(1/2)}*(-b^2*a)^{(1/3)}+2*b*x+(-b^2*a)^{(1/3)})*(I \\
& *3^{(1/2)}*(-b^2*a)^{(1/3)}-2*b*x-(-b^2*a)^{(1/3)})^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2} x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^8 + ax^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^8 + a*x^5), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}} \sqrt{x^2(a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2} x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

$$3.305 \quad \int \frac{1}{x^{7/2} \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=555

$$\frac{4(1 - \sqrt{3})b^{4/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}} \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1 - \sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{7\sqrt[4]{3}a^{5/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} - \frac{8(1 + \sqrt{3})b^{4/3}x^{3/2}}{7a^2(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})}$$

[Out] $(-8*(1 + \text{Sqrt}[3])*b^{(4/3)}*x^{(3/2)}*(a + b*x^3))/(7*a^2*(a^{(1/3)} + (1 + \text{Sqrt}[3]))*b^{(1/3)}*x)*\text{Sqrt}[a*x^2 + b*x^5] - (2*\text{Sqrt}[a*x^2 + b*x^5])/(7*a*x^{(9/2)}) + (8*b*\text{Sqrt}[a*x^2 + b*x^5])/(7*a^2*x^{(3/2)}) + (8*3^{(1/4)}*b^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(7*a^{(5/3)}*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x^2 + b*x^5] + (4*(1 - \text{Sqrt}[3])*b^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(7*3^{(1/4)}*a^{(5/3)}*\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{Sqrt}[a*x^2 + b*x^5])$

Rubi [A] time = 0.573181, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$\frac{8(1 + \sqrt{3})b^{4/3}x^{3/2}(a + bx^3)}{7a^2(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})\sqrt{ax^2 + bx^5}} + \frac{4(1 - \sqrt{3})b^{4/3}x^{3/2}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1 - \sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt{3})\sqrt[3]{bx} + \sqrt[3]{a}}\right), \frac{1}{4}\right)}{7\sqrt[4]{3}a^{5/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1 + \sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] $(-8*(1 + \text{Sqrt}[3])*b^{(4/3)}*x^{(3/2)}*(a + b*x^3))/(7*a^2*(a^{(1/3)} + (1 + \text{Sqrt}[3]))*b^{(1/3)}*x)*\text{Sqrt}[a*x^2 + b*x^5] - (2*\text{Sqrt}[a*x^2 + b*x^5])/(7*a*x^{(9/2)})$

$$\begin{aligned}
& + (8*b*\text{Sqrt}[a*x^2 + b*x^5])/(7*a^2*x^{(3/2)}) + (8*3^{(1/4)}*b^{(4/3)}*x^{(3/2)}*(\\
& a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + \\
& (1 + \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3]) \\
& *b^{(1/3)}*x)/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(7*a^{(5/3)} \\
& *\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)} \\
& *x)^2]*\text{Sqrt}[a*x^2 + b*x^5]) + (4*(1 - \text{Sqrt}[3])*b^{(4/3)}*x^{(3/2)}*(a^{(1/3)} + b \\
& ^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/(a^{(1/3)} + (1 + \\
& \text{Sqrt}[3])*b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcCos}[(a^{(1/3)} + (1 - \text{Sqrt}[3])*b^{(1/3)}*x) \\
& / (a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x)], (2 + \text{Sqrt}[3])/4])/(7*3^{(1/4)}*a^{(5/3)} \\
& *\text{Sqrt}[(b^{(1/3)}*x*(a^{(1/3)} + b^{(1/3)}*x))/(a^{(1/3)} + (1 + \text{Sqrt}[3])*b^{(1/3)}*x \\
& ^2]*\text{Sqrt}[a*x^2 + b*x^5])
\end{aligned}$$

Rule 2025

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 308

```

Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
+ b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]

```

Rule 225

```
Int[1/Sqrt[(a_) + (b_)*(x_)^6], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_)*(x_)^4)/Sqrt[(a_) + (b_)*(x_)^6], x_Symbol] := With[{r =
Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])]/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}} dx &= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} - \frac{(4b) \int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}} dx}{7a} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(8b^2) \int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}} dx}{7a^2} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(8b^2x\sqrt{a+bx^3}) \int \frac{x^{3/2}}{\sqrt{a+bx^3}} dx}{7a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(16b^2x\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{(8b^{4/3}x\sqrt{a+bx^3}) \text{Subst}\left(\int \frac{(-1+\sqrt{3})a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}} dx, x, \sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}} + \dots \\
&= -\frac{8(1+\sqrt{3})b^{4/3}x^{3/2}(a+bx^3)}{7a^2(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{bx})\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{8\sqrt[4]{3}b^{4/3}x^{3/2}(\sqrt[3]{a}+\sqrt[3]{bx})}{7a^2\sqrt{ax^2+bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0141385, size = 57, normalized size = 0.1

$$\frac{2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; -\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -(b*x^3)/a])/ (7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.043, size = 3048, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x)

[Out] 2/7*(-8*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b-I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3)))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*a^2-16*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b+24*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2)))/(I*3^(1/2)-3))^(1/2)*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b+3*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I

$$\frac{2}{3} \cdot 3^{1/2} \cdot x^4 + 3 \cdot \left(\frac{1}{b^2} \cdot x \cdot (-b \cdot x + (-b^2 \cdot a)^{1/3}) \cdot (I \cdot 3^{1/2} \cdot (-b^2 \cdot a)^{1/3}) + 2 \cdot b \cdot x + (-b^2 \cdot a)^{1/3} \right) \cdot (I \cdot 3^{1/2} \cdot (-b^2 \cdot a)^{1/3} - 2 \cdot b \cdot x - (-b^2 \cdot a)^{1/3})^{1/2} \cdot a^2 / (b \cdot x^5 + a \cdot x^2)^{1/2} / x^{5/2} / a^2 / (I \cdot 3^{1/2} - 3) / \left(\frac{1}{b^2} \cdot x \cdot (-b \cdot x + (-b^2 \cdot a)^{1/3}) \cdot (I \cdot 3^{1/2} \cdot (-b^2 \cdot a)^{1/3}) + 2 \cdot b \cdot x + (-b^2 \cdot a)^{1/3} \right) \cdot (I \cdot 3^{1/2} \cdot (-b^2 \cdot a)^{1/3} - 2 \cdot b \cdot x - (-b^2 \cdot a)^{1/3})^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^5 + ax^2} \sqrt{x}}{bx^9 + ax^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^9 + a*x^6), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{x^2 (a + bx^3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2} x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)

$$3.306 \quad \int \frac{1}{x^{9/2} \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2 + bx^5}}{9a^2x^{5/2}} - \frac{2\sqrt{ax^2 + bx^5}}{9ax^{11/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(9*a*x^{(11/2)}) + (4*b*\text{Sqrt}[a*x^2 + b*x^5])/(9*a^2*x^{(5/2)})$

Rubi [A] time = 0.0830774, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2 + bx^5}}{9a^2x^{5/2}} - \frac{2\sqrt{ax^2 + bx^5}}{9ax^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^5])/(9*a*x^{(11/2)}) + (4*b*\text{Sqrt}[a*x^2 + b*x^5])/(9*a^2*x^{(5/2)})$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x]
  - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x]
  /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x]
  /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}} dx = -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}} - \frac{(2b) \int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}} dx}{3a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}} + \frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}$$

Mathematica [A] time = 0.0173151, size = 35, normalized size = 0.62

$$-\frac{2(a-2bx^3)\sqrt{x^2(a+bx^3)}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)]/(9*a^2*x^(11/2))

Maple [A] time = 0.006, size = 37, normalized size = 0.7

$$-\frac{(2bx^3+2a)(-2bx^3+a)}{9a^2}x^{-\frac{7}{2}}\frac{1}{\sqrt{bx^5+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x)

[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)

Maxima [A] time = 1.10407, size = 51, normalized size = 0.91

$$\frac{2(2b^2x^7+abx^4-a^2x)}{9\sqrt{bx^3+aa^2x^{\frac{11}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] $2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(\sqrt{b*x^3 + a} * a^2*x^{(11/2)})$

Fricas [A] time = 0.822707, size = 73, normalized size = 1.3

$$\frac{2\sqrt{bx^5 + ax^2}(2bx^3 - a)}{9a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $2/9*\sqrt{b*x^5 + a*x^2}*(2*b*x^3 - a)/(a^2*x^{(11/2)})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.20074, size = 49, normalized size = 0.88

$$-\frac{4b^{\frac{3}{2}}}{9a^2} - \frac{2\left(\left(b + \frac{a}{x^3}\right)^{\frac{3}{2}} - 3\sqrt{b + \frac{a}{x^3}b}\right)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")

[Out] $-4/9*b^{(3/2)}/a^2 - 2/9*((b + a/x^3)^{(3/2)} - 3*\sqrt{(b + a/x^3)*b})/a^2$

$$3.307 \quad \int \frac{1}{x^{11/2} \sqrt{ax^2 + bx^5}} dx$$

Optimal. Leaf size=265

$$\frac{16b^2x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3}) \sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx}}\right), \frac{1}{4}(2 + \sqrt{3})\right)}{55\sqrt[4]{3}a^{7/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} - \frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}}$$

[Out] $(-2\sqrt{ax^2 + bx^5})/(11ax^{13/2}) + (16b\sqrt{ax^2 + bx^5})/(55a^2x^{7/2}) + (16b^2x^{3/2}(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3})b^{1/3}x + b^{2/3}x^2})/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2 \operatorname{EllipticF}[\operatorname{ArcCos}[(a^{1/3} + (1 - \sqrt{3})b^{1/3}x)/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)], (2 + \sqrt{3})/4])/(55 \cdot 3^{1/4} a^{7/3} \sqrt{(b^{1/3}x(a^{1/3} + b^{1/3}x))/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2}) \sqrt{ax^2 + bx^5}$

Rubi [A] time = 0.284104, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2025, 2032, 329, 225}

$$\frac{16b^2x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{bx} + \sqrt[3]{a}}\right) \middle| \frac{1}{4}(2 + \sqrt{3})\right)}{55\sqrt[4]{3}a^{7/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3}) \sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} - \frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x^{11/2} \sqrt{ax^2 + bx^5}), x]$

[Out] $(-2\sqrt{ax^2 + bx^5})/(11ax^{13/2}) + (16b\sqrt{ax^2 + bx^5})/(55a^2x^{7/2}) + (16b^2x^{3/2}(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3})b^{1/3}x + b^{2/3}x^2})/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2 \operatorname{EllipticF}[\operatorname{ArcCos}[(a^{1/3} + (1 - \sqrt{3})b^{1/3}x)/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)], (2 + \sqrt{3})/4])/(55 \cdot 3^{1/4} a^{7/3} \sqrt{(b^{1/3}x(a^{1/3} + b^{1/3}x))/(a^{1/3} + (1 + \sqrt{3})b^{1/3}x)^2}) \sqrt{ax^2 + bx^5}$

Rule 2025

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
+ 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
+ j*p + 1, 0]

```

Rule 2032

```

Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)* (a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]

```

Rule 329

```

Int[((c_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rule 225

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x] /; FreeQ[{a, b}, x
]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{11/2} \sqrt{ax^2 + bx^5}} dx &= -\frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}} - \frac{(8b) \int \frac{1}{x^{5/2} \sqrt{ax^2 + bx^5}} dx}{11a} \\
&= -\frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} + \frac{(16b^2) \int \frac{\sqrt{x}}{\sqrt{ax^2 + bx^5}} dx}{55a^2} \\
&= -\frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} + \frac{(16b^2x\sqrt{a + bx^3}) \int \frac{1}{\sqrt{x}\sqrt{a+bx^3}} dx}{55a^2\sqrt{ax^2 + bx^5}} \\
&= -\frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} + \frac{(32b^2x\sqrt{a + bx^3}) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx^6}} dx, x, \sqrt{x} \right)}{55a^2\sqrt{ax^2 + bx^5}} \\
&= -\frac{2\sqrt{ax^2 + bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2 + bx^5}}{55a^2x^{7/2}} + \frac{16b^2x^{3/2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{bx}}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx}}\right)\right)}{55\sqrt[4]{3}a^{7/3} \sqrt{\frac{\sqrt[3]{bx}(\sqrt[3]{a} + \sqrt[3]{bx})}{(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{bx})^2}} \sqrt{ax^2 + bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.0139006, size = 57, normalized size = 0.22

$$\frac{2\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(-\frac{11}{6}, \frac{1}{2}; -\frac{5}{6}; -\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2(a + bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])/(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])

Maple [C] time = 0.041, size = 2009, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x)

$a^{1/3}) * (I * 3^{1/2} * (-b^2 * a)^{1/3} - 2 * b * x - (-b^2 * a)^{1/3}))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2} x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^{11} + ax^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^11 + a*x^8), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2} x^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

3.308

$$\int \frac{x}{ax^3+bx^4} dx$$

Optimal. Leaf size=28

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Rubi [A] time = 0.0169484, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 44}

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(a*x^3 + b*x^4), x]$

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
  ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
  & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
  + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^3 + bx^4} dx &= \int \frac{1}{x^2(a + bx)} dx \\ &= \int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a + bx)} \right) dx \\ &= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0043284, size = 28, normalized size = 1.

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a + bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^3 + b*x^4),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2

Maple [A] time = 0.007, size = 29, normalized size = 1.

$$-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx + a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^4+a*x^3),x)

[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2

Maxima [A] time = 1.10428, size = 38, normalized size = 1.36

$$\frac{b \log(bx + a)}{a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")

[Out] $b \cdot \log(bx + a)/a^2 - b \cdot \log(x)/a^2 - 1/(a \cdot x)$

Fricas [A] time = 0.73642, size = 61, normalized size = 2.18

$$\frac{bx \log(bx + a) - bx \log(x) - a}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")`

[Out] $(b \cdot x \cdot \log(bx + a) - b \cdot x \cdot \log(x) - a)/(a^2 \cdot x)$

Sympy [A] time = 0.787333, size = 19, normalized size = 0.68

$$-\frac{1}{ax} + \frac{b \left(-\log(x) + \log\left(\frac{a}{b} + x\right) \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**4+a*x**3),x)`

[Out] $-1/(a \cdot x) + b \cdot (-\log(x) + \log(a/b + x))/a^2$

Giac [A] time = 1.20335, size = 41, normalized size = 1.46

$$\frac{b \log(|bx + a|)}{a^2} - \frac{b \log(|x|)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^4+a*x^3),x, algorithm="giac")`

[Out] $b \cdot \log(\text{abs}(bx + a))/a^2 - b \cdot \log(\text{abs}(x))/a^2 - 1/(a \cdot x)$

$$3.309 \quad \int \frac{1}{ax^3+bx^4} dx$$

Optimal. Leaf size=42

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b^2*\text{Log}[x])/a^3 - (b^2*\text{Log}[a + b*x])/a^3$

Rubi [A] time = 0.0191524, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 44}

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^4)^(-1), x]

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b^2*\text{Log}[x])/a^3 - (b^2*\text{Log}[a + b*x])/a^3$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{ax^3 + bx^4} dx &= \int \frac{1}{x^3(a + bx)} dx \\ &= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a + bx)} \right) dx \\ &= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a + bx)}{a^3} \end{aligned}$$

Mathematica [A] time = 0.0045704, size = 42, normalized size = 1.

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a + bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^4)^(-1),x]

[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3

Maple [A] time = 0.006, size = 41, normalized size = 1.

$$-\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(bx + a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^4+a*x^3),x)

[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3

Maxima [A] time = 1.06869, size = 54, normalized size = 1.29

$$-\frac{b^2 \log(bx + a)}{a^3} + \frac{b^2 \log(x)}{a^3} + \frac{2bx - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")

[Out] $-b^2 \log(bx + a)/a^3 + b^2 \log(x)/a^3 + 1/2(2bx - a)/(a^2 x^2)$

Fricas [A] time = 0.793161, size = 103, normalized size = 2.45

$$\frac{2b^2x^2 \log(bx + a) - 2b^2x^2 \log(x) - 2abx + a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")`

[Out] $-1/2(2b^2x^2 \log(bx + a) - 2b^2x^2 \log(x) - 2abx + a^2)/(a^3x^2)$

Sympy [A] time = 0.529053, size = 31, normalized size = 0.74

$$\frac{-a + 2bx}{2a^2x^2} + \frac{b^2 \left(\log(x) - \log\left(\frac{a}{b} + x\right) \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a*x**3),x)`

[Out] $(-a + 2bx)/(2a^2x^2) + b^2(\log(x) - \log(a/b + x))/a^3$

Giac [A] time = 1.30535, size = 61, normalized size = 1.45

$$-\frac{b^2 \log(|bx + a|)}{a^3} + \frac{b^2 \log(|x|)}{a^3} + \frac{2abx - a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^4+a*x^3),x, algorithm="giac")`

[Out] $-b^2 \log(\text{abs}(bx + a))/a^3 + b^2 \log(\text{abs}(x))/a^3 + 1/2(2abx - a^2)/(a^3 x^2)$

$$3.310 \quad \int \frac{x^4}{\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=112

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} - \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}} - \frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b}$$

[Out] $(-5*a*\text{Sqrt}[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*\text{Sqrt}[a*x^3 + b*x^4])/(8*b^3*x) + (x*\text{Sqrt}[a*x^3 + b*x^4])/(3*b) - (5*a^3*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a*x^3 + b*x^4]])/(8*b^{(7/2)})$

Rubi [A] time = 0.178822, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} - \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}} - \frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/\text{Sqrt}[a*x^3 + b*x^4], x]$

[Out] $(-5*a*\text{Sqrt}[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*\text{Sqrt}[a*x^3 + b*x^4])/(8*b^3*x) + (x*\text{Sqrt}[a*x^3 + b*x^4])/(3*b) - (5*a^3*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a*x^3 + b*x^4]])/(8*b^{(7/2)})$

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```


$x] /; \text{FreeQ}\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2 - 1] \&\& \text{NeQ}[n, j]$

Rule 206

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sqrt{ax^3 + bx^4}} dx &= \frac{x\sqrt{ax^3 + bx^4}}{3b} - \frac{(5a) \int \frac{x^3}{\sqrt{ax^3 + bx^4}} dx}{6b} \\ &= -\frac{5a\sqrt{ax^3 + bx^4}}{12b^2} + \frac{x\sqrt{ax^3 + bx^4}}{3b} + \frac{(5a^2) \int \frac{x^2}{\sqrt{ax^3 + bx^4}} dx}{8b^2} \\ &= -\frac{5a\sqrt{ax^3 + bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3 + bx^4}}{8b^3x} + \frac{x\sqrt{ax^3 + bx^4}}{3b} - \frac{(5a^3) \int \frac{x}{\sqrt{ax^3 + bx^4}} dx}{16b^3} \\ &= -\frac{5a\sqrt{ax^3 + bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3 + bx^4}}{8b^3x} + \frac{x\sqrt{ax^3 + bx^4}}{3b} - \frac{(5a^3) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{ax^3 + bx^4}}\right)}{8b^3} \\ &= -\frac{5a\sqrt{ax^3 + bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3 + bx^4}}{8b^3x} + \frac{x\sqrt{ax^3 + bx^4}}{3b} - \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3 + bx^4}}\right)}{8b^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.145038, size = 94, normalized size = 0.84

$$\frac{\sqrt{x^3(a + bx)} \left(\sqrt{b}\sqrt{x} (15a^2 - 10abx + 8b^2x^2) - \frac{15a^{5/2} \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a} + 1}} \right)}{24b^{7/2}x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4], x]

[Out] (Sqrt[x^3*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a])/(24*b^(7/2)*x^(3/2))

Maple [A] time = 0.004, size = 120, normalized size = 1.1

$$\frac{x}{48} \sqrt{x(bx+a)} \left(16x^2 \sqrt{bx^2+ax} b^{7/2} - 20 \sqrt{bx^2+ax} b^{5/2} xa + 30 \sqrt{bx^2+ax} b^{3/2} a^2 - 15 \ln \left(\frac{1}{2} \frac{2\sqrt{bx^2+ax}\sqrt{b} + 2bx+a}{\sqrt{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^4+a*x^3)^(1/2), x)

[Out] 1/48*x*(x*(b*x+a))^(1/2)*(16*x^2*(b*x^2+a*x)^(1/2)*b^(7/2)-20*(b*x^2+a*x)^(1/2)*b^(5/2)*x*a+30*(b*x^2+a*x)^(1/2)*b^(3/2)*a^2-15*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^3*b)/(b*x^4+a*x^3)^(1/2)/b^(9/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^4+ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x^3)^(1/2), x, algorithm="maxima")

[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)

Fricas [A] time = 0.91539, size = 390, normalized size = 3.48

$$\left[\frac{15a^3\sqrt{bx} \log\left(\frac{2bx^2+ax-2\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right) + 2(8b^3x^2 - 10ab^2x + 15a^2b)\sqrt{bx^4+ax^3} + 15a^3\sqrt{-bx} \arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right) + (8b^3x^2 - 10ab^2x + 15a^2b)\sqrt{bx^4+ax^3}}{48b^4x}, \frac{15a^3\sqrt{-bx} \arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right) + (8b^3x^2 - 10ab^2x + 15a^2b)\sqrt{bx^4+ax^3}}{24b^4x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x^3)^(1/2), x, algorithm="fricas")

[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(24*b^4*x)]

$3*x^2 - 10*a*b^2*x + 15*a^2*b)*\text{sqrt}(b*x^4 + a*x^3))/(b^4*x)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")

[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)

$$3.311 \quad \int \frac{x^3}{\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=86

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{\sqrt{ax^3+bx^4}}{2b}$$

[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))

Rubi [A] time = 0.127086, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2024, 2029, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{\sqrt{ax^3+bx^4}}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x^3 + b*x^4], x]

[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol]
:> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\sqrt{ax^3 + bx^4}} dx &= \frac{\sqrt{ax^3 + bx^4}}{2b} - \frac{(3a) \int \frac{x^2}{\sqrt{ax^3 + bx^4}} dx}{4b} \\
 &= \frac{\sqrt{ax^3 + bx^4}}{2b} - \frac{3a\sqrt{ax^3 + bx^4}}{4b^2x} + \frac{(3a^2) \int \frac{x}{\sqrt{ax^3 + bx^4}} dx}{8b^2} \\
 &= \frac{\sqrt{ax^3 + bx^4}}{2b} - \frac{3a\sqrt{ax^3 + bx^4}}{4b^2x} + \frac{(3a^2) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{ax^3 + bx^4}}\right)}{4b^2} \\
 &= \frac{\sqrt{ax^3 + bx^4}}{2b} - \frac{3a\sqrt{ax^3 + bx^4}}{4b^2x} + \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3 + bx^4}}\right)}{4b^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0515954, size = 92, normalized size = 1.07

$$\frac{\sqrt{bx^2}(-3a^2 - abx + 2b^2x^2) + 3a^{5/2}x^{3/2}\sqrt{\frac{bx}{a}} + 1 \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{x^3(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4], x]
```

```
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^3*(a + b*x)])
```

Maple [A] time = 0.004, size = 98, normalized size = 1.1

$$\frac{x}{8} \sqrt{x(bx + a)} \left(4x\sqrt{bx^2 + ax}b^{5/2} - 6\sqrt{bx^2 + ax}b^{3/2}a + 3 \ln\left(1/2 \frac{2\sqrt{bx^2 + ax}\sqrt{b} + 2bx + a}{\sqrt{b}}\right) a^2b \right) \frac{1}{\sqrt{bx^4 + ax^3}} b^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2), x)
```

[Out] $\frac{1}{8}x(x(bx+a))^{1/2}(4x(bx^2+ax))^{1/2}b^{5/2}-6(bx^2+ax)^{1/2}b^{3/2}a+3\ln\left(\frac{1}{2}(2(bx^2+ax))^{1/2}b^{1/2}+2bx+a\right)/b^{1/2})a^2b/(bx^4+ax^3)^{1/2}/b^{7/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/sqrt(b*x^4 + a*x^3), x)`

Fricas [A] time = 0.811266, size = 338, normalized size = 3.93

$$\left[\frac{3a^2\sqrt{bx} \log\left(\frac{2bx^2+ax+2\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right) + 2\sqrt{bx^4+ax^3}(2b^2x-3ab)}{8b^3x}, -\frac{3a^2\sqrt{-bx} \arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right) - \sqrt{bx^4+ax^3}(2b^2x-3ab)}{4b^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{8}(3a^2\sqrt{b}x \log((2bx^2 + ax + 2\sqrt{bx^4 + ax^3})\sqrt{b}))/x + 2\sqrt{bx^4 + ax^3}(2b^2x - 3ab)/(b^3x), -\frac{1}{4}(3a^2\sqrt{-b}x \arctan(\sqrt{bx^4 + ax^3}\sqrt{-b}/(bx^2)) - \sqrt{bx^4 + ax^3}(2b^2x - 3ab))/(b^3x) \right]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

$$3.312 \quad \int \frac{x^2}{\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=56

$$\frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b^(3/2)

Rubi [A] time = 0.0823131, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^3 + b*x^4], x]

[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b^(3/2)

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol]
:> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{ax^3 + bx^4}} dx &= \frac{\sqrt{ax^3 + bx^4}}{bx} - \frac{a \int \frac{x}{\sqrt{ax^3 + bx^4}} dx}{2b} \\ &= \frac{\sqrt{ax^3 + bx^4}}{bx} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{ax^3 + bx^4}}\right)}{b} \\ &= \frac{\sqrt{ax^3 + bx^4}}{bx} - \frac{a \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3 + bx^4}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0405259, size = 75, normalized size = 1.34

$$\frac{\sqrt{bx^2(a+bx)} - a^{3/2}x^{3/2}\sqrt{\frac{bx}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4], x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x) - a^(3/2)*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

Maple [A] time = 0.005, size = 78, normalized size = 1.4

$$\frac{x}{2} \sqrt{x(bx+a)} \left(2 \sqrt{bx^2 + ax} b^{3/2} - a \ln \left(\frac{1}{2} \left(2 \sqrt{bx^2 + ax} \sqrt{b} + 2bx + a \right) \frac{1}{\sqrt{b}} \right) b \right) \frac{1}{\sqrt{bx^4 + ax^3}} b^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2), x)
```

```
[Out] 1/2*x*(x*(b*x+a))^(1/2)*(2*(b*x^2+a*x)^(1/2)*b^(3/2)-a*ln(1/2*(2*(b*x^2+a*x)
)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^4+a*x^3)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)

Fricas [A] time = 0.809629, size = 277, normalized size = 4.95

$$\left[\frac{a\sqrt{bx} \log\left(\frac{2bx^2+ax-2\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right) + 2\sqrt{bx^4+ax^3}b}{2b^2x}, \frac{a\sqrt{-b} \arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right) + \sqrt{bx^4+ax^3}b}{b^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) + 2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)

Giac [A] time = 1.21226, size = 55, normalized size = 0.98

$$\frac{\sqrt{b + \frac{a}{x}}x}{b} + \frac{a \arctan\left(\frac{\sqrt{b + \frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-bb}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")

[Out] sqrt(b + a/x)*x/b + a*arctan(sqrt(b + a/x)/sqrt(-b))/(sqrt(-b)*b)

$$3.313 \quad \int \frac{x}{\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]

Rubi [A] time = 0.0343985, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x^3 + b*x^4],x]

[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{x}{\sqrt{ax^3 + bx^4}} dx = 2 \operatorname{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^2}{\sqrt{ax^3 + bx^4}} \right)$$

$$= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{ax^3 + bx^4}} \right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0166366, size = 59, normalized size = 1.84

$$\frac{2\sqrt{ax^{3/2}}\sqrt{\frac{bx}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}} \right)}{\sqrt{b}\sqrt{x^3(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x^3 + b*x^4], x]

[Out] (2*Sqrt[a]*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x^3*(a + b*x)])

Maple [B] time = 0.003, size = 56, normalized size = 1.8

$$x\sqrt{x(bx + a)} \ln \left(\frac{1}{2} \left(2\sqrt{bx^2 + ax}\sqrt{b} + 2bx + a \right) \frac{1}{\sqrt{b}} \right) \frac{1}{\sqrt{bx^4 + ax^3}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^4+a*x^3)^(1/2), x)

[Out] 1/(b*x^4+a*x^3)^(1/2)*x*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)

Fricas [A] time = 0.820835, size = 173, normalized size = 5.41

$$\left[\frac{\log\left(\frac{2bx^2+ax+2\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(x/sqrt(x**3*(a + b*x)), x)

Giac [A] time = 1.39866, size = 31, normalized size = 0.97

$$-\frac{2\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b + a/x)/sqrt(-b))/sqrt(-b)
```

$$3.314 \quad \int \frac{1}{\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(a*x^2)$

Rubi [A] time = 0.0050791, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2000}

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[a*x^3 + b*x^4], x]$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(a*x^2)$

Rule 2000

$\text{Int}[(a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)}]^{(p_*)}, x_Symbol] \text{ :> } \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(b*(n-j)*(p+1)*x^{(n-1)}), x] \text{ /; } \text{FreeQ}[\{a, b, j, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[j*p - n + j + 1, 0]$

Rubi steps

$$\int \frac{1}{\sqrt{ax^3+bx^4}} dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Mathematica [A] time = 0.0071141, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]

[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)

Maple [A] time = 0.002, size = 25, normalized size = 1.1

$$-2 \frac{x(bx + a)}{a\sqrt{bx^4 + ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^4+a*x^3)^(1/2),x)

[Out] -2*x*(b*x+a)/a/(b*x^4+a*x^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)

Fricas [A] time = 0.622897, size = 43, normalized size = 1.87

$$-\frac{2\sqrt{bx^4 + ax^3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] $-2\sqrt{bx^4 + ax^3}/(ax^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^3 + bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a*x**3)**(1/2),x)`

[Out] `Integral(1/sqrt(a*x**3 + b*x**4), x)`

Giac [A] time = 1.21362, size = 36, normalized size = 1.57

$$\frac{2}{\left(\sqrt{bx} - \sqrt{bx^2 + ax}\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")`

[Out] `2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))`

$$3.315 \quad \int \frac{1}{x\sqrt{ax^3+bx^4}} dx$$

Optimal. Leaf size=52

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2} - \frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*\text{Sqrt}[a*x^3 + b*x^4])/(3*a^2*x^2)$

Rubi [A] time = 0.0450667, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2000}

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2} - \frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[a*x^3 + b*x^4]), x]$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*\text{Sqrt}[a*x^3 + b*x^4])/(3*a^2*x^2)$

Rule 2016

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[m+j*p+1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2000

$\text{Int}[(a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(b*(n-j)*(p+1)*x^{(n-1)}), x] /; \text{FreeQ}\{a, b, j, n, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[j*p - n + j + 1, 0]$

Rubi steps

$$\int \frac{1}{x\sqrt{ax^3+bx^4}} dx = -\frac{2\sqrt{ax^3+bx^4}}{3ax^3} - \frac{(2b) \int \frac{1}{\sqrt{ax^3+bx^4}} dx}{3a}$$

$$= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3} + \frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}$$

Mathematica [A] time = 0.0110932, size = 29, normalized size = 0.56

$$-\frac{2(a-2bx)\sqrt{x^3(a+bx)}}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]

[Out] (-2*(a - 2*b*x)*Sqrt[x^3*(a + b*x)])/(3*a^2*x^3)

Maple [A] time = 0.004, size = 30, normalized size = 0.6

$$-\frac{(2bx+2a)(-2bx+a)}{3a^2} \frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^4+a*x^3)^(1/2),x)

[Out] -2/3*(b*x+a)*(-2*b*x+a)/a^2/(b*x^4+a*x^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+ax^3}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)

Fricas [A] time = 0.859516, size = 63, normalized size = 1.21

$$\frac{2\sqrt{bx^4 + ax^3}(2bx - a)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)

Giac [A] time = 1.20618, size = 36, normalized size = 0.69

$$\frac{2\left(\left(b + \frac{a}{x}\right)^{\frac{3}{2}} - 3\sqrt{b + \frac{a}{x}}\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")

[Out] -2/3*((b + a/x)^(3/2) - 3*sqrt(b + a/x)*b)/a^2

$$3.316 \quad \int \frac{1}{x^2 \sqrt{ax^3 + bx^4}} dx$$

Optimal. Leaf size=80

$$-\frac{16b^2\sqrt{ax^3 + bx^4}}{15a^3x^2} + \frac{8b\sqrt{ax^3 + bx^4}}{15a^2x^3} - \frac{2\sqrt{ax^3 + bx^4}}{5ax^4}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*\text{Sqrt}[a*x^3 + b*x^4])/(15*a^2*x^3) - (16*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(15*a^3*x^2)$

Rubi [A] time = 0.0860185, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2000}

$$-\frac{16b^2\sqrt{ax^3 + bx^4}}{15a^3x^2} + \frac{8b\sqrt{ax^3 + bx^4}}{15a^2x^3} - \frac{2\sqrt{ax^3 + bx^4}}{5ax^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[a*x^3 + b*x^4]), x]$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*\text{Sqrt}[a*x^3 + b*x^4])/(15*a^2*x^3) - (16*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(15*a^3*x^2)$

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{ax^3 + bx^4}} dx &= -\frac{2\sqrt{ax^3 + bx^4}}{5ax^4} - \frac{(4b) \int \frac{1}{x\sqrt{ax^3 + bx^4}} dx}{5a} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{5ax^4} + \frac{8b\sqrt{ax^3 + bx^4}}{15a^2x^3} + \frac{(8b^2) \int \frac{1}{\sqrt{ax^3 + bx^4}} dx}{15a^2} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{5ax^4} + \frac{8b\sqrt{ax^3 + bx^4}}{15a^2x^3} - \frac{16b^2\sqrt{ax^3 + bx^4}}{15a^3x^2}
\end{aligned}$$

Mathematica [A] time = 0.0122532, size = 42, normalized size = 0.52

$$-\frac{2\sqrt{x^3(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]

[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)

Maple [A] time = 0.003, size = 46, normalized size = 0.6

$$-\frac{(2bx+2a)(8b^2x^2-4abx+3a^2)}{15xa^3} \frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x)

[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x/a^3/(b*x^4+a*x^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)

Fricas [A] time = 0.763782, size = 90, normalized size = 1.12

$$\frac{2\sqrt{bx^4 + ax^3}(8b^2x^2 - 4abx + 3a^2)}{15a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^3(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)

Giac [A] time = 1.21621, size = 58, normalized size = 0.72

$$\frac{2\left(3\left(b + \frac{a}{x}\right)^{\frac{5}{2}} - 10\left(b + \frac{a}{x}\right)^{\frac{3}{2}}b + 15\sqrt{b + \frac{a}{x}}b^2\right)}{15a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")

[Out] -2/15*(3*(b + a/x)^(5/2) - 10*(b + a/x)^(3/2)*b + 15*sqrt(b + a/x)*b^2)/a^3

$$3.317 \quad \int \frac{1}{x^3 \sqrt{ax^3 + bx^4}} dx$$

Optimal. Leaf size=108

$$\frac{32b^3 \sqrt{ax^3 + bx^4}}{35a^4 x^2} - \frac{16b^2 \sqrt{ax^3 + bx^4}}{35a^3 x^3} + \frac{12b \sqrt{ax^3 + bx^4}}{35a^2 x^4} - \frac{2 \sqrt{ax^3 + bx^4}}{7ax^5}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^2*x^4) - (16*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^4*x^2)$

Rubi [A] time = 0.134198, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2000}

$$\frac{32b^3 \sqrt{ax^3 + bx^4}}{35a^4 x^2} - \frac{16b^2 \sqrt{ax^3 + bx^4}}{35a^3 x^3} + \frac{12b \sqrt{ax^3 + bx^4}}{35a^2 x^4} - \frac{2 \sqrt{ax^3 + bx^4}}{7ax^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^3*\text{Sqrt}[a*x^3 + b*x^4]), x]$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^2*x^4) - (16*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*\text{Sqrt}[a*x^3 + b*x^4])/(35*a^4*x^2)$

Rule 2016

$\text{Int}[(c*(x))^m*((a*(x))^{j*} + (b*(x))^{n*})^p, x_Symbol] \rightarrow \text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(m+j*p+1)), x] - \text{Dist}[(b*(m+n*p+n-j+1))/(a*c^{(n-j)}*(m+j*p+1)), \text{Int}[(c*x)^{(m+n-j)}*(a*x^j + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{ILtQ}[\text{Simplify}[(m+n*p+n-j+1)/(n-j)], 0] \&\& \text{NeQ}[m+j*p+1, 0] \&\& (\text{IntegersQ}[j, n] \parallel \text{GtQ}[c, 0])$

Rule 2000

$\text{Int}[(a*(x))^{j*} + (b*(x))^{n*})^p, x_Symbol] \rightarrow \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(b*(n-j)*(p+1)*x^{(n-1)}), x] /; \text{FreeQ}\{a, b, j, n, p\}, x \&\& \text{!IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[j*p - n + j + 1, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{ax^3 + bx^4}} dx &= -\frac{2\sqrt{ax^3 + bx^4}}{7ax^5} - \frac{(6b) \int \frac{1}{x^2 \sqrt{ax^3 + bx^4}} dx}{7a} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3 + bx^4}}{35a^2x^4} + \frac{(24b^2) \int \frac{1}{x\sqrt{ax^3 + bx^4}} dx}{35a^2} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3 + bx^4}}{35a^2x^4} - \frac{16b^2\sqrt{ax^3 + bx^4}}{35a^3x^3} - \frac{(16b^3) \int \frac{1}{\sqrt{ax^3 + bx^4}} dx}{35a^3} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3 + bx^4}}{35a^2x^4} - \frac{16b^2\sqrt{ax^3 + bx^4}}{35a^3x^3} + \frac{32b^3\sqrt{ax^3 + bx^4}}{35a^4x^2}
\end{aligned}$$

Mathematica [A] time = 0.0170488, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x^3(a+bx)}(6a^2bx - 5a^3 - 8ab^2x^2 + 16b^3x^3)}{35a^4x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]

[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35*a^4*x^5)

Maple [A] time = 0.005, size = 57, normalized size = 0.5

$$-\frac{(2bx + 2a)(-16b^3x^3 + 8ab^2x^2 - 6bxa^2 + 5a^3)}{35x^2a^4} \frac{1}{\sqrt{bx^4 + ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x)

[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^2/a^4/(b*x^4+a*x^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)

Fricas [A] time = 0.790084, size = 112, normalized size = 1.04

$$\frac{2(16b^3x^3 - 8ab^2x^2 + 6a^2bx - 5a^3)\sqrt{bx^4 + ax^3}}{35a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^4*x^5)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{x^3(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)

Giac [A] time = 1.2341, size = 77, normalized size = 0.71

$$\frac{2\left(5\left(b + \frac{a}{x}\right)^{\frac{7}{2}} - 21\left(b + \frac{a}{x}\right)^{\frac{5}{2}}b + 35\left(b + \frac{a}{x}\right)^{\frac{3}{2}}b^2 - 35\sqrt{b + \frac{a}{x}}b^3\right)}{35a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(b + a/x)^(7/2) - 21*(b + a/x)^(5/2)*b + 35*(b + a/x)^(3/2)*b^2 -  
35*sqrt(b + a/x)*b^3)/a^4
```

$$3.318 \quad \int \frac{1}{x^4 \sqrt{ax^3 + bx^4}} dx$$

Optimal. Leaf size=136

$$-\frac{256b^4 \sqrt{ax^3 + bx^4}}{315a^5x^2} + \frac{128b^3 \sqrt{ax^3 + bx^4}}{315a^4x^3} - \frac{32b^2 \sqrt{ax^3 + bx^4}}{105a^3x^4} + \frac{16b \sqrt{ax^3 + bx^4}}{63a^2x^5} - \frac{2\sqrt{ax^3 + bx^4}}{9ax^6}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*\text{Sqrt}[a*x^3 + b*x^4])/(63*a^2*x^5) - (32*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*\text{Sqrt}[a*x^3 + b*x^4])/(315*a^4*x^3) - (256*b^4*\text{Sqrt}[a*x^3 + b*x^4])/(315*a^5*x^2)$

Rubi [A] time = 0.174421, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2000}

$$-\frac{256b^4 \sqrt{ax^3 + bx^4}}{315a^5x^2} + \frac{128b^3 \sqrt{ax^3 + bx^4}}{315a^4x^3} - \frac{32b^2 \sqrt{ax^3 + bx^4}}{105a^3x^4} + \frac{16b \sqrt{ax^3 + bx^4}}{63a^2x^5} - \frac{2\sqrt{ax^3 + bx^4}}{9ax^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*\text{Sqrt}[a*x^3 + b*x^4])/(63*a^2*x^5) - (32*b^2*\text{Sqrt}[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*\text{Sqrt}[a*x^3 + b*x^4])/(315*a^4*x^3) - (256*b^4*\text{Sqrt}[a*x^3 + b*x^4])/(315*a^5*x^2)$

Rule 2016

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{ax^3 + bx^4}} dx &= -\frac{2\sqrt{ax^3 + bx^4}}{9ax^6} - \frac{(8b) \int \frac{1}{x^3 \sqrt{ax^3 + bx^4}} dx}{9a} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3 + bx^4}}{63a^2x^5} + \frac{(16b^2) \int \frac{1}{x^2 \sqrt{ax^3 + bx^4}} dx}{21a^2} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3 + bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3 + bx^4}}{105a^3x^4} - \frac{(64b^3) \int \frac{1}{x \sqrt{ax^3 + bx^4}} dx}{105a^3} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3 + bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3 + bx^4}}{105a^3x^4} + \frac{128b^3\sqrt{ax^3 + bx^4}}{315a^4x^3} + \frac{(128b^4) \int \frac{1}{\sqrt{ax^3 + bx^4}} dx}{315a^4} \\
&= -\frac{2\sqrt{ax^3 + bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3 + bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3 + bx^4}}{105a^3x^4} + \frac{128b^3\sqrt{ax^3 + bx^4}}{315a^4x^3} - \frac{256b^4\sqrt{ax^3 + bx^4}}{315a^5x^2}
\end{aligned}$$

Mathematica [A] time = 0.0189052, size = 64, normalized size = 0.47

$$-\frac{2\sqrt{x^3(a+bx)}(48a^2b^2x^2 - 40a^3bx + 35a^4 - 64ab^3x^3 + 128b^4x^4)}{315a^5x^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]

[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^3 + 128*b^4*x^4))/(315*a^5*x^6)

Maple [A] time = 0.003, size = 68, normalized size = 0.5

$$\frac{(2bx + 2a)(128b^4x^4 - 64ab^3x^3 + 48b^2x^2a^2 - 40xa^3b + 35a^4)}{315x^3a^5} \frac{1}{\sqrt{bx^4 + ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x)

[Out] -2/315*(b*x+a)*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/x^3/a^5/(b*x^4+a*x^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)

Fricas [A] time = 0.821276, size = 143, normalized size = 1.05

$$-\frac{2(128b^4x^4 - 64ab^3x^3 + 48a^2b^2x^2 - 40a^3bx + 35a^4)\sqrt{bx^4 + ax^3}}{315a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt{x^3(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)

Giac [A] time = 1.27836, size = 96, normalized size = 0.71

$$\frac{2 \left(35 \left(b + \frac{a}{x} \right)^{\frac{9}{2}} - 180 \left(b + \frac{a}{x} \right)^{\frac{7}{2}} b + 378 \left(b + \frac{a}{x} \right)^{\frac{5}{2}} b^2 - 420 \left(b + \frac{a}{x} \right)^{\frac{3}{2}} b^3 + 315 \sqrt{b + \frac{a}{x}} b^4 \right)}{315 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")

[Out] -2/315*(35*(b + a/x)^(9/2) - 180*(b + a/x)^(7/2)*b + 378*(b + a/x)^(5/2)*b^2 - 420*(b + a/x)^(3/2)*b^3 + 315*sqrt(b + a/x)*b^4)/a^5

$$3.319 \quad \int \frac{1}{x^3+bx^5} dx$$

Optimal. Leaf size=26

$$\frac{1}{2}b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

[Out] $-1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 + b*x^2])/2$

Rubi [A] time = 0.0176425, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1593, 266, 44}

$$\frac{1}{2}b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3 + b*x^5)^{-1}, x]$

[Out] $-1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 + b*x^2])/2$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)})^n, x] /; \text{FreeQ}\{a, b, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 44

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 + bx^5} dx &= \int \frac{1}{x^3(1 + bx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1 + bx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{1 + bx} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} - b \log(x) + \frac{1}{2} b \log(1 + bx^2)
\end{aligned}$$

Mathematica [A] time = 0.00508, size = 26, normalized size = 1.

$$\frac{1}{2} b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3 + b*x^5)^(-1), x]

[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2

Maple [A] time = 0.005, size = 23, normalized size = 0.9

$$-\frac{1}{2x^2} - b \ln(x) + \frac{b \ln(bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^5+x^3), x)

[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)

Maxima [A] time = 1.1516, size = 30, normalized size = 1.15

$$\frac{1}{2} b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")

[Out] $1/2*b*\log(b*x^2 + 1) - b*\log(x) - 1/2/x^2$

Fricas [A] time = 0.846842, size = 72, normalized size = 2.77

$$\frac{bx^2 \log(bx^2 + 1) - 2bx^2 \log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")

[Out] $1/2*(b*x^2*\log(b*x^2 + 1) - 2*b*x^2*\log(x) - 1)/x^2$

Sympy [A] time = 0.522277, size = 22, normalized size = 0.85

$$-b \log(x) + \frac{b \log\left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**5+x**3),x)

[Out] $-b*\log(x) + b*\log(x**2 + 1/b)/2 - 1/(2*x**2)$

Giac [A] time = 1.23451, size = 43, normalized size = 1.65

$$-\frac{1}{2} b \log(x^2) + \frac{1}{2} b \log(|bx^2 + 1|) + \frac{bx^2 - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")

[Out] $-1/2*b*\log(x^2) + 1/2*b*\log(\text{abs}(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2$

$$3.320 \quad \int \frac{1}{-x^3+bx^5} dx$$

Optimal. Leaf size=27

$$\frac{1}{2}b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2

Rubi [A] time = 0.0184613, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 44}

$$\frac{1}{2}b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(-x^3 + b*x^5)^(-1), x]

[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3 + bx^5} dx &= \int \frac{1}{x^3(-1 + bx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(-1 + bx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{-1 + bx} \right) dx, x, x^2 \right) \\
&= \frac{1}{2x^2} - b \log(x) + \frac{1}{2} b \log(1 - bx^2)
\end{aligned}$$

Mathematica [A] time = 0.005098, size = 27, normalized size = 1.

$$\frac{1}{2} b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(-x^3 + b*x^5)^(-1), x]

[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2

Maple [A] time = 0.004, size = 23, normalized size = 0.9

$$\frac{1}{2x^2} - b \ln(x) + \frac{b \ln(bx^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^5-x^3), x)

[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)

Maxima [A] time = 1.03757, size = 30, normalized size = 1.11

$$\frac{1}{2} b \log(bx^2 - 1) - b \log(x) + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")

[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2

Fricas [A] time = 0.764366, size = 72, normalized size = 2.67

$$\frac{bx^2 \log(bx^2 - 1) - 2bx^2 \log(x) + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")

[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2

Sympy [A] time = 0.263068, size = 22, normalized size = 0.81

$$-b \log(x) + \frac{b \log\left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**5-x**3),x)

[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)

Giac [A] time = 1.2824, size = 43, normalized size = 1.59

$$-\frac{1}{2} b \log(x^2) + \frac{1}{2} b \log(|bx^2 - 1|) + \frac{bx^2 + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")

[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2

$$3.321 \quad \int \frac{1}{ax+bx} dx$$

Optimal. Leaf size=8

$$\frac{\log(x)}{a+b}$$

[Out] Log[x]/(a + b)

Rubi [A] time = 0.0031854, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6, 12, 29}

$$\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-1), x]

[Out] Log[x]/(a + b)

Rule 6

Int[(u_)*((w_) + (a_)*(v_) + (b_)*(v_))^(p_), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned}\int \frac{1}{ax + bx} dx &= \int \frac{1}{(a + b)x} dx \\ &= \frac{\int \frac{1}{x} dx}{a + b} \\ &= \frac{\log(x)}{a + b}\end{aligned}$$

Mathematica [A] time = 0.0025627, size = 14, normalized size = 1.75

$$\frac{\log(ax + bx)}{a + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x)^(-1), x]

[Out] Log[a*x + b*x]/(a + b)

Maple [A] time = 0., size = 9, normalized size = 1.1

$$\frac{\ln(x)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x), x)

[Out] ln(x)/(a+b)

Maxima [A] time = 1.02556, size = 19, normalized size = 2.38

$$\frac{\log(ax + bx)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x), x, algorithm="maxima")

[Out] $\log(a*x + b*x)/(a + b)$

Fricas [A] time = 0.84249, size = 22, normalized size = 2.75

$$\frac{\log(x)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x),x, algorithm="fricas")`

[Out] $\log(x)/(a + b)$

Sympy [A] time = 0.093372, size = 5, normalized size = 0.62

$$\frac{\log(x)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x),x)`

[Out] $\log(x)/(a + b)$

Giac [A] time = 1.18924, size = 20, normalized size = 2.5

$$\frac{\log(|ax + bx|)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x),x, algorithm="giac")`

[Out] $\log(\text{abs}(a*x + b*x))/(a + b)$

$$3.322 \quad \int \frac{1}{(ax+bx)^2} dx$$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)^2}$$

[Out] -(1/((a + b)^2*x))

Rubi [A] time = 0.0032584, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-2), x]

[Out] -(1/((a + b)^2*x))

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax + bx)^2} dx &= \int \frac{1}{(a + b)^2 x^2} dx \\ &= \frac{\int \frac{1}{x^2} dx}{(a + b)^2} \\ &= -\frac{1}{(a + b)^2 x}\end{aligned}$$

Mathematica [A] time = 0.0030123, size = 10, normalized size = 1.

$$-\frac{1}{x(a + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x)^(-2), x]

[Out] -(1/((a + b)^2*x))

Maple [A] time = 0., size = 11, normalized size = 1.1

$$-\frac{1}{(a + b)^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x)^2, x)

[Out] -1/(a+b)^2/x

Maxima [A] time = 1.11015, size = 22, normalized size = 2.2

$$-\frac{1}{(ax + bx)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

Fricas [A] time = 0.800912, size = 38, normalized size = 3.8

$$-\frac{1}{(a^2 + 2ab + b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

Sympy [A] time = 0.111881, size = 15, normalized size = 1.5

$$-\frac{1}{x(a^2 + 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**2,x)
```

```
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

Giac [A] time = 1.12956, size = 22, normalized size = 2.2

$$-\frac{1}{(ax + bx)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

$$3.323 \quad \int \frac{1}{(ax+bx)^3} dx$$

Optimal. Leaf size=12

$$-\frac{1}{2x^2(a+b)^3}$$

[Out] -1/(2*(a + b)^3*x^2)

Rubi [A] time = 0.0030896, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6, 12, 30}

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-3), x]

[Out] -1/(2*(a + b)^3*x^2)

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 12

Int[(a_.)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_) /; FreeQ[b, x]]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax + bx)^3} dx &= \int \frac{1}{(a + b)^3 x^3} dx \\ &= \frac{\int \frac{1}{x^3} dx}{(a + b)^3} \\ &= -\frac{1}{2(a + b)^3 x^2}\end{aligned}$$

Mathematica [A] time = 0.0034175, size = 12, normalized size = 1.

$$-\frac{1}{2x^2(a + b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x)^(-3), x]

[Out] -1/(2*(a + b)^3*x^2)

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$-\frac{1}{2(a + b)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x)^3, x)

[Out] -1/2/(a+b)^3/x^2

Maxima [A] time = 0.995037, size = 22, normalized size = 1.83

$$-\frac{1}{2(ax + bx)^2(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")

[Out] -1/2/((a*x + b*x)^2*(a + b))

Fricas [B] time = 0.820803, size = 59, normalized size = 4.92

$$-\frac{1}{2(a^3 + 3a^2b + 3ab^2 + b^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")

[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)

Sympy [B] time = 0.107827, size = 27, normalized size = 2.25

$$-\frac{1}{2x^2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x)**3,x)

[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))

Giac [A] time = 1.15199, size = 22, normalized size = 1.83

$$-\frac{1}{2(ax + bx)^2(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")

[Out] -1/2/((a*x + b*x)^2*(a + b))

$$3.324 \quad \int \frac{1}{ax^2+bx^2} dx$$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)}$$

[Out] -(1/((a + b)*x))

Rubi [A] time = 0.0016056, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^2)^(-1),x]

[Out] -(1/((a + b)*x))

Rule 6

Int[(u_)*((w_) + (a_)*(v_) + (b_)*(v_))^(p_), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2 + bx^2} dx &= \int \frac{1}{(a+b)x^2} dx \\ &= \frac{\int \frac{1}{x^2} dx}{a+b} \\ &= -\frac{1}{(a+b)x}\end{aligned}$$

Mathematica [A] time = 0.0006774, size = 10, normalized size = 1.

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^2)^(-1),x]

[Out] -(1/((a + b)*x))

Maple [A] time = 0., size = 11, normalized size = 1.1

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^2+b*x^2),x)

[Out] -1/(a+b)/x

Maxima [A] time = 1.08684, size = 14, normalized size = 1.4

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")

[Out] $-1/((a + b)*x)$

Fricas [A] time = 0.639769, size = 22, normalized size = 2.2

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")`

[Out] $-1/((a + b)*x)$

Sympy [A] time = 0.081292, size = 7, normalized size = 0.7

$$-\frac{1}{x(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x**2+b*x**2),x)`

[Out] $-1/(x*(a + b))$

Giac [A] time = 1.13651, size = 14, normalized size = 1.4

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^2+b*x^2),x, algorithm="giac")`

[Out] $-1/((a + b)*x)$

$$3.325 \quad \int \frac{1}{ax^n + bx^n} dx$$

Optimal. Leaf size=20

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

[Out] $x^{(1-n)/((a+b)*(1-n))}$

Rubi [A] time = 0.0058103, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^n + b*x^n)^{-1}, x]$

[Out] $x^{(1-n)/((a+b)*(1-n))}$

Rule 6

$\text{Int}[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[u*((a + b)*v + w)^p, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{!FreeQ}[v, x]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ \text{!Match}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{ax^n + bx^n} dx &= \int \frac{x^{-n}}{a + b} dx \\ &= \frac{\int x^{-n} dx}{a + b} \\ &= \frac{x^{1-n}}{(a + b)(1 - n)} \end{aligned}$$

Mathematica [A] time = 0.0038062, size = 20, normalized size = 1.

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^n + b*x^n)^(-1),x]

[Out] x^(1 - n)/((a + b)*(1 - n))

Maple [A] time = 0.002, size = 19, normalized size = 1.

$$-\frac{x}{(-1+n)x^n(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^n+b*x^n),x)

[Out] -x/(-1+n)/(x^n)/(a+b)

Maxima [A] time = 1.07105, size = 28, normalized size = 1.4

$$-\frac{x}{(a(n-1)+b(n-1))x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")

[Out] $-x/((a*(n - 1) + b*(n - 1))*x^n)$

Fricas [A] time = 0.872954, size = 41, normalized size = 2.05

$$-\frac{x}{((a + b)n - a - b)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")`

[Out] $-x/(((a + b)*n - a - b)*x^n)$

Sympy [A] time = 0.839191, size = 32, normalized size = 1.6

$$\begin{cases} -\frac{x}{ax^n - ax^n + bnx^n - bx^n} & \text{for } n \neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x**n+b*x**n),x)`

[Out] `Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(a + b), True))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^n + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^n+b*x^n),x, algorithm="giac")`

[Out] `integrate(1/(a*x^n + b*x^n), x)`

$$3.326 \quad \int \frac{1}{(ax^n + bx^n)^2} dx$$

Optimal. Leaf size=20

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

[Out] $x^{(1 - 2*n)/((a + b)^{2*(1 - 2*n)})}$

Rubi [A] time = 0.0082571, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^n + b*x^n)^{-2}, x]$

[Out] $x^{(1 - 2*n)/((a + b)^{2*(1 - 2*n)})}$

Rule 6

$\text{Int}[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[u*((a + b)*v + w)^p, x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{!FreeQ}[v, x]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[u, (b_)*(v_) \text{ ; FreeQ}[b, x]]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ ; FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^n + bx^n)^2} dx &= \int \frac{x^{-2n}}{(a+b)^2} dx \\ &= \frac{\int x^{-2n} dx}{(a+b)^2} \\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)} \end{aligned}$$

Mathematica [A] time = 0.0032169, size = 20, normalized size = 1.

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^n + b*x^n)^(-2), x]

[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))

Maple [A] time = 0.001, size = 21, normalized size = 1.1

$$-\frac{x}{(-1+2n)(x^n)^2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^n+b*x^n)^2, x)

[Out] -x/(-1+2*n)/(x^n)^2/(a+b)^2

Maxima [A] time = 1.05347, size = 54, normalized size = 2.7

$$-\frac{x}{(a^2(2n-1) + 2ab(2n-1) + b^2(2n-1))x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")

[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))

Fricas [A] time = 0.916805, size = 80, normalized size = 4.

$$\frac{x}{(a^2 + 2ab + b^2 - 2(a^2 + 2ab + b^2)n)x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")

[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))

Sympy [A] time = 1.38359, size = 82, normalized size = 4.1

$$\begin{cases} \frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**n+b*x**n)**2,x)

[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2 + 2*a*b + b**2), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^n + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")


```
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

$$3.327 \quad \int \frac{1}{(ax^n + bx^n)^3} dx$$

Optimal. Leaf size=20

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

[Out] $x^{(1-3n)/((a+b)^{3*(1-3n)})}$

Rubi [A] time = 0.0089884, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^n + b*x^n)^{-3}, x]$

[Out] $x^{(1-3n)/((a+b)^{3*(1-3n)})}$

Rule 6

$\text{Int}[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[u*((a + b)*v + w)^p, x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{!FreeQ}[v, x]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[u, (b_)*(v_) \text{ ; FreeQ}[b, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] \text{ ; FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^n + bx^n)^3} dx &= \int \frac{x^{-3n}}{(a+b)^3} dx \\ &= \frac{\int x^{-3n} dx}{(a+b)^3} \\ &= \frac{x^{1-3n}}{(a+b)^3(1-3n)} \end{aligned}$$

Mathematica [A] time = 0.0032928, size = 20, normalized size = 1.

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^n + b*x^n)^(-3), x]

[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))

Maple [A] time = 0.002, size = 21, normalized size = 1.1

$$-\frac{x}{(-1+3n)(x^n)^3(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^n+b*x^n)^3, x)

[Out] -x/(-1+3*n)/(x^n)^3/(a+b)^3

Maxima [B] time = 1.01413, size = 72, normalized size = 3.6

$$-\frac{x}{(a^3(3n-1) + 3a^2b(3n-1) + 3ab^2(3n-1) + b^3(3n-1))x^{3n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")

[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))*x^(3*n))

Fricas [B] time = 0.918571, size = 112, normalized size = 5.6

$$\frac{x}{(a^3 + 3a^2b + 3ab^2 + b^3 - 3(a^3 + 3a^2b + 3ab^2 + b^3)n)x^{3n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")

[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^(3*n))

Sympy [A] time = 1.80554, size = 119, normalized size = 5.95

$$\begin{cases} -\frac{x}{\frac{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}}{\log(x)}} & \text{for } n \neq \frac{1}{3} \\ \frac{x}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**n+b*x**n)**3,x)

[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**3), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^n + bx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

$$3.328 \quad \int (ax + bx^{14})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{13})^{13}}{169b}$$

[Out] (a + b*x^13)^13/(169*b)

Rubi [A] time = 0.0044672, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 261}

$$\frac{(a + bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^14)^12,x]

[Out] (a + b*x^13)^13/(169*b)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (ax + bx^{14})^{12} dx &= \int x^{12} (a + bx^{13})^{12} dx \\ &= \frac{(a + bx^{13})^{13}}{169b} \end{aligned}$$

Mathematica [B] time = 0.0043833, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^14)^12,x]

[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169

Maple [B] time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169} + \frac{b^{11}ax^{156}}{13} + \frac{6b^{10}a^2x^{143}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{a^{12}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^14+a*x)^12,x)

[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*x^26+1/13*a^12*x^13

Maxima [B] time = 1.02378, size = 181, normalized size = 11.31

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")

[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13

$$x^{39} + 6/13 a^{11} b x^{26} + 1/13 a^{12} x^{13}$$

Fricas [B] time = 0.597755, size = 365, normalized size = 22.81

$$\frac{1}{169} x^{169} b^{12} + \frac{1}{13} x^{156} b^{11} a + \frac{6}{13} x^{143} b^{10} a^2 + \frac{22}{13} x^{130} b^9 a^3 + \frac{55}{13} x^{117} b^8 a^4 + \frac{99}{13} x^{104} b^7 a^5 + \frac{132}{13} x^{91} b^6 a^6 + \frac{132}{13} x^{78} b^5 a^7 + \frac{99}{13} x^{65} b^4 a^8 + \frac{55}{13} x^{52} b^3 a^9 + \frac{22}{13} x^{39} b^2 a^{10} + \frac{6}{13} x^{26} b a^{11} + \frac{1}{13} x^{13} a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")

[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 132/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12

Sympy [B] time = 0.122485, size = 160, normalized size = 10.

$$\frac{a^{12} x^{13}}{13} + \frac{6 a^{11} b x^{26}}{13} + \frac{22 a^{10} b^2 x^{39}}{13} + \frac{55 a^9 b^3 x^{52}}{13} + \frac{99 a^8 b^4 x^{65}}{13} + \frac{132 a^7 b^5 x^{78}}{13} + \frac{132 a^6 b^6 x^{91}}{13} + \frac{99 a^5 b^7 x^{104}}{13} + \frac{55 a^4 b^8 x^{117}}{13} + \frac{22 a^3 b^9 x^{130}}{13} + \frac{6 a^2 b^{10} x^{143}}{13} + \frac{a b^{11} x^{156}}{13} + \frac{b^{12} x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**14+a*x)**12,x)

[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169

Giac [B] time = 1.15551, size = 181, normalized size = 11.31

$$\frac{1}{169} b^{12} x^{169} + \frac{1}{13} a b^{11} x^{156} + \frac{6}{13} a^2 b^{10} x^{143} + \frac{22}{13} a^3 b^9 x^{130} + \frac{55}{13} a^4 b^8 x^{117} + \frac{99}{13} a^5 b^7 x^{104} + \frac{132}{13} a^6 b^6 x^{91} + \frac{132}{13} a^7 b^5 x^{78} + \frac{99}{13} a^8 b^4 x^{65} + \frac{55}{13} a^9 b^3 x^{52} + \frac{22}{13} a^{10} b^2 x^{39} + \frac{6}{13} a^{11} b x^{26} + \frac{1}{13} a^{12} x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")

[Out] $\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$

$$3.329 \quad \int x^{12} (ax + bx^{26})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{25})^{13}}{325b}$$

[Out] (a + b*x^25)^13/(325*b)

Rubi [A] time = 0.0114282, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{(a + bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

[In] Int[x^12*(a*x + b*x^26)^12,x]

[Out] (a + b*x^25)^13/(325*b)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^{12} (ax + bx^{26})^{12} dx &= \int x^{24} (a + bx^{25})^{12} dx \\ &= \frac{(a + bx^{25})^{13}}{325b} \end{aligned}$$

Mathematica [B] time = 0.0066342, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{325}b^{12}x^{325}$$

Antiderivative was successfully verified.

[In] Integrate[x^12*(a*x + b*x^26)^12,x]

[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325

Maple [B] time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325} + \frac{b^{11}ax^{300}}{25} + \frac{6b^{10}a^2x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{99a^8b^4x^{125}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{22a^{10}b^2x^{75}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12*(b*x^26+a*x)^12,x)

[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*b*x^50+1/325*b^12*x^325

Maxima [B] time = 1.11586, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{325}b^{12}x^{325}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")

[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/325*b^12*x^325

$$*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25$$

Fricas [B] time = 0.682468, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}b^1a^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")

[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 132/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12

Sympy [B] time = 0.240242, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**12*(b*x**26+a*x)**12,x)

[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325

Giac [B] time = 1.15535, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")

[Out] $\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{32}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$

$$3.330 \quad \int x^{24} (ax + bx^{38})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{37})^{13}}{481b}$$

[Out] (a + b*x^37)^13/(481*b)

Rubi [A] time = 0.0117595, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{(a + bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^24*(a*x + b*x^38)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^{24} (ax + bx^{38})^{12} dx &= \int x^{36} (a + bx^{37})^{12} dx \\ &= \frac{(a + bx^{37})^{13}}{481b} \end{aligned}$$

Mathematica [B] time = 0.008164, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Antiderivative was successfully verified.

[In] Integrate[x^24*(a*x + b*x^38)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

Maple [B] time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6b^{10}a^2x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^24*(b*x^38+a*x)^12,x)

[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^11*b*x^74+1/481*b^12*x^481

Maxima [B] time = 1.05626, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")

[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 + 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/481*b^12*x^481

$$10*b^2*x^{111} + 6/37*a^{11}*b*x^{74} + 1/37*a^{12}*x^{37}$$

Fricas [B] time = 0.760761, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}b*a^{11} + \frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")

[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 + 132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12

Sympy [B] time = 0.23361, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{99a^2b^{10}x^{407}}{37} + \frac{132ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**24*(b*x**38+a*x)**12,x)

[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/481

Giac [B] time = 1.11851, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*  
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +  
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^  
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

$$3.331 \quad \int x^{12(-1+m)} (ax + bx^{2+12m})^{12} dx$$

Optimal. Leaf size=27

$$\frac{(a + bx^{12m+1})^{13}}{13b(12m + 1)}$$

[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))

Rubi [A] time = 0.0147372, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1584, 261}

$$\frac{(a + bx^{12m+1})^{13}}{13b(12m + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]

[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
  ^ (p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
  NeQ[p, -1]
```

Rubi steps

$$\int x^{12(-1+m)} (ax + bx^{2+12m})^{12} dx = \int x^{12+12(-1+m)} (a + bx^{1+12m})^{12} dx$$

$$= \frac{(a + bx^{1+12m})^{13}}{13b(1 + 12m)}$$

Mathematica [A] time = 0.013241, size = 24, normalized size = 0.89

$$\frac{(a + bx^{12m+1})^{13}}{156bm + 13b}$$

Antiderivative was successfully verified.

[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]

[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)

Maple [B] time = 0.106, size = 339, normalized size = 12.6

$$\frac{b^{12} (x^{2+12m})^{13}}{(13 + 156m)x^{13}} + \frac{ab^{11} (x^{2+12m})^{12}}{(1 + 12m)x^{12}} + 6 \frac{a^2 b^{10} (x^{2+12m})^{11}}{(1 + 12m)x^{11}} + 22 \frac{a^3 b^9 (x^{2+12m})^{10}}{(1 + 12m)x^{10}} + 55 \frac{a^4 b^8 (x^{2+12m})^9}{(1 + 12m)x^9} + 99 \frac{a^5 b^7 (x^{2+12m})^8}{(1 + 12m)x^8} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x)

[Out] 1/13/(1+12*m)*b^12/x^13*(x^(2+12*m))^13+1/(1+12*m)*a*b^11/x^12*(x^(2+12*m))^12+6/(1+12*m)*a^2*b^10/x^11*(x^(2+12*m))^11+22/(1+12*m)*a^3*b^9/x^10*(x^(2+12*m))^10+55/(1+12*m)*a^4*b^8/x^9*(x^(2+12*m))^9+99/(1+12*m)*a^5*b^7/x^8*(x^(2+12*m))^8+132/(1+12*m)*a^6*b^6/x^7*(x^(2+12*m))^7+132/(1+12*m)*a^7*b^5/x^6*(x^(2+12*m))^6+99/(1+12*m)*a^8*b^4/x^5*(x^(2+12*m))^5+55/(1+12*m)*a^9*b^3/x^4*(x^(2+12*m))^4+22/(1+12*m)*a^10*b^2/x^3*(x^(2+12*m))^3+6/(1+12*m)*a^11*b/x^2*(x^(2+12*m))^2+1/(1+12*m)*a^12/x*x^(2+12*m)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))¹²,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 0.723535, size = 578, normalized size = 21.41

$$13a^{12}x^{12}x^{12m+2} + 78a^{11}bx^{11}x^{24m+4} + 286a^{10}b^2x^{10}x^{36m+6} + 715a^9b^3x^9x^{48m+8} + 1287a^8b^4x^8x^{60m+10} + 1716a^7b^5x^7x^{72m+12} + 1287a^6b^6x^6x^{84m+14} + 715a^5b^7x^5x^{96m+16} + 286a^4b^8x^4x^{108m+18} + 13a^3b^9x^3x^{120m+20} + 78a^2b^{10}x^2x^{132m+22} + 13ab^{11}xx^{144m+24} + b^{12}x^{156m+26} / ((12m+1)x^{13})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))¹²,x, algorithm="fricas")

[Out] 1/13*(13*a¹²*x¹²*x^(12*m + 2) + 78*a¹¹*b*x¹¹*x^(24*m + 4) + 286*a¹⁰*b²*x¹⁰*x^(36*m + 6) + 715*a⁹*b³*x⁹*x^(48*m + 8) + 1287*a⁸*b⁴*x⁸*x^(60*m + 10) + 1716*a⁷*b⁵*x⁷*x^(72*m + 12) + 1716*a⁶*b⁶*x⁶*x^(84*m + 14) + 1287*a⁵*b⁷*x⁵*x^(96*m + 16) + 715*a⁴*b⁸*x⁴*x^(108*m + 18) + 286*a³*b⁹*x³*x^(120*m + 20) + 78*a²*b¹⁰*x²*x^(132*m + 22) + 13*a*b¹¹*x*x^(144*m + 24) + b¹²*x^(156*m + 26))/((12*m + 1)*x¹³)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))¹²,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax + bx^{12m+2})^{12} x^{12m-12} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(12*m + 2))^12*x^(12*m - 12), x)
```

$$3.332 \quad \int (ax + bx^{14})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{13})^{13}}{169b}$$

[Out] (a + b*x^13)^13/(169*b)

Rubi [A] time = 0.0044802, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 261}

$$\frac{(a + bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^14)^12,x]

[Out] (a + b*x^13)^13/(169*b)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (ax + bx^{14})^{12} dx &= \int x^{12} (a + bx^{13})^{12} dx \\ &= \frac{(a + bx^{13})^{13}}{169b} \end{aligned}$$

Mathematica [B] time = 0.0031746, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^14)^12,x]

[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169

Maple [B] time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169} + \frac{b^{11}ax^{156}}{13} + \frac{6b^{10}a^2x^{143}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{a^{12}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^14+a*x)^12,x)

[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*x^26+1/13*a^12*x^13

Maxima [B] time = 1.10368, size = 181, normalized size = 11.31

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")

[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13

$$\frac{1}{169}x^{169}b^{12} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Fricas [B] time = 0.596537, size = 365, normalized size = 22.81

$$\frac{1}{169}x^{169}b^{12} + \frac{1}{13}x^{156}b^{11}a + \frac{6}{13}x^{143}b^{10}a^2 + \frac{22}{13}x^{130}b^9a^3 + \frac{55}{13}x^{117}b^8a^4 + \frac{99}{13}x^{104}b^7a^5 + \frac{132}{13}x^{91}b^6a^6 + \frac{132}{13}x^{78}b^5a^7 + \frac{99}{13}x^{65}b^4a^8 + \frac{55}{13}x^{52}b^3a^9 + \frac{22}{13}x^{39}b^2a^{10} + \frac{6}{13}x^{26}ba^{11} + \frac{1}{13}x^{13}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")

[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 132/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12

Sympy [B] time = 0.222354, size = 160, normalized size = 10.

$$\frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{a^{11}bx^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**14+a*x)**12,x)

[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169

Giac [B] time = 1.15819, size = 181, normalized size = 11.31

$$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")

[Out] $\frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$

$$\mathbf{3.333} \quad \int (ax^2 + bx^{27})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{25})^{13}}{325b}$$

[Out] (a + b*x^25)^13/(325*b)

Rubi [A] time = 0.0045659, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 261}

$$\frac{(a + bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^27)^12,x]

[Out] (a + b*x^25)^13/(325*b)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (ax^2 + bx^{27})^{12} dx &= \int x^{24} (a + bx^{25})^{12} dx \\ &= \frac{(a + bx^{25})^{13}}{325b} \end{aligned}$$

Mathematica [B] time = 0.0047925, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{325}b^{12}x^{325}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^27)^12,x]

[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325

Maple [B] time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325} + \frac{b^{11}ax^{300}}{25} + \frac{6b^{10}a^2x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{99a^8b^4x^{125}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{22a^{10}b^2x^{75}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^27+a*x^2)^12,x)

[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*b*x^50+1/325*b^12*x^325

Maxima [B] time = 1.02941, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{325}b^{12}x^{325}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")

[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/325*b^12*x^325

$$*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25$$

Fricas [B] time = 0.659142, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}b^1a^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")

[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 132/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12

Sympy [B] time = 0.171689, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**27+a*x**2)**12,x)

[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325

Giac [B] time = 1.12765, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")

[Out] $\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{32}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$

$$\mathbf{3.334} \quad \int (ax^3 + bx^{40})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{37})^{13}}{481b}$$

[Out] (a + b*x^37)^13/(481*b)

Rubi [A] time = 0.0044845, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 261}

$$\frac{(a + bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^40)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (ax^3 + bx^{40})^{12} dx &= \int x^{36} (a + bx^{37})^{12} dx \\ &= \frac{(a + bx^{37})^{13}}{481b} \end{aligned}$$

Mathematica [B] time = 0.0059641, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^40)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

Maple [B] time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6b^{10}a^2x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^40+a*x^3)^12,x)

[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^11*b*x^74+1/37*a^12*x^37

Maxima [B] time = 1.06418, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")

[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 + 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/481*b^12*x^481

$$10*b^2*x^{111} + 6/37*a^{11}*b*x^{74} + 1/37*a^{12}*x^{37}$$

Fricas [B] time = 0.664963, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}b*a^{11} + \frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")

[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 + 132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12

Sympy [B] time = 0.163629, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**40+a*x**3)**12,x)

[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/481

Giac [B] time = 1.14712, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*  
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +  
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^  
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

$$3.335 \quad \int (ax^m + bx^{1+13m})^{12} dx$$

Optimal. Leaf size=27

$$\frac{(a + bx^{12m+1})^{13}}{13b(12m + 1)}$$

[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))

Rubi [A] time = 0.0077874, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 261}

$$\frac{(a + bx^{12m+1})^{13}}{13b(12m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]

[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int (ax^m + bx^{1+13m})^{12} dx = \int x^{12m} (a + bx^{1+12m})^{12} dx$$

$$= \frac{(a + bx^{1+12m})^{13}}{13b(1 + 12m)}$$

Mathematica [A] time = 0.0040226, size = 24, normalized size = 0.89

$$\frac{(a + bx^{12m+1})^{13}}{156bm + 13b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]

[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)

Maple [B] time = 0.041, size = 287, normalized size = 10.6

$$\frac{b^{12}x^{13}(x^m)^{156}}{13 + 156m} + \frac{ab^{11}x^{12}(x^m)^{144}}{1 + 12m} + 6\frac{a^2b^{10}x^{11}(x^m)^{132}}{1 + 12m} + 22\frac{a^3b^9x^{10}(x^m)^{120}}{1 + 12m} + 55\frac{a^4b^8x^9(x^m)^{108}}{1 + 12m} + 99\frac{a^5b^7x^8(x^m)^{96}}{1 + 12m} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m+b*x^(1+13*m))^12,x)

[Out] 1/13*b^12*x^13/(1+12*m)*(x^m)^156+a*b^11*x^12/(1+12*m)*(x^m)^144+6*a^2*b^10*x^11/(1+12*m)*(x^m)^132+22*a^3*b^9*x^10/(1+12*m)*(x^m)^120+55*a^4*b^8*x^9/(1+12*m)*(x^m)^108+99*a^5*b^7*x^8/(1+12*m)*(x^m)^96+132*a^6*b^6*x^7/(1+12*m)*(x^m)^84+132*a^7*b^5*x^6/(1+12*m)*(x^m)^72+99*a^8*b^4*x^5/(1+12*m)*(x^m)^60+55*a^9*b^3*x^4/(1+12*m)*(x^m)^48+22*a^10*b^2*x^3/(1+12*m)*(x^m)^36+6*a^11*b*x^2/(1+12*m)*(x^m)^24+a^12/(1+12*m)*x*(x^m)^12

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 0.862526, size = 493, normalized size = 18.26

$$\frac{b^{12}x^{13}x^{156m} + 13ab^{11}x^{12}x^{144m} + 78a^2b^{10}x^{11}x^{132m} + 286a^3b^9x^{10}x^{120m} + 715a^4b^8x^9x^{108m} + 1287a^5b^7x^8x^{96m} + 1716a^6b^6x^7x^{84m} + 1716a^7b^5x^6x^{72m} + 1287a^8b^4x^5x^{60m} + 715a^9b^3x^4x^{48m} + 286a^{10}b^2x^3x^{36m} + 78a^{11}bx^2x^{24m} + 13a^{12}xx^{12m}}{13(12m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")

[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) + 1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**m+b*x**(1+13*m))**12,x)

[Out] Timed out

Giac [B] time = 1.25509, size = 277, normalized size = 10.26

$$\frac{b^{12}x^{13}x^{156m} + 13ab^{11}x^{12}x^{144m} + 78a^2b^{10}x^{11}x^{132m} + 286a^3b^9x^{10}x^{120m} + 715a^4b^8x^9x^{108m} + 1287a^5b^7x^8x^{96m} + 1716a^6b^6x^7x^{84m} + 1716a^7b^5x^6x^{72m} + 1287a^8b^4x^5x^{60m} + 715a^9b^3x^4x^{48m} + 286a^{10}b^2x^3x^{36m} + 78a^{11}bx^2x^{24m} + 13a^{12}xx^{12m}}{13(12m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) + 1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

$$3.336 \quad \int (ax^m + bx^{1+6m})^5 dx$$

Optimal. Leaf size=27

$$\frac{(a + bx^{5m+1})^6}{6b(5m+1)}$$

[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))

Rubi [A] time = 0.0102364, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 261}

$$\frac{(a + bx^{5m+1})^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]

[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int (ax^m + bx^{1+6m})^5 dx = \int x^{5m} (a + bx^{1+5m})^5 dx$$

$$= \frac{(a + bx^{1+5m})^6}{6b(1 + 5m)}$$

Mathematica [A] time = 0.0133221, size = 27, normalized size = 1.

$$\frac{(a + bx^{5m+1})^6}{6b(5m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]

[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))

Maple [B] time = 0.018, size = 126, normalized size = 4.7

$$\frac{b^5 x^6 (x^m)^{30}}{6 + 30 m} + \frac{ab^4 x^5 (x^m)^{25}}{1 + 5 m} + \frac{5 a^2 b^3 x^4 (x^m)^{20}}{2 + 10 m} + \frac{10 a^3 b^2 x^3 (x^m)^{15}}{3 + 15 m} + \frac{5 a^4 b x^2 (x^m)^{10}}{2 + 10 m} + \frac{a^5 x (x^m)^5}{1 + 5 m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m+b*x^(1+6*m))^5,x)

[Out] 1/6*b^5*x^6/(1+5*m)*(x^m)^30+a*b^4*x^5/(1+5*m)*(x^m)^25+5/2*a^2*b^3*x^4/(1+5*m)*(x^m)^20+10/3*a^3*b^2*x^3/(1+5*m)*(x^m)^15+5/2*a^4*b*x^2/(1+5*m)*(x^m)^10+a^5/(1+5*m)*x*(x^m)^5

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 0.85108, size = 204, normalized size = 7.56

$$\frac{b^5 x^6 x^{30m} + 6 a b^4 x^5 x^{25m} + 15 a^2 b^3 x^4 x^{20m} + 20 a^3 b^2 x^3 x^{15m} + 15 a^4 b x^2 x^{10m} + 6 a^5 x x^{5m}}{6(5m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")

[Out] $1/6*(b^5*x^6*x^{(30*m)} + 6*a*b^4*x^5*x^{(25*m)} + 15*a^2*b^3*x^4*x^{(20*m)} + 20*a^3*b^2*x^3*x^{(15*m)} + 15*a^4*b*x^2*x^{(10*m)} + 6*a^5*x*x^{(5*m)})/(5*m + 1)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**m+b*x**(1+6*m))**5,x)

[Out] Timed out

Giac [B] time = 1.12316, size = 126, normalized size = 4.67

$$\frac{b^5 x^6 x^{30m} + 6 a b^4 x^5 x^{25m} + 15 a^2 b^3 x^4 x^{20m} + 20 a^3 b^2 x^3 x^{15m} + 15 a^4 b x^2 x^{10m} + 6 a^5 x x^{5m}}{6(5m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")

[Out] $1/6*(b^5*x^6*x^{(30*m)} + 6*a*b^4*x^5*x^{(25*m)} + 15*a^2*b^3*x^4*x^{(20*m)} + 20*a^3*b^2*x^3*x^{(15*m)} + 15*a^4*b*x^2*x^{(10*m)} + 6*a^5*x*x^{(5*m)})/(5*m + 1)$

$$3.337 \quad \int \frac{1}{(bx^{1-2m} + ax^m)^3} dx$$

Optimal. Leaf size=27

$$-\frac{1}{2b(1-3m)(a+bx^{1-3m})^2}$$

[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)

Rubi [A] time = 0.0137646, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 261}

$$-\frac{1}{2b(1-3m)(a+bx^{1-3m})^2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3), x]

[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{1}{(bx^{1-2m} + ax^m)^3} dx = \int \frac{x^{-3m}}{(a + bx^{1-3m})^3} dx$$

$$= -\frac{1}{2b(1-3m)(a + bx^{1-3m})^2}$$

Mathematica [A] time = 0.0185168, size = 27, normalized size = 1.

$$-\frac{1}{2b(1-3m)(a + bx^{1-3m})^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3), x]

[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)

Maple [A] time = 0.02, size = 39, normalized size = 1.4

$$-\frac{(2a(x^m)^3 + bx)x}{(-2 + 6m)a^2(a(x^m)^3 + bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^(1-2*m)+a*x^m)^3, x)

[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(-1+3*m)/a^2/(a*(x^m)^3+b*x)^2

Maxima [B] time = 1.10417, size = 89, normalized size = 3.3

$$-\frac{2axx^{3m} + bx^2}{2(2a^3b(3m-1)xx^{3m} + a^2b^2(3m-1)x^2 + a^4(3m-1)x^{6m})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")

[Out] $-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m - 1)*x^2 + a^4*(3*m - 1)*x^(6*m))$

Fricas [B] time = 0.893229, size = 162, normalized size = 6.

$$\frac{2axx^{3m} + bx^2}{2\left(2\left(3a^3bm - a^3b\right)xx^{3m} + \left(3a^2b^2m - a^2b^2\right)x^2 + \left(3a^4m - a^4\right)x^{6m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")

[Out] $-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^m + bx^{-2m+1})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")

[Out] integrate((a*x^m + b*x^(-2*m + 1))^-3, x)

$$3.338 \quad \int \frac{1}{\frac{b}{x} + ax} dx$$

Optimal. Leaf size=15

$$\frac{\log(ax^2 + b)}{2a}$$

[Out] Log[b + a*x^2]/(2*a)

Rubi [A] time = 0.004795, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{\log(ax^2 + b)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(b/x + a*x)^(-1), x]

[Out] Log[b + a*x^2]/(2*a)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{b}{x} + ax} dx &= \int \frac{x}{b + ax^2} dx \\ &= \frac{\log(b + ax^2)}{2a} \end{aligned}$$

Mathematica [A] time = 0.0026931, size = 15, normalized size = 1.

$$\frac{\log(ax^2 + b)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x + a*x)^(-1),x]

[Out] Log[b + a*x^2]/(2*a)

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln(ax^2 + b)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/x+a*x),x)

[Out] 1/2*ln(a*x^2+b)/a

Maxima [A] time = 1.26634, size = 18, normalized size = 1.2

$$\frac{\log(ax^2 + b)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x),x, algorithm="maxima")

[Out] 1/2*log(a*x^2 + b)/a

Fricas [A] time = 0.651248, size = 30, normalized size = 2.

$$\frac{\log(ax^2 + b)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x),x, algorithm="fricas")

[Out] 1/2*log(a*x^2 + b)/a

Sympy [A] time = 0.1141, size = 10, normalized size = 0.67

$$\frac{\log(ax^2 + b)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x),x)

[Out] log(a*x**2 + b)/(2*a)

Giac [A] time = 1.12146, size = 19, normalized size = 1.27

$$\frac{\log(|ax^2 + b|)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x),x, algorithm="giac")

[Out] 1/2*log(abs(a*x^2 + b))/a

$$3.339 \quad \int \frac{1}{\frac{b}{x^2} + ax} dx$$

Optimal. Leaf size=15

$$\frac{\log(ax^3 + b)}{3a}$$

[Out] Log[b + a*x^3]/(3*a)

Rubi [A] time = 0.0060296, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{\log(ax^3 + b)}{3a}$$

Antiderivative was successfully verified.

[In] Int[(b/x^2 + a*x)^(-1), x]

[Out] Log[b + a*x^3]/(3*a)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{1}{\frac{b}{x^2} + ax} dx = \int \frac{x^2}{b + ax^3} dx$$

$$= \frac{\log(b + ax^3)}{3a}$$

Mathematica [A] time = 0.0039968, size = 15, normalized size = 1.

$$\frac{\log(ax^3 + b)}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^2 + a*x)^(-1),x]

[Out] Log[b + a*x^3]/(3*a)

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$\frac{\ln(ax^3 + b)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^2*b+a*x),x)

[Out] 1/3*ln(a*x^3+b)/a

Maxima [A] time = 1.08087, size = 18, normalized size = 1.2

$$\frac{\log(ax^3 + b)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")

[Out] $\frac{1}{3} \log(ax^3 + b)/a$

Fricas [A] time = 0.728851, size = 30, normalized size = 2.

$$\frac{\log(ax^3 + b)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b/x^2+a*x),x, algorithm="fricas")`

[Out] $\frac{1}{3} \log(ax^3 + b)/a$

Sympy [A] time = 0.188406, size = 10, normalized size = 0.67

$$\frac{\log(ax^3 + b)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b/x**2+a*x),x)`

[Out] $\log(ax^{**3} + b)/(3*a)$

Giac [A] time = 1.15672, size = 19, normalized size = 1.27

$$\frac{\log(|ax^3 + b|)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b/x^2+a*x),x, algorithm="giac")`

[Out] $\frac{1}{3} \log(\text{abs}(ax^3 + b))/a$

$$3.340 \quad \int \frac{1}{\frac{b}{x^3} + ax} dx$$

Optimal. Leaf size=15

$$\frac{\log(ax^4 + b)}{4a}$$

[Out] Log[b + a*x^4]/(4*a)

Rubi [A] time = 0.0053283, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{\log(ax^4 + b)}{4a}$$

Antiderivative was successfully verified.

[In] Int[(b/x^3 + a*x)^(-1), x]

[Out] Log[b + a*x^4]/(4*a)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{b}{x^3} + ax} dx &= \int \frac{x^3}{b + ax^4} dx \\ &= \frac{\log(b + ax^4)}{4a} \end{aligned}$$

Mathematica [A] time = 0.0047679, size = 15, normalized size = 1.

$$\frac{\log(ax^4 + b)}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^3 + a*x)^(-1),x]

[Out] Log[b + a*x^4]/(4*a)

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$\frac{\ln(ax^4 + b)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/x^3+a*x),x)

[Out] 1/4*ln(a*x^4+b)/a

Maxima [A] time = 1.11956, size = 18, normalized size = 1.2

$$\frac{\log(ax^4 + b)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")

[Out] 1/4*log(a*x^4 + b)/a

Fricas [A] time = 0.705642, size = 30, normalized size = 2.

$$\frac{\log(ax^4 + b)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")

[Out] 1/4*log(a*x^4 + b)/a

Sympy [A] time = 0.17412, size = 10, normalized size = 0.67

$$\frac{\log(ax^4 + b)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x**3+a*x),x)

[Out] log(a*x**4 + b)/(4*a)

Giac [A] time = 1.09987, size = 19, normalized size = 1.27

$$\frac{\log(|ax^4 + b|)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")

[Out] 1/4*log(abs(a*x^4 + b))/a

$$3.341 \quad \int \frac{1}{\left(\frac{b}{x} + ax\right)^3} dx$$

Optimal. Leaf size=19

$$\frac{x^4}{4b(ax^2 + b)^2}$$

[Out] x^4/(4*b*(b + a*x^2)^2)

Rubi [A] time = 0.0054465, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 264}

$$\frac{x^4}{4b(ax^2 + b)^2}$$

Antiderivative was successfully verified.

[In] Int[(b/x + a*x)^(-3), x]

[Out] x^4/(4*b*(b + a*x^2)^2)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\left(\frac{b}{x} + ax\right)^3} dx = \int \frac{x^3}{(b + ax^2)^3} dx$$

$$= \frac{x^4}{4b(b + ax^2)^2}$$

Mathematica [A] time = 0.0084775, size = 24, normalized size = 1.26

$$-\frac{2ax^2 + b}{4a^2(ax^2 + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x + a*x)^(-3), x]

[Out] -(b + 2*a*x^2)/(4*a^2*(b + a*x^2)^2)

Maple [A] time = 0.007, size = 31, normalized size = 1.6

$$\frac{b}{4a^2(ax^2 + b)^2} - \frac{1}{2a^2(ax^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/x+a*x)^3, x)

[Out] 1/4*b/a^2/(a*x^2+b)^2-1/2/a^2/(a*x^2+b)

Maxima [B] time = 1.15915, size = 49, normalized size = 2.58

$$-\frac{2ax^2 + b}{4(a^4x^4 + 2a^3bx^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")

[Out] $-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)$

Fricas [B] time = 0.784184, size = 73, normalized size = 3.84

$$-\frac{2ax^2 + b}{4(a^4x^4 + 2a^3bx^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")

[Out] $-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)$

Sympy [B] time = 0.482015, size = 36, normalized size = 1.89

$$-\frac{2ax^2 + b}{4a^4x^4 + 8a^3bx^2 + 4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x)**3,x)

[Out] $-(2*a*x**2 + b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)$

Giac [A] time = 1.15051, size = 30, normalized size = 1.58

$$-\frac{2ax^2 + b}{4(ax^2 + b)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")

[Out] $-1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)$

$$3.342 \quad \int \frac{1}{\left(\frac{b}{x^3} + ax^2\right)^3} dx$$

Optimal. Leaf size=19

$$\frac{x^{10}}{10b(ax^5 + b)^2}$$

[Out] x^10/(10*b*(b + a*x^5)^2)

Rubi [A] time = 0.006112, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 264}

$$\frac{x^{10}}{10b(ax^5 + b)^2}$$

Antiderivative was successfully verified.

[In] Int[(b/x^3 + a*x^2)^(-3), x]

[Out] x^10/(10*b*(b + a*x^5)^2)

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\left(\frac{b}{x^3} + ax^2\right)^3} dx = \int \frac{x^9}{(b + ax^5)^3} dx$$

$$= \frac{x^{10}}{10b(b + ax^5)^2}$$

Mathematica [A] time = 0.0087693, size = 24, normalized size = 1.26

$$-\frac{2ax^5 + b}{10a^2(ax^5 + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^3 + a*x^2)^(-3), x]

[Out] -(b + 2*a*x^5)/(10*a^2*(b + a*x^5)^2)

Maple [A] time = 0.004, size = 31, normalized size = 1.6

$$\frac{b}{10a^2(ax^5 + b)^2} - \frac{1}{5a^2(ax^5 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/x^3+a*x^2)^3, x)

[Out] 1/10*b/a^2/(a*x^5+b)^2-1/5/a^2/(a*x^5+b)

Maxima [B] time = 1.0571, size = 49, normalized size = 2.58

$$-\frac{2ax^5 + b}{10(a^4x^{10} + 2a^3bx^5 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")

[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)

Fricas [B] time = 0.631051, size = 76, normalized size = 4.

$$-\frac{2ax^5 + b}{10(a^4x^{10} + 2a^3bx^5 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")

[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)

Sympy [B] time = 5.42171, size = 36, normalized size = 1.89

$$-\frac{2ax^5 + b}{10a^4x^{10} + 20a^3bx^5 + 10a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x**3+a*x**2)**3,x)

[Out] -(2*a*x**5 + b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)

Giac [A] time = 1.1578, size = 30, normalized size = 1.58

$$-\frac{2ax^5 + b}{10(ax^5 + b)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")

[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)

$$3.343 \quad \int \frac{1}{\left(\frac{b}{x^5} + ax^3\right)^3} dx$$

Optimal. Leaf size=19

$$\frac{x^{16}}{16b(ax^8 + b)^2}$$

[Out] x¹⁶/(16*b*(b + a*x⁸)²)

Rubi [A] time = 0.0057949, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 264}

$$\frac{x^{16}}{16b(ax^8 + b)^2}$$

Antiderivative was successfully verified.

[In] Int[(b/x⁵ + a*x³)⁽⁻³⁾, x]

[Out] x¹⁶/(16*b*(b + a*x⁸)²)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))ⁿ, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*xⁿ)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\left(\frac{b}{x^5} + ax^3\right)^3} dx = \int \frac{x^{15}}{(b + ax^8)^3} dx$$

$$= \frac{x^{16}}{16b(b + ax^8)^2}$$

Mathematica [A] time = 0.0111089, size = 24, normalized size = 1.26

$$-\frac{2ax^8 + b}{16a^2(ax^8 + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^5 + a*x^3)^(-3),x]

[Out] -(b + 2*a*x^8)/(16*a^2*(b + a*x^8)^2)

Maple [A] time = 0.008, size = 31, normalized size = 1.6

$$\frac{b}{16a^2(ax^8 + b)^2} - \frac{1}{8a^2(ax^8 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/x^5+a*x^3)^3,x)

[Out] 1/16*b/a^2/(a*x^8+b)^2-1/8/a^2/(a*x^8+b)

Maxima [B] time = 1.10908, size = 49, normalized size = 2.58

$$-\frac{2ax^8 + b}{16(a^4x^{16} + 2a^3bx^8 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")

[Out] $-1/16*(2*a*x^8 + b)/(a^4*x^{16} + 2*a^3*b*x^8 + a^2*b^2)$

Fricas [B] time = 0.820367, size = 76, normalized size = 4.

$$-\frac{2ax^8 + b}{16(a^4x^{16} + 2a^3bx^8 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")

[Out] $-1/16*(2*a*x^8 + b)/(a^4*x^{16} + 2*a^3*b*x^8 + a^2*b^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x**5+a*x**3)**3,x)

[Out] Timed out

Giac [A] time = 1.17288, size = 30, normalized size = 1.58

$$-\frac{2ax^8 + b}{16(ax^8 + b)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")

[Out] $-1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)$

$$3.344 \quad \int \left(\frac{a}{x} + bx \right)^2 dx$$

Optimal. Leaf size=24

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

[Out] $-(a^2/x) + 2*a*b*x + (b^2*x^3)/3$

Rubi [A] time = 0.0109929, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 270}

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a/x + b*x)^2, x]$

[Out] $-(a^2/x) + 2*a*b*x + (b^2*x^3)/3$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 270

$\text{Int}[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] \rightarrow \text{Int}[\text{Exp andIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x} + bx\right)^2 dx &= \int \frac{(a + bx^2)^2}{x^2} dx \\ &= \int \left(2ab + \frac{a^2}{x^2} + b^2x^2\right) dx \\ &= -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.001072, size = 24, normalized size = 1.

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a/x + b*x)^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

Maple [A] time = 0.003, size = 23, normalized size = 1.

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x+b*x)^2,x)

[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3

Maxima [A] time = 1.10582, size = 30, normalized size = 1.25

$$\frac{1}{3}b^2x^3 + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A] time = 0.726116, size = 50, normalized size = 2.08

$$\frac{b^2x^4 + 6abx^2 - 3a^2}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] time = 0.317238, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [A] time = 1.17389, size = 30, normalized size = 1.25

$$\frac{1}{3}b^2x^3 + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```


$$3.345 \quad \int \left(\frac{a}{x} + bx \right)^3 dx$$

Optimal. Leaf size=40

$$3a^2b \log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

[Out] $-a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]$

Rubi [A] time = 0.0225053, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1593, 266, 43}

$$3a^2b \log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a/x + b*x)^3, x]

[Out] $-a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \left(\frac{a}{x} + bx\right)^3 dx &= \int \frac{(a + bx^2)^3}{x^3} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx)^3}{x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2b}{x} + b^3x \right) dx, x, x^2 \right) \\
&= -\frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4} + 3a^2b \log(x)
\end{aligned}$$

Mathematica [A] time = 0.0068147, size = 40, normalized size = 1.

$$3a^2b \log(x) - \frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a/x + b*x)^3,x]

[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{a^3}{2x^2} + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} + 3a^2b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x+b*x)^3,x)

[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)

Maxima [A] time = 1.15406, size = 46, normalized size = 1.15

$$\frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + 3a^2b \log(x) - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^3,x, algorithm="maxima")

[Out] $1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*\log(x) - 1/2*a^3/x^2$

Fricas [A] time = 0.770119, size = 85, normalized size = 2.12

$$\frac{b^3x^6 + 6ab^2x^4 + 12a^2bx^2\log(x) - 2a^3}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^3,x, algorithm="fricas")

[Out] $1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*\log(x) - 2*a^3)/x^2$

Sympy [A] time = 0.419012, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)**3,x)

[Out] $-a**3/(2*x**2) + 3*a**2*b*\log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4$

Giac [A] time = 1.11189, size = 62, normalized size = 1.55

$$\frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\log(x^2) - \frac{3a^2bx^2 + a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^3,x, algorithm="giac")

[Out] $1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*\log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/x^2$

$$3.346 \quad \int \left(\frac{a}{x} + bx \right)^4 dx$$

Optimal. Leaf size=50

$$6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3} + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

[Out] $-a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5$

Rubi [A] time = 0.0211109, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 270}

$$6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3} + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a/x + b*x)^4, x]

[Out] $-a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 270

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \left(\frac{a}{x} + bx\right)^4 dx &= \int \frac{(a + bx^2)^4}{x^4} dx \\
&= \int \left(6a^2b^2 + \frac{a^4}{x^4} + \frac{4a^3b}{x^2} + 4ab^3x^2 + b^4x^4\right) dx \\
&= -\frac{a^4}{3x^3} - \frac{4a^3b}{x} + 6a^2b^2x + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.0060251, size = 50, normalized size = 1.

$$6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3} + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a/x + b*x)^4,x]

[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5

Maple [A] time = 0.006, size = 45, normalized size = 0.9

$$-\frac{a^4}{3x^3} - 4\frac{a^3b}{x} + 6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x+b*x)^4,x)

[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5

Maxima [A] time = 1.17249, size = 59, normalized size = 1.18

$$\frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^4,x, algorithm="maxima")

[Out] $1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3$

Fricas [A] time = 0.757692, size = 104, normalized size = 2.08

$$\frac{3b^4x^8 + 20ab^3x^6 + 90a^2b^2x^4 - 60a^3bx^2 - 5a^4}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^4,x, algorithm="fricas")

[Out] $1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3$

Sympy [A] time = 0.350488, size = 48, normalized size = 0.96

$$6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} - \frac{a^4 + 12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)**4,x)

[Out] $6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 - (a**4 + 12*a**3*b*x**2)/(3*x**3)$

Giac [A] time = 1.11432, size = 61, normalized size = 1.22

$$\frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{12a^3bx^2 + a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x)^4,x, algorithm="giac")

[Out] $1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3$

$$3.347 \quad \int \frac{1}{\frac{1}{x^2} + x^3} dx$$

Optimal. Leaf size=185

$$-\frac{1}{20}(1 + \sqrt{5}) \log\left(x^2 - \frac{1}{2}(1 - \sqrt{5})x + 1\right) - \frac{1}{20}(1 - \sqrt{5}) \log\left(x^2 - \frac{1}{2}(1 + \sqrt{5})x + 1\right) + \frac{1}{5} \log(x + 1) - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{arctan}\left(\frac{x}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}}\right)$$

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[(2*(5 + Sqrt[5])/5)*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sqrt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

Rubi [A] time = 0.345543, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {1593, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}(1 + \sqrt{5}) \log\left(x^2 - \frac{1}{2}(1 - \sqrt{5})x + 1\right) - \frac{1}{20}(1 - \sqrt{5}) \log\left(x^2 - \frac{1}{2}(1 + \sqrt{5})x + 1\right) + \frac{1}{5} \log(x + 1) - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{arctan}\left(\frac{x}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-2) + x^3)^(-1), x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[(2*(5 + Sqrt[5])/5)*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sqrt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[(2*k
```

```

- 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((r)^m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]

```

Rule 634

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rule 628

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

Rule 31

```

Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\frac{1}{x^2} + x^3} dx &= \int \frac{x^2}{1 + x^5} dx \\
&= \frac{2}{5} \int \frac{\frac{1}{4}(-1 - \sqrt{5}) - \frac{1}{4}(1 + \sqrt{5})x}{1 - \frac{1}{2}(1 - \sqrt{5})x + x^2} dx + \frac{2}{5} \int \frac{\frac{1}{4}(-1 + \sqrt{5}) - \frac{1}{4}(1 - \sqrt{5})x}{1 - \frac{1}{2}(1 + \sqrt{5})x + x^2} dx + \frac{1}{5} \int \frac{1}{1 + x} dx \\
&= \frac{1}{5} \log(1 + x) + \frac{\int \frac{1}{1 + \frac{1}{2}(-1 - \sqrt{5})x + x^2} dx}{2\sqrt{5}} - \frac{\int \frac{1}{1 + \frac{1}{2}(-1 + \sqrt{5})x + x^2} dx}{2\sqrt{5}} + \frac{1}{20}(-1 - \sqrt{5}) \int \frac{\frac{1}{2}(-1 + \sqrt{5}) + 2x}{1 + \frac{1}{2}(-1 + \sqrt{5})x + x^2} dx \\
&= \frac{1}{5} \log(1 + x) - \frac{1}{20}(1 - \sqrt{5}) \log(2 - x - \sqrt{5}x + 2x^2) - \frac{1}{20}(1 + \sqrt{5}) \log(2 - x + \sqrt{5}x + 2x^2) + \frac{\text{Subst}}{\dots} \\
&= \sqrt{\frac{2}{5(5 + \sqrt{5})}} \tan^{-1} \left(\frac{1 - \sqrt{5} - 4x}{\sqrt{2(5 + \sqrt{5})}} \right) - \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tan^{-1} \left(\frac{1}{2} \sqrt{\frac{1}{10}(5 + \sqrt{5})} (1 + \sqrt{5} - 4x) \right) + \frac{1}{5} \log
\end{aligned}$$

Mathematica [A] time = 0.125276, size = 144, normalized size = 0.78

$$\frac{1}{20} \left(-(1 + \sqrt{5}) \log \left(x^2 + \frac{1}{2}(\sqrt{5} - 1)x + 1 \right) + (\sqrt{5} - 1) \log \left(x^2 - \frac{1}{2}(1 + \sqrt{5})x + 1 \right) + 4 \log(x + 1) - 2\sqrt{2(5 + \sqrt{5})} \tan^{-1} \left(\frac{1 - \sqrt{5} - 4x}{\sqrt{2(5 + \sqrt{5})}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^(-2) + x^3)^(-1), x]

[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]] + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20

Maple [A] time = 0.019, size = 156, normalized size = 0.8

$$\frac{\ln(1 + x)}{5} + \frac{\ln(-x\sqrt{5} + 2x^2 - x + 2)\sqrt{5}}{20} - \frac{\ln(-x\sqrt{5} + 2x^2 - x + 2)}{20} + \frac{2\sqrt{5}}{5\sqrt{10 - 2\sqrt{5}}} \arctan \left(\frac{-\sqrt{5} + 4x - 1}{\sqrt{10 - 2\sqrt{5}}} \right) - \frac{\ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^2+x^3),x)

[Out] $\frac{1}{5}\ln(1+x) + \frac{1}{20}\ln(-x\sqrt{5} + 2x^2 - x + 2)\sqrt{5} - \frac{1}{20}\ln(-x\sqrt{5} + 2x^2 - x + 2) + \frac{2}{5}\sqrt{10 - 2\sqrt{5}} \arctan\left(\frac{-\sqrt{5} + 4x - 1}{\sqrt{10 - 2\sqrt{5}}}\right) - \frac{1}{20}\ln(x\sqrt{5} + 2x^2 - x + 2)\sqrt{5} - \frac{1}{20}\ln(x\sqrt{5} + 2x^2 - x + 2) - \frac{2}{5}\sqrt{10 + 2\sqrt{5}} \arctan\left(\frac{\sqrt{5} + 4x - 1}{\sqrt{10 + 2\sqrt{5}}}\right)\sqrt{5}$

Maxima [A] time = 1.54593, size = 167, normalized size = 0.9

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}} + \frac{\log(2x^2 - x(\sqrt{5}+1) + 2)}{5(\sqrt{5}+1)} - \frac{\log(2x^2 + x(\sqrt{5}-1) + 2)}{5(\sqrt{5}-1)} + \frac{1}{5}\log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")

[Out] $-\frac{2}{5}\sqrt{5}\arctan\left(\frac{4x + \sqrt{5} - 1}{\sqrt{2\sqrt{5} + 10}}\right) + \frac{2}{5}\sqrt{5}\arctan\left(\frac{4x - \sqrt{5} - 1}{\sqrt{-2\sqrt{5} + 10}}\right) + \frac{1}{5}\log(2x^2 - x(\sqrt{5} + 1) + 2) - \frac{1}{5}\log(2x^2 + x(\sqrt{5} - 1) + 2)$

Fricas [B] time = 5.63503, size = 2210, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")

[Out] $-\frac{1}{20}(2\sqrt{1/2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)\log(1/16(2\sqrt{1/2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 + x) + \frac{1}{20}(\sqrt{5} + 2\sqrt{-3/16(2\sqrt{1/2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 + 1/8(2\sqrt{1/2}\sqrt{\sqrt{5}-5} - 5) + \sqrt{5} - 3)}(2\sqrt{1/2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1) - \frac{3}{16}(2\sqrt{1/2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + \sqrt{1/2}\sqrt{\sqrt{5}-5} + 1/2\sqrt{5} - 5/2) - 1)\log(-1/16(2\sqrt{1/2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 - 1/16(2\sqrt{1/2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + 1/2\sqrt{5} - 5/2)$

$$\begin{aligned}
& (-3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 + 1/8*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} - 3)*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1) - 3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + \sqrt{1/2}*\sqrt{\sqrt{5}-5} + 1/2*\sqrt{5} - 5/2)*(\sqrt{5} - 1) + 2*x - 1) + 1/20*(\sqrt{5} - 2*\sqrt{-3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 + 1/8*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} - 3)*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1) - 3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + \sqrt{1/2}*\sqrt{\sqrt{5}-5} + 1/2*\sqrt{5} - 5/2) - 1)*\log(-1/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 - 1/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 - 1/2*\sqrt{-3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} + 1)^2 + 1/8*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} + \sqrt{5} - 3)*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1) - 3/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + \sqrt{1/2}*\sqrt{\sqrt{5}-5} + 1/2*\sqrt{5} - 5/2)*(\sqrt{5} - 1) + 2*x - 1) + 1/20*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)*\log(1/16*(2*\sqrt{1/2}*\sqrt{\sqrt{5}-5} - \sqrt{5} - 1)^2 + x) + 1/5*\log(x + 1)
\end{aligned}$$

Sympy [A] time = 1.6551, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5} + \text{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, (t \mapsto t \log(25t^2 + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x**2+x**3),x)

[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(25*_t**2 + x)))

Giac [A] time = 1.15905, size = 151, normalized size = 0.82

$$\frac{1}{20}(\sqrt{5}-1)\log\left(x^2 - \frac{1}{2}x(\sqrt{5}+1) + 1\right) - \frac{1}{20}(\sqrt{5}+1)\log\left(x^2 + \frac{1}{2}x(\sqrt{5}-1) + 1\right) - \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x + \sqrt{2\sqrt{5}}}{\sqrt{2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")

[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x

$$\begin{aligned} & + \sqrt{5} - 1)/\sqrt{2\sqrt{5} + 10}) + 1/10\sqrt{2\sqrt{5} + 10})\arctan((4 \\ *x - \sqrt{5} - 1)/\sqrt{-2\sqrt{5} + 10})) + 1/5*\log(\text{abs}(x + 1)) \end{aligned}$$

$$3.348 \quad \int x^p (ax^n + bx^{1+13n+p})^{12} dx$$

Optimal. Leaf size=29

$$\frac{(a + bx^{12n+p+1})^{13}}{13b(12n + p + 1)}$$

[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))

Rubi [A] time = 0.0172572, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1584, 261}

$$\frac{(a + bx^{12n+p+1})^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]

[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^p (ax^n + bx^{1+13n+p})^{12} dx = \int x^{12n+p} (a + bx^{1+12n+p})^{12} dx$$

$$= \frac{(a + bx^{1+12n+p})^{13}}{13b(1 + 12n + p)}$$

Mathematica [A] time = 0.0155429, size = 29, normalized size = 1.

$$\frac{(a + bx^{12n+p+1})^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]

[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))

Maple [B] time = 0.168, size = 363, normalized size = 12.5

$$\frac{(x^n)^{156} x^{13} b^{12} (x^p)^{13}}{13 + 156n + 13p} + \frac{(x^n)^{144} x^{12} a b^{11} (x^p)^{12}}{1 + 12n + p} + 6 \frac{(x^n)^{132} x^{11} a^2 b^{10} (x^p)^{11}}{1 + 12n + p} + 22 \frac{(x^n)^{120} x^{10} a^3 b^9 (x^p)^{10}}{1 + 12n + p} + 55 \frac{(x^n)^{108} x^9 a^4 b^8 (x^p)^9}{1 + 12n + p} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)

[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*a^11*b*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+a^12/(1+12*n+p)*x*(x^n)^12*x^p

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 0.853519, size = 753, normalized size = 25.97

$$78 a^2 b^{10} x^{2n} x^{143n+11p+11} + 286 a^3 b^9 x^{3n} x^{130n+10p+10} + 715 a^4 b^8 x^{4n} x^{117n+9p+9} + 1287 a^5 b^7 x^{5n} x^{104n+8p+8} + 1716 a^6 b^6 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")`

[Out]
$$\frac{1}{13} (78 a^2 b^{10} x^{2n} x^{(143n+11p+11)} + 286 a^3 b^9 x^{3n} x^{(130n+10p+10)} + 715 a^4 b^8 x^{4n} x^{(117n+9p+9)} + 1287 a^5 b^7 x^{5n} x^{(104n+8p+8)} + 1716 a^6 b^6 x^{6n} x^{(91n+7p+7)} + 1716 a^7 b^5 x^{7n} x^{(78n+6p+6)} + 1287 a^8 b^4 x^{8n} x^{(65n+5p+5)} + 715 a^9 b^3 x^{9n} x^{(52n+4p+4)} + 286 a^{10} b^2 x^{10n} x^{(39n+3p+3)} + 78 a^{11} b x^{11n} x^{(26n+2p+2)} + 13 a^{12} x^{12n} x^{(13n+p+1)} + 13 a b^{11} x^{(156n+12p+12)} x^n + b^{12} x^{(169n+13p+13)}) / ((12n+p+1) x^{(13n)})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)`

[Out] Timed out

Giac [B] time = 4.31688, size = 363, normalized size = 12.52

$$b^{12} x^{13} x^{156n} x^{13p} + 13 a b^{11} x^{12} x^{144n} x^{12p} + 78 a^2 b^{10} x^{11} x^{132n} x^{11p} + 286 a^3 b^9 x^{10} x^{120n} x^{10p} + 715 a^4 b^8 x^9 x^{108n} x^{9p} + 1287 a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^p*(a*xⁿ+b*x^(1+13*n+p))¹²,x, algorithm="giac")

[Out] $\frac{1}{13}*(b^{12}*x^{13}*x^{(156*n)}*x^{(13*p)} + 13*a*b^{11}*x^{12}*x^{(144*n)}*x^{(12*p)} + 78*a^2*b^{10}*x^{11}*x^{(132*n)}*x^{(11*p)} + 286*a^3*b^9*x^{10}*x^{(120*n)}*x^{(10*p)} + 715*a^4*b^8*x^9*x^{(108*n)}*x^{(9*p)} + 1287*a^5*b^7*x^8*x^{(96*n)}*x^{(8*p)} + 1716*a^6*b^6*x^7*x^{(84*n)}*x^{(7*p)} + 1716*a^7*b^5*x^6*x^{(72*n)}*x^{(6*p)} + 1287*a^8*b^4*x^5*x^{(60*n)}*x^{(5*p)} + 715*a^9*b^3*x^4*x^{(48*n)}*x^{(4*p)} + 286*a^{10}*b^2*x^3*x^{(36*n)}*x^{(3*p)} + 78*a^{11}*b*x^2*x^{(24*n)}*x^{(2*p)} + 13*a^{12}*x*x^{(12*n)}*x^p)/(12*n + p + 1)$

$$3.349 \quad \int x^{12} (a + bx^{13})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{13})^{13}}{169b}$$

[Out] (a + b*x^13)^13/(169*b)

Rubi [A] time = 0.0029452, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{(a + bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

[In] Int[x^12*(a + b*x^13)^12,x]

[Out] (a + b*x^13)^13/(169*b)

Rule 261

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^{12} (a + bx^{13})^{12} dx = \frac{(a + bx^{13})^{13}}{169b}$$

Mathematica [B] time = 0.0047348, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{40} + \frac{6}{13}a^{11}bx^{27} + \frac{2}{13}a^{12}$$

Antiderivative was successfully verified.

[In] Integrate[x^12*(a + b*x^13)^12,x]

[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169

Maple [B] time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169} + \frac{b^{11}ax^{156}}{13} + \frac{6b^{10}a^2x^{143}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{99a^8b^4x^{65}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12*(b*x^13+a)^12,x)

[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*x^26+1/13*a^12*x^13

Maxima [A] time = 1.0264, size = 19, normalized size = 1.19

$$\frac{(bx^{13} + a)^{13}}{169b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")

[Out] 1/169*(b*x^13 + a)^13/b

Fricas [B] time = 0.612211, size = 365, normalized size = 22.81

$$\frac{1}{169}x^{169}b^{12} + \frac{1}{13}x^{156}b^{11}a + \frac{6}{13}x^{143}b^{10}a^2 + \frac{22}{13}x^{130}b^9a^3 + \frac{55}{13}x^{117}b^8a^4 + \frac{99}{13}x^{104}b^7a^5 + \frac{132}{13}x^{91}b^6a^6 + \frac{132}{13}x^{78}b^5a^7 + \frac{99}{13}x^{65}b^4a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")`

[Out] $\frac{1}{169}x^{169}b^{12} + \frac{1}{13}x^{156}b^{11}a + \frac{6}{13}x^{143}b^{10}a^2 + \frac{22}{13}x^{130}b^9a^3 + \frac{55}{13}x^{117}b^8a^4 + \frac{99}{13}x^{104}b^7a^5 + \frac{132}{13}x^{91}b^6a^6 + \frac{132}{13}x^{78}b^5a^7 + \frac{99}{13}x^{65}b^4a^8 + \frac{55}{13}x^{52}b^3a^9 + \frac{22}{13}x^{39}b^2a^{10} + \frac{6}{13}x^{26}b^1a^{11} + \frac{1}{13}x^{13}a^{12}$

Sympy [B] time = 0.122232, size = 160, normalized size = 10.

$$\frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**12*(b*x**13+a)**12,x)`

[Out] $a^{12}x^{13}/13 + 6a^{11}bx^{26}/13 + 22a^{10}b^2x^{39}/13 + 55a^9b^3x^{52}/13 + 99a^8b^4x^{65}/13 + 132a^7b^5x^{78}/13 + 132a^6b^6x^{91}/13 + 99a^5b^7x^{104}/13 + 55a^4b^8x^{117}/13 + 22a^3b^9x^{130}/13 + 6a^2b^{10}x^{143}/13 + ab^{11}x^{156}/13 + b^{12}x^{169}/169$

Giac [A] time = 1.21248, size = 19, normalized size = 1.19

$$\frac{(bx^{13} + a)^{13}}{169b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")`

[Out] $\frac{1}{169}(b*x^{13} + a)^{13}/b$

$$3.350 \quad \int x^{12} (ax + bx^{26})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{25})^{13}}{325b}$$

[Out] (a + b*x^25)^13/(325*b)

Rubi [A] time = 0.0086186, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{(a + bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

[In] Int[x^12*(a*x + b*x^26)^12,x]

[Out] (a + b*x^25)^13/(325*b)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^{12} (ax + bx^{26})^{12} dx &= \int x^{24} (a + bx^{25})^{12} dx \\ &= \frac{(a + bx^{25})^{13}}{325b} \end{aligned}$$

Mathematica [B] time = 0.0060576, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}$$

Antiderivative was successfully verified.

[In] Integrate[x^12*(a*x + b*x^26)^12,x]

[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325

Maple [B] time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325} + \frac{b^{11}ax^{300}}{25} + \frac{6b^{10}a^2x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{99a^8b^4x^{125}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{22a^{10}b^2x^{75}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{a^{12}x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12*(b*x^26+a*x)^12,x)

[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*b*x^50+1/25*a^12*x^25

Maxima [B] time = 1.05386, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{a^{12}x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")

[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10*b^2*x^75 + 6/25*a^11*b*x^50 + a^12*x^25/25

$$*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25$$

Fricas [B] time = 0.619714, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}b^1a^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")

[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 132/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12

Sympy [B] time = 0.151775, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**12*(b*x**26+a*x)**12,x)

[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325

Giac [B] time = 1.1787, size = 181, normalized size = 11.31

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")

[Out] $\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{32}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$

$$3.351 \quad \int x^{12} (ax^2 + bx^{39})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{37})^{13}}{481b}$$

[Out] (a + b*x^37)^13/(481*b)

Rubi [A] time = 0.0119976, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 261}

$$\frac{(a + bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^12*(a*x^2 + b*x^39)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
  :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
  && IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^{12} (ax^2 + bx^{39})^{12} dx &= \int x^{36} (a + bx^{37})^{12} dx \\ &= \frac{(a + bx^{37})^{13}}{481b} \end{aligned}$$

Mathematica [B] time = 0.0071078, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Antiderivative was successfully verified.

[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

Maple [B] time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6b^{10}a^2x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12*(b*x^39+a*x^2)^12,x)

[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^11*b*x^74+1/481*b^12*x^481

Maxima [B] time = 1.06426, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")

[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 + 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/481*b^12*x^481

$$10*b^2*x^{111} + 6/37*a^{11}*b*x^{74} + 1/37*a^{12}*x^{37}$$

Fricas [B] time = 0.58102, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}b*a^{11} + \frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")

[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 + 132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12

Sympy [B] time = 0.126827, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**12*(b*x**39+a*x**2)**12,x)

[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/481

Giac [B] time = 1.19325, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*  
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +  
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^  
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

$$3.352 \quad \int x^{24} (a + bx^{25})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{25})^{13}}{325b}$$

[Out] (a + b*x^25)^13/(325*b)

Rubi [A] time = 0.0027836, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{(a + bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

[In] Int[x^24*(a + b*x^25)^12,x]

[Out] (a + b*x^25)^13/(325*b)

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^{24} (a + bx^{25})^{12} dx = \frac{(a + bx^{25})^{13}}{325b}$$

Mathematica [B] time = 0.0042827, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}b^{10}x^{275}$$

Antiderivative was successfully verified.

[In] Integrate[x^24*(a + b*x^25)^12,x]

[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325

Maple [B] time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325} + \frac{b^{11}ax^{300}}{25} + \frac{6b^{10}a^2x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{99a^8b^4x^{125}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{22a^{10}b^2x^{75}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{a^{12}x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^24*(b*x^25+a)^12,x)

[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*b*x^50+1/25*a^12*x^25

Maxima [A] time = 1.10246, size = 19, normalized size = 1.19

$$\frac{(bx^{25} + a)^{13}}{325b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")

[Out] 1/325*(b*x^25 + a)^13/b

Fricas [B] time = 0.680386, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{125}b^4a^8 + \frac{11}{5}x^{100}b^3a^9 + \frac{22}{25}x^{75}b^2a^{10} + \frac{6}{25}x^{50}ba^{11} + \frac{1}{25}x^{25}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²⁴*(b*x²⁵+a)¹²,x, algorithm="fricas")

[Out] 1/325*x³²⁵*b¹² + 1/25*x³⁰⁰*b¹¹*a + 6/25*x²⁷⁵*b¹⁰*a² + 22/25*x²⁵⁰*b⁹*a³ + 11/5*x²²⁵*b⁸*a⁴ + 99/25*x²⁰⁰*b⁷*a⁵ + 132/25*x¹⁷⁵*b⁶*a⁶ + 132/25*x¹⁵⁰*b⁵*a⁷ + 99/25*x¹²⁵*b⁴*a⁸ + 11/5*x¹⁰⁰*b³*a⁹ + 22/25*x⁷⁵*b²*a¹⁰ + 6/25*x⁵⁰*b*a¹¹ + 1/25*x²⁵*a¹²

Sympy [B] time = 0.121134, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**24*(b*x**25+a)**12,x)

[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325

Giac [A] time = 1.20377, size = 19, normalized size = 1.19

$$\frac{(bx^{25} + a)^{13}}{325b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²⁴*(b*x²⁵+a)¹²,x, algorithm="giac")

[Out] 1/325*(b*x²⁵ + a)¹³/b

$$3.353 \quad \int x^{24} (ax + bx^{38})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{37})^{13}}{481b}$$

[Out] (a + b*x^37)^13/(481*b)

Rubi [A] time = 0.0083321, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{(a + bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^24*(a*x + b*x^38)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x^{24} (ax + bx^{38})^{12} dx &= \int x^{36} (a + bx^{37})^{12} dx \\ &= \frac{(a + bx^{37})^{13}}{481b} \end{aligned}$$

Mathematica [B] time = 0.0057769, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{11}{37}a^{11}bx^{74} + \frac{1}{481}b^{12}x^{481}$$

Antiderivative was successfully verified.

[In] Integrate[x^24*(a*x + b*x^38)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

Maple [B] time = 0.002, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6b^{10}a^2x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{11ab^{11}x^{74}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^24*(b*x^38+a*x)^12,x)

[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^11*b*x^74+1/37*a^12*x^37

Maxima [B] time = 1.01604, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{11}{37}ab^{11}x^{74} + \frac{1}{481}b^{12}x^{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")

[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 + 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/481*b^12*x^481

$$10*b^2*x^{111} + 6/37*a^{11}*b*x^{74} + 1/37*a^{12}*x^{37}$$

Fricas [B] time = 0.644098, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}b^1a^{11} + \frac{1}{37}x^{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")

[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 + 132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12

Sympy [B] time = 0.145064, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{a^{11}bx^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**24*(b*x**38+a*x)**12,x)

[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/481

Giac [B] time = 1.17673, size = 181, normalized size = 11.31

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*  
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +  
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^  
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

$$3.354 \quad \int x^{36} (a + bx^{37})^{12} dx$$

Optimal. Leaf size=16

$$\frac{(a + bx^{37})^{13}}{481b}$$

[Out] (a + b*x^37)^13/(481*b)

Rubi [A] time = 0.0029089, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{(a + bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^36*(a + b*x^37)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 261

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^{36} (a + bx^{37})^{12} dx = \frac{(a + bx^{37})^{13}}{481b}$$

Mathematica [B] time = 0.0058375, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}b^1x^{74} + \frac{6}{37}a^{12}b^0x^{37}$$

Antiderivative was successfully verified.

[In] Integrate[x^36*(a + b*x^37)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

Maple [B] time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6b^{10}a^2x^{407}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{a^{12}x^{37}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^36*(b*x^37+a)^12,x)

[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^11*b*x^74+1/37*a^12*x^37

Maxima [A] time = 0.981064, size = 19, normalized size = 1.19

$$\frac{(bx^{37} + a)^{13}}{481b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")

[Out] 1/481*(b*x^37 + a)^13/b

Fricas [B] time = 0.589219, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}ba^{11} + \frac{a^{12}x^{37}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³⁶*(b*x³⁷+a)¹²,x, algorithm="fricas")

[Out] $\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 + \frac{99}{37}x^{185}b^4a^8 + \frac{55}{37}x^{148}b^3a^9 + \frac{22}{37}x^{111}b^2a^{10} + \frac{6}{37}x^{74}b^1a^{11} + \frac{1}{37}x^{37}a^{12}$

Sympy [B] time = 0.133362, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**36*(b*x**37+a)**12,x)

[Out] $a^{12}x^{37}/37 + 6a^{11}bx^{74}/37 + 22a^{10}b^2x^{111}/37 + 55a^9b^3x^{148}/37 + 99a^8b^4x^{185}/37 + 132a^7b^5x^{222}/37 + 132a^6b^6x^{259}/37 + 99a^5b^7x^{296}/37 + 55a^4b^8x^{333}/37 + 22a^3b^9x^{370}/37 + 6a^2b^{10}x^{407}/37 + ab^{11}x^{444}/37 + b^{12}x^{481}/481$

Giac [A] time = 1.18695, size = 19, normalized size = 1.19

$$\frac{(bx^{37} + a)^{13}}{481b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³⁶*(b*x³⁷+a)¹²,x, algorithm="giac")

[Out] $\frac{1}{481}(b*x^{37} + a)^{13}/b$

$$3.355 \quad \int \frac{1}{ax+bx^n} dx$$

Optimal. Leaf size=23

$$\frac{\log(ax^{1-n} + b)}{a(1-n)}$$

[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))

Rubi [A] time = 0.0108914, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{\log(ax^{1-n} + b)}{a(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^n)^(-1), x]

[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{ax + bx^n} dx &= \int \frac{x^{-n}}{b + ax^{1-n}} dx \\ &= \frac{\log(b + ax^{1-n})}{a(1-n)} \end{aligned}$$

Mathematica [A] time = 0.0081442, size = 23, normalized size = 1.

$$\frac{\log(ax^{1-n} + b)}{a(1-n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^n)^(-1), x]

[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))

Maple [A] time = 0.01, size = 36, normalized size = 1.6

$$\frac{n \ln(x)}{a(-1+n)} - \frac{\ln(ax + be^{n \ln(x)})}{a(-1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x^n), x)

[Out] n/a/(-1+n)*ln(x)-1/a/(-1+n)*ln(a*x+b*exp(n*ln(x)))

Maxima [A] time = 1.00649, size = 50, normalized size = 2.17

$$\frac{n \log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x^n), x, algorithm="maxima")

[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))

Fricas [A] time = 0.930557, size = 55, normalized size = 2.39

$$\frac{n \log(x) - \log(ax + bx^n)}{an - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

Sympy [A] time = 0.693469, size = 48, normalized size = 2.09

$$\begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 1 \\ -\frac{x}{b(nx^n - x^n)} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 1 \\ \frac{n \log(x)}{an-a} - \frac{\log\left(x + \frac{bx^n}{a}\right)}{an-a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 1)), (-x/(b*(n*x**n - x**n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (n*log(x)/(a*n - a) - log(x + b*x**n/a)/(a*n - a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^n), x)
```


$$3.356 \quad \int \frac{1}{ax+bx^{1+n}} dx$$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a + bx^n)}{an}$$

[Out] Log[x]/a - Log[a + b*x^n]/(a*n)

Rubi [A] time = 0.0131801, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a + bx^n)}{an}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^(1 + n))^(-1), x]

[Out] Log[x]/a - Log[a + b*x^n]/(a*n)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{ax + bx^{1+n}} dx &= \int \frac{1}{x(a + bx^n)} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right)}{an} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, x^n\right)}{an} \\ &= \frac{\log(x)}{a} - \frac{\log(a + bx^n)}{an} \end{aligned}$$

Mathematica [A] time = 0.0062417, size = 22, normalized size = 0.96

$$\frac{n \log(x) - \log(a + bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 + n))^-1, x]
```

```
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

Maple [A] time = 0.008, size = 39, normalized size = 1.7

$$\frac{\ln(x)}{an} + \frac{\ln(x)}{a} - \frac{\ln(ax + be^{(1+n)\ln(x)})}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1+n)), x)
```

[Out] $1/a/n*\ln(x)+\ln(x)/a-1/a/n*\ln(a*x+b*\exp((1+n)*\ln(x)))$

Maxima [A] time = 1.03796, size = 36, normalized size = 1.57

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")`

[Out] $\log(x)/a - \log((b*x^n + a)/b)/(a*n)$

Fricas [A] time = 0.877325, size = 66, normalized size = 2.87

$$\frac{(n+1)\log(x) - \log(ax + bx^{n+1})}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")`

[Out] $((n+1)*\log(x) - \log(a*x + b*x^(n+1)))/(a*n)$

Sympy [A] time = 1.71174, size = 41, normalized size = 1.78

$$\begin{cases} \infty \log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a} & \text{for } n = 0 \\ -\frac{a+b}{x^n} & \text{for } a = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+b*x**(1+n)),x)`

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (log(x)/(a + b), Eq(n, 0)), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

$$3.357 \quad \int \frac{1}{ax+bx^{1-n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(ax^n + b)}{an}$$

[Out] Log[b + a*x^n]/(a*n)

Rubi [A] time = 0.0076386, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 260}

$$\frac{\log(ax^n + b)}{an}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^(1 - n))^(-1), x]

[Out] Log[b + a*x^n]/(a*n)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{1}{ax+bx^{1-n}} dx = \int \frac{x^{-1+n}}{b+ax^n} dx = \frac{\log(b+ax^n)}{an}$$

Mathematica [A] time = 0.0044175, size = 15, normalized size = 1.

$$\frac{\log(ax^n + b)}{an}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^(1 - n))^(-1), x]

[Out] Log[b + a*x^n]/(a*n)

Maple [B] time = 0.01, size = 41, normalized size = 2.7

$$-\frac{\ln(x)}{an} + \frac{\ln(x)}{a} + \frac{\ln(ax + be^{(1-n)\ln(x)})}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x^(1-n)), x)

[Out] -1/a/n*ln(x)+ln(x)/a+1/a/n*ln(a*x+b*exp((1-n)*ln(x)))

Maxima [A] time = 1.02379, size = 26, normalized size = 1.73

$$\frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x^(1-n)), x, algorithm="maxima")

[Out] log((a*x^n + b)/a)/(a*n)

Fricas [A] time = 0.90674, size = 68, normalized size = 4.53

$$\frac{(n-1)\log(x) + \log(ax + bx^{-n+1})}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

Sympy [A] time = 1.99376, size = 39, normalized size = 2.6

$$\left\{ \begin{array}{ll} \infty \log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a} & \text{for } n = 0 \\ x^{\frac{a}{b}+b} & \text{for } a = 0 \\ \frac{bn}{a} & \text{for } a = 0 \\ \frac{\log(x)}{a} + \frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (log(x)/(a + b), Eq(n, 0)), (x**n/(b*n), Eq(a, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^{-n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

$$3.358 \quad \int \frac{1}{2x+3x^{1+n}} dx$$

Optimal. Leaf size=22

$$\frac{\log(x)}{2} - \frac{\log(3x^n + 2)}{2n}$$

[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)

Rubi [A] time = 0.0097024, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{2} - \frac{\log(3x^n + 2)}{2n}$$

Antiderivative was successfully verified.

[In] Int[(2*x + 3*x^(1 + n))^(-1), x]

[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29


```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{2x + 3x^{1+n}} dx &= \int \frac{1}{x(2 + 3x^n)} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{x(2+3x)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right)}{2n} - \frac{3 \text{Subst}\left(\int \frac{1}{2+3x} dx, x, x^n\right)}{2n} \\ &= \frac{\log(x)}{2} - \frac{\log(2 + 3x^n)}{2n} \end{aligned}$$

Mathematica [A] time = 0.0059449, size = 22, normalized size = 1.

$$\frac{n \log(x) - \log(3x^n + 2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 + n))^-1, x]
```

```
[Out] (n*Log[x] - Log[2 + 3*x^n])/(2*n)
```

Maple [A] time = 0.028, size = 32, normalized size = 1.5

$$\frac{\ln(x)}{2n} + \frac{\ln(x)}{2} - \frac{\ln(2x + 3e^{(1+n)\ln(x)})}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(1+n)), x)
```

[Out] $1/2/n*\ln(x)+1/2*\ln(x)-1/2/n*\ln(2*x+3*\exp((1+n)*\ln(x)))$

Maxima [A] time = 1.11082, size = 22, normalized size = 1.

$$-\frac{\log\left(x^n + \frac{2}{3}\right)}{2n} + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")`

[Out] $-1/2*\log(x^n + 2/3)/n + 1/2*\log(x)$

Fricas [A] time = 0.831635, size = 66, normalized size = 3.

$$\frac{(n+1)\log(x) - \log(3x^{n+1} + 2x)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")`

[Out] $1/2*((n+1)*\log(x) - \log(3*x^{(n+1)} + 2*x))/n$

Sympy [A] time = 1.989, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\log(x)}{2} - \frac{\log\left(x^n + \frac{2}{3}\right)}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x+3*x**(1+n)),x)`

[Out] `Piecewise((log(x)/2 - log(x**n + 2/3)/(2*n), Ne(n, 0)), (log(x)/5, True))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3x^{n+1} + 2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")`

[Out] `integrate(1/(3*x^(n + 1) + 2*x), x)`

$$3.359 \quad \int \frac{1}{2x+3x^{1-n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(2x^n + 3)}{2n}$$

[Out] Log[3 + 2*x^n]/(2*n)

Rubi [A] time = 0.0063023, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 260}

$$\frac{\log(2x^n + 3)}{2n}$$

Antiderivative was successfully verified.

[In] Int[(2*x + 3*x^(1 - n))^(-1), x]

[Out] Log[3 + 2*x^n]/(2*n)

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{2x + 3x^{1-n}} dx &= \int \frac{x^{-1+n}}{3 + 2x^n} dx \\ &= \frac{\log(3 + 2x^n)}{2n} \end{aligned}$$

Mathematica [A] time = 0.0033049, size = 15, normalized size = 1.

$$\frac{\log(2x^n + 3)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + 3*x^(1 - n))^(-1), x]

[Out] Log[3 + 2*x^n]/(2*n)

Maple [B] time = 0.009, size = 34, normalized size = 2.3

$$-\frac{\ln(x)}{2n} + \frac{\ln(x)}{2} + \frac{\ln(2x + 3e^{(1-n)\ln(x)})}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x+3*x^(1-n)), x)

[Out] -1/2/n*ln(x)+1/2*ln(x)+1/2/n*ln(2*x+3*exp((1-n)*ln(x)))

Maxima [A] time = 1.04812, size = 15, normalized size = 1.

$$\frac{\log\left(x^n + \frac{3}{2}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x+3*x^(1-n)), x, algorithm="maxima")

[Out] 1/2*log(x^n + 3/2)/n

Fricas [A] time = 0.722041, size = 68, normalized size = 4.53

$$\frac{(n-1)\log(x) + \log(3x^{-n+1} + 2x)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")

[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n

Sympy [A] time = 1.64235, size = 22, normalized size = 1.47

$$\begin{cases} \frac{\log(x)}{2} + \frac{\log\left(\frac{2}{3} + x^{-n}\right)}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x+3*x**(1-n)),x)

[Out] Piecewise((log(x)/2 + log(2/3 + x**(-n))/(2*n), Ne(n, 0)), (log(x)/5, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3x^{-n+1} + 2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")

[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)

$$3.360 \quad \int \frac{1}{-\sqrt{x+x}} dx$$

Optimal. Leaf size=12

$$2 \log(1 - \sqrt{x})$$

[Out] 2*Log[1 - Sqrt[x]]

Rubi [A] time = 0.003826, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$2 \log(1 - \sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[x] + x)^(-1), x]

[Out] 2*Log[1 - Sqrt[x]]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt{x+x}} dx &= \int \frac{1}{(-1 + \sqrt{x})\sqrt{x}} dx \\ &= 2 \log(1 - \sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.0022968, size = 12, normalized size = 1.

$$2 \log(1 - \sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[x] + x)^(-1), x]

[Out] 2*Log[1 - Sqrt[x]]

Maple [A] time = 0.003, size = 12, normalized size = 1.

$$\ln(-1 + x) - 2 \operatorname{Artanh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-x^(1/2)), x)

[Out] ln(-1+x)-2*arctanh(x^(1/2))

Maxima [A] time = 1.04106, size = 11, normalized size = 0.92

$$2 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-x^(1/2)), x, algorithm="maxima")

[Out] 2*log(sqrt(x) - 1)

Fricas [A] time = 0.82284, size = 27, normalized size = 2.25

$$2 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-x^(1/2)),x, algorithm="fricas")`

[Out] `2*log(sqrt(x) - 1)`

Sympy [A] time = 0.152274, size = 8, normalized size = 0.67

$$2 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-x**(1/2)),x)`

[Out] `2*log(sqrt(x) - 1)`

Giac [A] time = 1.17205, size = 12, normalized size = 1.

$$2 \log(|\sqrt{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-x^(1/2)),x, algorithm="giac")`

[Out] `2*log(abs(sqrt(x) - 1))`

$$3.361 \quad \int \frac{1}{-x^{3/5}+x} dx$$

Optimal. Leaf size=14

$$\frac{5}{2} \log(1 - x^{2/5})$$

[Out] (5*Log[1 - x^(2/5)])/2

Rubi [A] time = 0.0046696, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{5}{2} \log(1 - x^{2/5})$$

Antiderivative was successfully verified.

[In] Int[(-x^(3/5) + x)^(-1), x]

[Out] (5*Log[1 - x^(2/5)])/2

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{-x^{3/5}+x} dx &= \int \frac{1}{(-1+x^{2/5})x^{3/5}} dx \\ &= \frac{5}{2} \log(1 - x^{2/5}) \end{aligned}$$

Mathematica [A] time = 0.0024372, size = 14, normalized size = 1.

$$\frac{5}{2} \log(1 - x^{2/5})$$

Antiderivative was successfully verified.

[In] Integrate[(-x^(3/5) + x)^(-1), x]

[Out] (5*Log[1 - x^(2/5)])/2

Maple [B] time = 0.096, size = 116, normalized size = 8.3

$$\frac{\ln(-1+x)}{2} + \frac{\ln(1+x)}{2} - \frac{1}{2} \ln(-\sqrt{5}\sqrt[5]{x} + 2x^{2/5} + \sqrt[5]{x} + 2) - \frac{1}{2} \ln(\sqrt{5}\sqrt[5]{x} + 2x^{2/5} + \sqrt[5]{x} + 2) + 2 \ln(-1 + \sqrt[5]{x}) + 2 \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(3/5)+x), x)

[Out] 1/2*ln(-1+x)+1/2*ln(1+x)-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)+2*ln(-1+x^(1/5))+2*ln(1+x^(1/5))-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)-x^(1/5)+2)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)-x^(1/5)+2)

Maxima [A] time = 1.0348, size = 23, normalized size = 1.64

$$\frac{5}{2} \log(x^{1/5} + 1) + \frac{5}{2} \log(x^{1/5} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(3/5)+x), x, algorithm="maxima")

[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)

Fricas [A] time = 0.787572, size = 30, normalized size = 2.14

$$\frac{5}{2} \log(x^{2/5} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")

[Out] 5/2*log(x^(2/5) - 1)

Sympy [B] time = 0.385201, size = 22, normalized size = 1.57

$$\frac{5 \log(\sqrt[5]{x} - 1)}{2} + \frac{5 \log(\sqrt[5]{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(3/5)+x),x)

[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2

Giac [A] time = 1.17703, size = 24, normalized size = 1.71

$$\frac{5}{2} \log\left(x^{\frac{1}{5}} + 1\right) + \frac{5}{2} \log\left(\left|x^{\frac{1}{5}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")

[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))

$$3.362 \quad \int \frac{1}{\frac{1}{\sqrt[3]{x}} + x} dx$$

Optimal. Leaf size=12

$$\frac{3}{4} \log(x^{4/3} + 1)$$

[Out] (3*Log[1 + x^(4/3)])/4

Rubi [A] time = 0.0035075, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1593, 260}

$$\frac{3}{4} \log(x^{4/3} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x^(-1/3) + x)^(-1), x]

[Out] (3*Log[1 + x^(4/3)])/4

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{1}{\sqrt[3]{x}} + x} dx &= \int \frac{\sqrt[3]{x}}{1 + x^{4/3}} dx \\ &= \frac{3}{4} \log(1 + x^{4/3}) \end{aligned}$$

Mathematica [A] time = 0.0020223, size = 12, normalized size = 1.

$$\frac{3}{4} \log(x^{4/3} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1/3) + x)^(-1), x]

[Out] (3*Log[1 + x^(4/3)])/4

Maple [A] time = 0., size = 9, normalized size = 0.8

$$\frac{3}{4} \ln(1 + x^{4/3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^(1/3)+x), x)

[Out] 3/4*ln(1+x^(4/3))

Maxima [A] time = 1.68293, size = 11, normalized size = 0.92

$$\frac{3}{4} \log(x^{4/3} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^(1/3)+x), x, algorithm="maxima")

[Out] 3/4*log(x^(4/3) + 1)

Fricas [A] time = 0.846989, size = 30, normalized size = 2.5

$$\frac{3}{4} \log(x^{4/3} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A] time = 0.261243, size = 10, normalized size = 0.83

$$\frac{3 \log\left(x^{\frac{4}{3}} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

Giac [B] time = 1.17784, size = 43, normalized size = 3.58

$$\frac{3}{4} \log\left(\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{3}{4} \log\left(-\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
+ 1)
```

$$3.363 \quad \int \frac{1}{x+x\sqrt{2}} dx$$

Optimal. Leaf size=24

$$\log(x) - (1 + \sqrt{2}) \log(x^{\sqrt{2}-1} + 1)$$

[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]

Rubi [A] time = 0.0163587, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1593, 266, 36, 29, 31}

$$\log(x) - (1 + \sqrt{2}) \log(x^{\sqrt{2}-1} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x + x^Sqrt[2])^(-1), x]

[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29


```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x + x\sqrt{2}} dx &= \int \frac{1}{x(1 + x^{-1+\sqrt{2}})} dx \\ &= (1 + \sqrt{2}) \operatorname{Subst} \left(\int \frac{1}{x(1+x)} dx, x, x^{-1+\sqrt{2}} \right) \\ &= (-1 - \sqrt{2}) \operatorname{Subst} \left(\int \frac{1}{1+x} dx, x, x^{-1+\sqrt{2}} \right) + (1 + \sqrt{2}) \operatorname{Subst} \left(\int \frac{1}{x} dx, x, x^{-1+\sqrt{2}} \right) \\ &= \log(x) - (1 + \sqrt{2}) \log(1 + x^{-1+\sqrt{2}}) \end{aligned}$$

Mathematica [A] time = 0.0208126, size = 24, normalized size = 1.

$$\log(x) - (1 + \sqrt{2}) \log(x^{\sqrt{2}-1} + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^Sqrt[2])^-1, x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^-1 + Sqrt[2]]
```

Maple [A] time = 0.02, size = 39, normalized size = 1.6

$$\sqrt{2} \ln(x) + 2 \ln(x) - \ln\left(x + e^{\sqrt{2} \ln(x)}\right) \sqrt{2} - \ln\left(x + e^{\sqrt{2} \ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(2^(1/2))), x)
```

[Out] $2^{(1/2)}*\ln(x)+2*\ln(x)-\ln(x+\exp(2^{(1/2)}*\ln(x)))*2^{(1/2)}-\ln(x+\exp(2^{(1/2)}*\ln(x)))$

Maxima [A] time = 1.61263, size = 42, normalized size = 1.75

$$\frac{\sqrt{2}\log(x)}{\sqrt{2}-1} - \frac{\log\left(x + x^{(\sqrt{2})}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")`

[Out] $\text{sqrt}(2)*\log(x)/(\text{sqrt}(2) - 1) - \log(x + x^{\text{sqrt}(2)})/(\text{sqrt}(2) - 1)$

Fricas [A] time = 0.774054, size = 78, normalized size = 3.25

$$-\left(\sqrt{2} + 1\right)\log\left(x + x^{(\sqrt{2})}\right) + \left(\sqrt{2} + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")`

[Out] $-(\text{sqrt}(2) + 1)*\log(x + x^{\text{sqrt}(2)}) + (\text{sqrt}(2) + 2)*\log(x)$

Sympy [B] time = 0.680891, size = 76, normalized size = 3.17

$$\frac{140\sqrt{2}\log(x)}{-338 + 239\sqrt{2}} - \frac{198\log(x)}{-338 + 239\sqrt{2}} + \frac{99\sqrt{2}\log\left(x + x^{\sqrt{2}}\right)}{-338 + 239\sqrt{2}} - \frac{140\log\left(x + x^{\sqrt{2}}\right)}{-338 + 239\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+x**(2**(1/2))),x)`

[Out] $140*\text{sqrt}(2)*\log(x)/(-338 + 239*\text{sqrt}(2)) - 198*\log(x)/(-338 + 239*\text{sqrt}(2)) + 99*\text{sqrt}(2)*\log(x + x**(\text{sqrt}(2)))/(-338 + 239*\text{sqrt}(2)) - 140*\log(x + x**(sq$

rt(2)))/(-338 + 239*sqrt(2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + x^{\sqrt{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")

[Out] integrate(1/(x + x^sqrt(2)), x)

3.364 $\int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j + bx^n}}\right)}{j-n} - \frac{2x^{-j/2}\sqrt{ax^j + bx^n}}{j-n}$$

[Out] $(-2*\text{Sqrt}[a*x^j + b*x^n])/((j - n)*x^{(j/2)}) + (2*\text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j + b*x^n]])/(j - n)$

Rubi [A] time = 0.114247, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {2028, 2029, 206}

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j + bx^n}}\right)}{j-n} - \frac{2x^{-j/2}\sqrt{ax^j + bx^n}}{j-n}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(-1 - j/2)}*\text{Sqrt}[a*x^j + b*x^n], x]$

[Out] $(-2*\text{Sqrt}[a*x^j + b*x^n])/((j - n)*x^{(j/2)}) + (2*\text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j + b*x^n]])/(j - n)$

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
  n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
  & (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx &= -\frac{2x^{-j/2} \sqrt{ax^j + bx^n}}{j-n} + a \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j + bx^n}} dx \\ &= -\frac{2x^{-j/2} \sqrt{ax^j + bx^n}}{j-n} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{j/2}}{\sqrt{ax^j + bx^n}}\right)}{j-n} \\ &= -\frac{2x^{-j/2} \sqrt{ax^j + bx^n}}{j-n} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax^j/2}}{\sqrt{ax^j + bx^n}}\right)}{j-n} \end{aligned}$$

Mathematica [A] time = 0.200335, size = 104, normalized size = 1.39

$$\frac{2x^{-j/2} \left(-\sqrt{a}\sqrt{bx^{\frac{j+n}{2}}} \sqrt{\frac{ax^{j-n}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{ax^{\frac{j-n}{2}}}}{\sqrt{b}}\right) + ax^j + bx^n \right)}{(j-n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n], x]

[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]) * ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]) / ((j - n)*x^(j/2)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.532, size = 0, normalized size = 0.

$$\int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2), x)

[Out] `int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} x^{-\frac{1}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} x^{-\frac{1}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

3.365 $\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{ax^{j/2}}(cx)^{-j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

[Out] $(-2\sqrt{a}x^{j/2}\sqrt{cx^{-j/2}}\operatorname{tanh}^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right) + 2\sqrt{a}x^{j/2}\operatorname{ArcTanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right))/(c(j-n)(cx)^{j/2})$

Rubi [A] time = 0.157338, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2031, 2028, 2029, 206}

$$\frac{2\sqrt{ax^{j/2}}(cx)^{-j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

[In] $\int (cx)^{-1-j/2}\sqrt{ax^j+bx^n}, x]$

[Out] $(-2\sqrt{a}x^{j/2}\sqrt{cx^{-j/2}}\operatorname{tanh}^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right) + 2\sqrt{a}x^{j/2}\operatorname{ArcTanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right))/(c(j-n)(cx)^{j/2})$

Rule 2031

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2028

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c*x)^(m+1)*(a*x^j + b*x^n)^p/(c*p*(n-j)), x] + Dist[a/c^j, Int[(c*x)^(m+j)*(a*x^j + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029


```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx &= \frac{(x^{j/2}(cx)^{-j/2}) \int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx}{c} \\ &= -\frac{2(cx)^{-j/2} \sqrt{ax^j + bx^n}}{c(j-n)} + \frac{(ax^{j/2}(cx)^{-j/2}) \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j + bx^n}} dx}{c} \\ &= -\frac{2(cx)^{-j/2} \sqrt{ax^j + bx^n}}{c(j-n)} + \frac{(2ax^{j/2}(cx)^{-j/2}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{j/2}}{\sqrt{ax^j + bx^n}}\right)}{c(j-n)} \\ &= -\frac{2(cx)^{-j/2} \sqrt{ax^j + bx^n}}{c(j-n)} + \frac{2\sqrt{ax^{j/2}}(cx)^{-j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j + bx^n}}\right)}{c(j-n)} \end{aligned}$$

Mathematica [A] time = 0.06444413, size = 109, normalized size = 1.1

$$\frac{2(cx)^{-j/2} \left(-\sqrt{a}\sqrt{bx}^{\frac{j+n}{2}} \sqrt{\frac{ax^{j-n}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{ax}^{\frac{j-n}}{\sqrt{b}}}\right) + ax^j + bx^n \right)}{c(j-n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n], x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]
)*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]])/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

Maple [F] time = 0.584, size = 0, normalized size = 0.

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)

[Out] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} (cx)^{-\frac{1}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (cx)^{-\frac{j}{2}-1} \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)

[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} (cx)^{-\frac{1}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)

$$3.366 \quad \int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}} dx$$

Optimal. Leaf size=91

$$\frac{2\sqrt{a}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}} - \frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^{(3/2)}) + (2*\text{Sqrt}[a]*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3 + b*x^n]])/(c^3*(3 - n)*\text{Sqrt}[x])$

Rubi [A] time = 0.155487, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}} - \frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^3 + b*x^n]/(c*x)^{(5/2)}, x]$

[Out] $(-2*\text{Sqrt}[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^{(3/2)}) + (2*\text{Sqrt}[a]*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3 + b*x^n]])/(c^3*(3 - n)*\text{Sqrt}[x])$

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
  n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}} dx &= -\frac{2\sqrt{ax^3 + bx^n}}{c(3-n)(cx)^{3/2}} + \frac{a \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx}{c^3} \\ &= -\frac{2\sqrt{ax^3 + bx^n}}{c(3-n)(cx)^{3/2}} + \frac{(a\sqrt{cx}) \int \frac{\sqrt{x}}{\sqrt{ax^3 + bx^n}} dx}{c^3\sqrt{x}} \\ &= -\frac{2\sqrt{ax^3 + bx^n}}{c(3-n)(cx)^{3/2}} + \frac{(2a\sqrt{cx}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^3 + bx^n}}\right)}{c^3(3-n)\sqrt{x}} \\ &= -\frac{2\sqrt{ax^3 + bx^n}}{c(3-n)(cx)^{3/2}} + \frac{2\sqrt{a}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3 + bx^n}}\right)}{c^3(3-n)\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.246594, size = 103, normalized size = 1.13

$$\frac{2x \left(-\sqrt{a}\sqrt{bx}^{\frac{n+3}{2}} \sqrt{\frac{ax^{3-n}}{b}} + 1 \sinh^{-1}\left(\frac{\sqrt{ax}^{\frac{3-n}{2}}}{\sqrt{b}}\right) + ax^3 + bx^n \right)}{(n-3)(cx)^{5/2}\sqrt{ax^3 + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2), x]

[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]])/((-3 + n)*(c*x)^(5/2)*Sqrt[a*x^3 + b*x^n])

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int \sqrt{ax^3 + bx^n} (cx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)`

[Out] `int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2), x)

[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2), x, algorithm="giac")

[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)

$$3.367 \quad \int \frac{\sqrt{ax^2+bx^n}}{c^2x^2} dx$$

Optimal. Leaf size=71

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)} - \frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*\text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^n]])/(c^2*(2 - n))$

Rubi [A] time = 0.0755826, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2028, 2008, 206}

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)} - \frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^2 + b*x^n]/(c^2*x^2), x]$

[Out] $(-2*\text{Sqrt}[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*\text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^n]])/(c^2*(2 - n))$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2028

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*p*(n-j)), x] + \text{Dist}[a/c^j, \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, j, m, n\}, x \ \&\& \ \text{IGtQ}[p + 1/2, 0] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[\text{Simplify}[m + j*p + 1], 0] \ \& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rule 2008


```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^n}}{c^2 x^2} dx &= \frac{\int \frac{\sqrt{ax^2 + bx^n}}{x^2} dx}{c^2} \\ &= -\frac{2\sqrt{ax^2 + bx^n}}{c^2(2-n)x} + \frac{a \int \frac{1}{\sqrt{ax^2 + bx^n}} dx}{c^2} \\ &= -\frac{2\sqrt{ax^2 + bx^n}}{c^2(2-n)x} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^n}}\right)}{c^2(2-n)} \\ &= -\frac{2\sqrt{ax^2 + bx^n}}{c^2(2-n)x} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^n}}\right)}{c^2(2-n)} \end{aligned}$$

Mathematica [A] time = 0.144159, size = 99, normalized size = 1.39

$$\frac{2\left(-\sqrt{a}\sqrt{bx^{\frac{n}{2}+1}}\sqrt{\frac{ax^{2-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{ax}^{1-\frac{n}{2}}}{\sqrt{b}}\right)+ax^2+bx^n\right)}{c^2(n-2)x\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2), x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \frac{1}{c^2 x^2} \sqrt{ax^2 + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)

[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)

[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")

[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)

$$3.368 \quad \int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}} dx$$

Optimal. Leaf size=87

$$\frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} - \frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

[Out] $(-2*\text{Sqrt}[a*x + b*x^n])/(c*(1 - n)*\text{Sqrt}[c*x]) + (2*\text{Sqrt}[a]*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(c*(1 - n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.137653, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} - \frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x + b*x^n]/(c*x)^{(3/2)}, x]$

[Out] $(-2*\text{Sqrt}[a*x + b*x^n])/(c*(1 - n)*\text{Sqrt}[c*x]) + (2*\text{Sqrt}[a]*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(c*(1 - n)*\text{Sqrt}[c*x])$

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
  n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}} dx &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{a \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx}{c} \\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{(a\sqrt{x}) \int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}} dx}{c\sqrt{cx}} \\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{(2a\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} \\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.250924, size = 100, normalized size = 1.15

$$\frac{x \left(-2\sqrt{a}\sqrt{bx}^{\frac{n+1}{2}} \sqrt{\frac{ax^{1-n}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{ax}^{\frac{1}{2}} \sqrt{bx}^{\frac{n}{2}}}{\sqrt{b}}\right) + 2ax + 2bx^n \right)}{(n-1)(cx)^{3/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2), x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

Maple [F] time = 0.383, size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^n} (cx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)`

[Out] `int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2), x)`

[Out] `Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2), x, algorithm="giac")`

[Out] `integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)`

$$3.369 \quad \int \frac{\sqrt{a+bx^n}}{cx} dx$$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)

Rubi [A] time = 0.0291036, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^n]/(c*x), x]

[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx^n}}{cx} dx &= \frac{\int \frac{\sqrt{a+bx^n}}{x} dx}{c} \\
 &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, x^n\right)}{cn} \\
 &= \frac{2\sqrt{a+bx^n}}{cn} + \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^n\right)}{cn} \\
 &= \frac{2\sqrt{a+bx^n}}{cn} + \frac{(2a) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx^n}\right)}{bcn} \\
 &= \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
 \end{aligned}$$

Mathematica [A] time = 0.012858, size = 46, normalized size = 0.9

$$\frac{2\sqrt{a+bx^n} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^n]/(c*x), x]

[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)

Maple [A] time = 0.002, size = 39, normalized size = 0.8

$$\frac{1}{cn} \left(2\sqrt{a + bx^n} - 2\sqrt{a} \operatorname{Artanh} \left(\frac{\sqrt{a + bx^n}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^n)^(1/2)/c/x, x)

[Out] 1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(1/2)/c/x, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0.9044, size = 223, normalized size = 4.37

$$\left[\frac{\sqrt{a} \log \left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a+2a}}{x^n} \right) + 2\sqrt{bx^n+a}}{cn}, \frac{2 \left(\sqrt{-a} \arctan \left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a} \right) + \sqrt{bx^n+a} \right)}{cn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(1/2)/c/x, x, algorithm="fricas")

```
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a))/(c*n)]
```

Sympy [A] time = 2.23138, size = 78, normalized size = 1.53

$$\frac{2\sqrt{a} \operatorname{asinh}\left(\frac{\sqrt{ax^{-\frac{n}{2}}}}{\sqrt{b}}\right)}{n} + \frac{2ax^{-\frac{n}{2}}}{\sqrt{bn}\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{2\sqrt{bx^{\frac{n}{2}}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*sqrt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1)))/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + a}}{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

$$3.370 \quad \int \frac{\sqrt{\frac{a}{x} + bx^n}}{\sqrt{cx}} dx$$

Optimal. Leaf size=84

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(n+1)} - \frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(n+1)\sqrt{cx}}$$

[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])

Rubi [A] time = 0.171002, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(n+1)} - \frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x], x]

[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
```

`eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]`

Rule 2029

`Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]`

Rule 206

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\frac{a}{x} + bx^n}}{\sqrt{cx}} dx &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(1+n)} + (ac) \int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x} + bx^n}} dx \\
 &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(1+n)} + \frac{(a\sqrt{x}) \int \frac{1}{x^{3/2}\sqrt{\frac{a}{x} + bx^n}} dx}{\sqrt{cx}} \\
 &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(1+n)} - \frac{(2a\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(1+n)\sqrt{cx}} \\
 &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{c(1+n)} - \frac{2\sqrt{a}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(1+n)\sqrt{cx}}
 \end{aligned}$$

Mathematica [A] time = 0.062124, size = 85, normalized size = 1.01

$$\frac{x\sqrt{\frac{a}{x} + bx^n} \left(2\sqrt{a + bx^{n+1}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right) \right)}{(n+1)\sqrt{cx}\sqrt{a + bx^{n+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x], x]

[Out] $(x\sqrt{a/x + b*x^n}*(2*\sqrt{a + b*x^{(1 + n)}} - 2*\sqrt{a}*\text{ArcTanh}[\sqrt{a + b*x^{(1 + n)}}/\sqrt{a}]))/((1 + n)*\sqrt{c*x}*\sqrt{a + b*x^{(1 + n)}})$

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \sqrt{\frac{a}{x} + bx^n} \frac{1}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)`

[Out] `int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + \frac{a}{x}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x} + bx^n}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)

[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + \frac{a}{x}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)

$$3.371 \quad \int \sqrt{\frac{a}{x^2} + bx^n} dx$$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2} + bx^n}}{n+2} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{n+2}$$

[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)

Rubi [A] time = 0.0834905, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2} + bx^n}}{n+2} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{n+2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^2 + b*x^n], x]

[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{a}{x^2} + bx^n} dx &= \frac{2x\sqrt{\frac{a}{x^2} + bx^n}}{2+n} + a \int \frac{1}{x^2\sqrt{\frac{a}{x^2} + bx^n}} dx \\ &= \frac{2x\sqrt{\frac{a}{x^2} + bx^n}}{2+n} - \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{2+n} \\ &= \frac{2x\sqrt{\frac{a}{x^2} + bx^n}}{2+n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{2+n} \end{aligned}$$

Mathematica [A] time = 0.0532376, size = 78, normalized size = 1.28

$$\frac{x\sqrt{\frac{a}{x^2} + bx^n} \left(2\sqrt{a + bx^{n+2}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right) \right)}{(n+2)\sqrt{a + bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2 + b*x^n], x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*(2*Sqrt[a + b*x^(2 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a
+ b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

Maple [F] time = 0.34, size = 0, normalized size = 0.

$$\int \sqrt{\frac{a}{x^2} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2*a+b*x^n)^(1/2), x)
```

[Out] `int((1/x^2*a+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + \frac{a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^n + a/x^2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a}{x^2} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x**2+b*x**n)**(1/2),x)`

[Out] `Integral(sqrt(a/x**2 + b*x**n), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + \frac{a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

$$3.372 \quad \int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx$$

Optimal. Leaf size=85

$$\frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(n+3)} - \frac{2\sqrt{ac}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3} + bx^n}}\right)}{(n+3)\sqrt{cx}}$$

[Out] $(2*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])/(c*(3 + n)) - (2*\text{Sqrt}[a]*c*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(c*(3 + n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.198622, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(n+3)} - \frac{2\sqrt{ac}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3} + bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c*x]*\text{Sqrt}[a/x^3 + b*x^n], x]$

[Out] $(2*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])/(c*(3 + n)) - (2*\text{Sqrt}[a]*c*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(c*(3 + n)*\text{Sqrt}[c*x])$

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m+1)*(a*x^j + b*x^n)^p)/(c*p*(n-j)), x] + Dist[a/c^j,
  Int[(c*x)^(m+j)*(a*x^j + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, j, m,
  n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
  & (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
  x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
```

eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx &= \frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(3+n)} + (ac^3) \int \frac{1}{(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx \\
 &= \frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(3+n)} + \frac{(ac\sqrt{x}) \int \frac{1}{x^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx}{\sqrt{cx}} \\
 &= \frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(3+n)} - \frac{(2ac\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{(3+n)\sqrt{cx}} \\
 &= \frac{2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{c(3+n)} - \frac{2\sqrt{ac}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{(3+n)\sqrt{cx}}
 \end{aligned}$$

Mathematica [A] time = 0.0676533, size = 85, normalized size = 1.

$$\frac{x\sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} \left(2\sqrt{a + bx^{n+3}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right) \right)}{(n+3)\sqrt{a + bx^{n+3}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n], x]

[Out] $(x*\sqrt{c*x}*\sqrt{a/x^3 + b*x^n}*(2*\sqrt{a + b*x^{(3 + n)}} - 2*\sqrt{a}*\operatorname{Arctanh}[\sqrt{a + b*x^{(3 + n)}}/\sqrt{a}]))/((3 + n)*\sqrt{a + b*x^{(3 + n)}})$

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)`

[Out] `int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + \frac{a}{x^3}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)

[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + \frac{a}{x^3}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)

$$3.373 \quad \int (cx)^{-1-\frac{3j}{2}} (ax^j + bx^n)^{3/2} dx$$

Optimal. Leaf size=141

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}(ax^j+bx^n)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

[Out] $(-2*a*x^j*\text{Sqrt}[a*x^j + b*x^n])/(c*(j - n)*(c*x)^{((3*j)/2)}) - (2*(a*x^j + b*x^n)^{(3/2)})/(3*c*(j - n)*(c*x)^{((3*j)/2)}) + (2*a^{(3/2)}*x^{((3*j)/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^{((3*j)/2)})$

Rubi [A] time = 0.225356, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2031, 2028, 2029, 206}

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}(ax^j+bx^n)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*x)^{-1 - (3*j)/2}*(a*x^j + b*x^n)^{(3/2)}, x]$

[Out] $(-2*a*x^j*\text{Sqrt}[a*x^j + b*x^n])/(c*(j - n)*(c*x)^{((3*j)/2)}) - (2*(a*x^j + b*x^n)^{(3/2)})/(3*c*(j - n)*(c*x)^{((3*j)/2)}) + (2*a^{(3/2)}*x^{((3*j)/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^{((3*j)/2)})$

Rule 2031

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2028

```
Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j, Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
```


& (IntegerQ[j] || GtQ[c, 0])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int (cx)^{-1-\frac{3j}{2}} (ax^j + bx^n)^{3/2} dx &= \frac{(x^{3j/2}(cx)^{-3j/2}) \int x^{-1-\frac{3j}{2}} (ax^j + bx^n)^{3/2} dx}{c} \\ &= -\frac{2(cx)^{-3j/2} (ax^j + bx^n)^{3/2}}{3c(j-n)} + \frac{(ax^{3j/2}(cx)^{-3j/2}) \int x^{-1-\frac{j}{2}} \sqrt{ax^j + bx^n} dx}{c} \\ &= -\frac{2ax^j (cx)^{-3j/2} \sqrt{ax^j + bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2} (ax^j + bx^n)^{3/2}}{3c(j-n)} + \frac{(a^2 x^{3j/2} (cx)^{-3j/2}) \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j + bx^n}} dx}{c} \\ &= -\frac{2ax^j (cx)^{-3j/2} \sqrt{ax^j + bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2} (ax^j + bx^n)^{3/2}}{3c(j-n)} + \frac{(2a^2 x^{3j/2} (cx)^{-3j/2}) \text{Subst}\left(\int \frac{1}{1-ax} dx\right)}{c(j-n)} \\ &= -\frac{2ax^j (cx)^{-3j/2} \sqrt{ax^j + bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2} (ax^j + bx^n)^{3/2}}{3c(j-n)} + \frac{2a^{3/2} x^{3j/2} (cx)^{-3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^j}}{\sqrt{ax^j + bx^n}}\right)}{c(j-n)} \end{aligned}$$

Mathematica [A] time = 0.233284, size = 131, normalized size = 0.93

$$\frac{2(cx)^{-3j/2} \left(-3a^{3/2} \sqrt{bx^{\frac{1}{2}(3j+n)}} \sqrt{\frac{ax^{j-n}}{b} + 1} \sinh^{-1}\left(\frac{\sqrt{ax^{\frac{j-n}{2}}}}{\sqrt{b}}\right) + 4a^2 x^{2j} + 5abx^{j+n} + b^2 x^{2n} \right)}{3c(j-n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2), x]

[Out] $(-2*(4*a^2*x^{(2*j)} + b^2*x^{(2*n)} + 5*a*b*x^{(j+n)} - 3*a^{(3/2)}*\text{Sqrt}[b]*x^{((3*j+n)/2)}*\text{Sqrt}[1 + (a*x^{(j-n)})/b]*\text{ArcSinh}[(\text{Sqrt}[a]*x^{((j-n)/2)})/\text{Sqrt}[b]]))/(3*c*(j-n)*(c*x)^{((3*j)/2)}*\text{Sqrt}[a*x^j + b*x^n])$

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int (cx)^{-1-\frac{3j}{2}} (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)`

[Out] `int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)

$$3.374 \quad \int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx$$

Optimal. Leaf size=128

$$\frac{2a^{3/2}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}} - \frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

[Out] $(-2*a*\text{Sqrt}[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^{(3/2)}) - (2*(a*x^3 + b*x^n)^{(3/2)})/(3*c*(3 - n)*(c*x)^{(9/2)}) + (2*a^{(3/2)}*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3 + b*x^n]])/(c^6*(3 - n)*\text{Sqrt}[x])$

Rubi [A] time = 0.208872, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}} - \frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^3 + b*x^n)^{(3/2)}/(c*x)^{(11/2)}, x]$

[Out] $(-2*a*\text{Sqrt}[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^{(3/2)}) - (2*(a*x^3 + b*x^n)^{(3/2)})/(3*c*(3 - n)*(c*x)^{(9/2)}) + (2*a^{(3/2)}*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3 + b*x^n]])/(c^6*(3 - n)*\text{Sqrt}[x])$

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
```

`eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]`

Rule 2029

`Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]`

Rule 206

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned} \int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}} dx &= -\frac{2(ax^3 + bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}} dx}{c^3} \\ &= -\frac{2a\sqrt{ax^3 + bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3 + bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a^2 \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx}{c^6} \\ &= -\frac{2a\sqrt{ax^3 + bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3 + bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{(a^2\sqrt{cx}) \int \frac{\sqrt{x}}{\sqrt{ax^3 + bx^n}} dx}{c^6\sqrt{x}} \\ &= -\frac{2a\sqrt{ax^3 + bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3 + bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{(2a^2\sqrt{cx}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^3 + bx^n}}\right)}{c^6(3-n)\sqrt{x}} \\ &= -\frac{2a\sqrt{ax^3 + bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3 + bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^3/2}}{\sqrt{ax^3 + bx^n}}\right)}{c^6(3-n)\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.293409, size = 126, normalized size = 0.98

$$\frac{2\sqrt{cx} \left(-3a^{3/2}\sqrt{bx} \frac{n+9}{2} \sqrt{\frac{ax^{3-n}}{b}} + 1 \sinh^{-1}\left(\frac{\sqrt{ax} \frac{3-n}{2}}{\sqrt{b}}\right) + 4a^2x^6 + 5abx^{n+3} + b^2x^{2n} \right)}{3c^6(n-3)x^5\sqrt{ax^3 + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x]

[Out] $(2\sqrt{c*x}*(4*a^2*x^6 + b^2*x^{(2*n)} + 5*a*b*x^{(3 + n)} - 3*a^{(3/2)}*\sqrt{b}*x^{((9 + n)/2)}*\sqrt{1 + (a*x^{(3 - n)})/b}*\text{ArcSinh}[(\sqrt{a}*x^{(3/2 - n/2)})/\sqrt{b}]))/(3*c^6*(-3 + n)*x^5*\sqrt{a*x^3 + b*x^n})$

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (ax^3 + bx^n)^{\frac{3}{2}} (cx)^{-\frac{11}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)`

[Out] `int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")`

[Out] `integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2), x, algorithm="giac")

[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)

$$3.375 \quad \int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4} dx$$

Optimal. Leaf size=104

$$\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)} - \frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}$$

[Out] $(-2*a*\text{Sqrt}[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^{(3/2)})/(3*c^4*(2 - n)*x^3) + (2*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^n]])/(c^4*(2 - n))$

Rubi [A] time = 0.133196, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2028, 2008, 206}

$$\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)} - \frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^n)^{(3/2)}/(c^4*x^4), x]$

[Out] $(-2*a*\text{Sqrt}[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^{(3/2)})/(3*c^4*(2 - n)*x^3) + (2*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2 + b*x^n]])/(c^4*(2 - n))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 2028

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*p*(n-j)), x] + \text{Dist}[a/c^j, \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}[\{a, b, c, j, m, n\}, x] \ \&\& \ \text{IGtQ}[p + 1/2, 0] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[\text{Simplify}[m + j*p + 1], 0] \ \&\& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rule 2008

`Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^n)^{3/2}}{c^4 x^4} dx &= \int \frac{(ax^2 + bx^n)^{3/2}}{c^4} dx \\
 &= -\frac{2(ax^2 + bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a \int \frac{\sqrt{ax^2 + bx^n}}{x^2} dx}{c^4} \\
 &= -\frac{2a\sqrt{ax^2 + bx^n}}{c^4(2-n)x} - \frac{2(ax^2 + bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a^2 \int \frac{1}{\sqrt{ax^2 + bx^n}} dx}{c^4} \\
 &= -\frac{2a\sqrt{ax^2 + bx^n}}{c^4(2-n)x} - \frac{2(ax^2 + bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{(2a^2) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^n}}\right)}{c^4(2-n)} \\
 &= -\frac{2a\sqrt{ax^2 + bx^n}}{c^4(2-n)x} - \frac{2(ax^2 + bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^n}}\right)}{c^4(2-n)}
 \end{aligned}$$

Mathematica [A] time = 0.157735, size = 117, normalized size = 1.12

$$\frac{2 \left(-3a^{3/2} \sqrt{bx^{\frac{n}{2}+3}} \sqrt{\frac{ax^{2-n}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{ax}^{1-\frac{n}{2}}}{\sqrt{b}} \right) + 4a^2 x^4 + 5abx^{n+2} + b^2 x^{2n} \right)}{3c^4(n-2)x^3 \sqrt{ax^2 + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x]

[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]])/(3*c^

$4*(-2 + n)*x^3*\text{Sqrt}[a*x^2 + b*x^n]$

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{1}{c^4 x^4} (ax^2 + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)`

[Out] `int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{x^4} dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")`

[Out] `integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2} dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4} dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)

[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 + b*x**n)/x**4, x))/c**4

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax^2 + bx^n)^{\frac{3}{2}}}{c^4 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")

[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)

$$3.376 \quad \int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} dx$$

Optimal. Leaf size=122

$$\frac{2a^{3/2}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} - \frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

[Out] $(-2*a*\text{Sqrt}[a*x + b*x^n])/(c^2*(1 - n)*\text{Sqrt}[c*x]) - (2*(a*x + b*x^n)^{(3/2)})/(3*c*(1 - n)*(c*x)^{(3/2)}) + (2*a^{(3/2)}*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(c^2*(1 - n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.188308, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} - \frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^n)^{(3/2)}/(c*x)^{(5/2)}, x]$

[Out] $(-2*a*\text{Sqrt}[a*x + b*x^n])/(c^2*(1 - n)*\text{Sqrt}[c*x]) - (2*(a*x + b*x^n)^{(3/2)})/(3*c*(1 - n)*(c*x)^{(3/2)}) + (2*a^{(3/2)}*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(c^2*(1 - n)*\text{Sqrt}[c*x])$

Rule 2028

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*p*(n-j)), x] + \text{Dist}[a/c^j, \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2031

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N

`eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]`

Rule 2029

`Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]`

Rule 206

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}} dx &= -\frac{2(ax + bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a \int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}} dx}{c} \\
 &= -\frac{2a\sqrt{ax + bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax + bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a^2 \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx}{c^2} \\
 &= -\frac{2a\sqrt{ax + bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax + bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{(a^2\sqrt{x}) \int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}} dx}{c^2\sqrt{cx}} \\
 &= -\frac{2a\sqrt{ax + bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax + bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{(2a^2\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} \\
 &= -\frac{2a\sqrt{ax + bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax + bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}
 \end{aligned}$$

Mathematica [A] time = 0.250859, size = 120, normalized size = 0.98

$$\frac{x \left(-6a^{3/2} \sqrt{bx^{\frac{n+3}{2}}} \sqrt{\frac{ax^{1-n}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{ax^{\frac{1-n}{2}}}}{\sqrt{b}} \right) + 8a^2 x^2 + 10abx^{n+1} + 2b^2 x^{2n} \right)}{3(n-1)(cx)^{5/2} \sqrt{ax + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2), x]

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (ax + bx^n)^{\frac{3}{2}} (cx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")

[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)

$$3.377 \quad \int \frac{(a+bx^n)^{3/2}}{cx} dx$$

Optimal. Leaf size=73

$$-\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn} + \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn}$$

[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)

Rubi [A] time = 0.0408936, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 266, 50, 63, 208}

$$-\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn} + \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^n)^(3/2)/(c*x), x]

[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^n)^{3/2}}{cx} dx &= \frac{\int \frac{(a+bx^n)^{3/2}}{x} dx}{c} \\
 &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, x^n\right)}{cn} \\
 &= \frac{2(a + bx^n)^{3/2}}{3cn} + \frac{a \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, x^n\right)}{cn} \\
 &= \frac{2a\sqrt{a + bx^n}}{cn} + \frac{2(a + bx^n)^{3/2}}{3cn} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^n\right)}{cn} \\
 &= \frac{2a\sqrt{a + bx^n}}{cn} + \frac{2(a + bx^n)^{3/2}}{3cn} + \frac{(2a^2) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^n}\right)}{bcn} \\
 &= \frac{2a\sqrt{a + bx^n}}{cn} + \frac{2(a + bx^n)^{3/2}}{3cn} - \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
 \end{aligned}$$

Mathematica [A] time = 0.0257814, size = 58, normalized size = 0.79

$$\frac{2\sqrt{a + bx^n} (4a + bx^n) - 6a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^n)^(3/2)/(c*x), x]

[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(3*c*n)

Maple [A] time = 0.002, size = 51, normalized size = 0.7

$$\frac{1}{cn} \left(\frac{2}{3} (a + bx^n)^{\frac{3}{2}} + 2a\sqrt{a + bx^n} - 2a^{3/2} \operatorname{Artanh} \left(\frac{\sqrt{a + bx^n}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^n)^(3/2)/c/x, x)

[Out] 1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(3/2)/c/x, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0.967679, size = 277, normalized size = 3.79

$$\left[\frac{3a^{\frac{3}{2}} \log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a+2a}}{x^n}\right) + 2(bx^n + 4a)\sqrt{bx^n+a}}{3cn}, \frac{2\left(3\sqrt{-aa} \arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + (bx^n + 4a)\sqrt{bx^n+a}\right)}{3cn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")

[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]

Sympy [A] time = 4.75678, size = 88, normalized size = 1.21

$$\frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{2\sqrt{abx^n}\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n)**(3/2)/c/x,x)

[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n))/c

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")

[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)

$$3.378 \quad \int \sqrt{cx} \left(\frac{a}{x} + bx^n \right)^{3/2} dx$$

Optimal. Leaf size=117

$$-\frac{2a^{3/2}c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}} + \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])

Rubi [A] time = 0.230101, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}} + \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2), x]

[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])

Rule 2028

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
  n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
  & (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  := Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
```

$x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{IntegerQ}[p + 1/2] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[\text{Simplify}[m + j*p + 1], 0]$

Rule 2029

$\text{Int}[(x_)^{(m_.)}/\text{Sqrt}[(a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /; \text{FreeQ}\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2 - 1] \&\& \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \sqrt{cx} \left(\frac{a}{x} + bx^n\right)^{3/2} dx &= \frac{2(cx)^{3/2} \left(\frac{a}{x} + bx^n\right)^{3/2}}{3c(1+n)} + (ac) \int \frac{\sqrt{\frac{a}{x} + bx^n}}{\sqrt{cx}} dx \\ &= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{1+n} + \frac{2(cx)^{3/2} \left(\frac{a}{x} + bx^n\right)^{3/2}}{3c(1+n)} + (a^2c^2) \int \frac{1}{(cx)^{3/2} \sqrt{\frac{a}{x} + bx^n}} dx \\ &= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{1+n} + \frac{2(cx)^{3/2} \left(\frac{a}{x} + bx^n\right)^{3/2}}{3c(1+n)} + \frac{(a^2c\sqrt{x}) \int \frac{1}{x^{3/2} \sqrt{\frac{a}{x} + bx^n}} dx}{\sqrt{cx}} \\ &= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{1+n} + \frac{2(cx)^{3/2} \left(\frac{a}{x} + bx^n\right)^{3/2}}{3c(1+n)} - \frac{(2a^2c\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(1+n)\sqrt{cx}} \\ &= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}{1+n} + \frac{2(cx)^{3/2} \left(\frac{a}{x} + bx^n\right)^{3/2}}{3c(1+n)} - \frac{2a^{3/2}c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{(1+n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.0714926, size = 97, normalized size = 0.83

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x} + bx^n} \left(\sqrt{a + bx^{n+1}} (4a + bx^{n+1}) - 3a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right) \right)}{3(n+1)\sqrt{a + bx^{n+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]

[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x^(1 + n)])

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int \sqrt{cx} \left(\frac{a}{x} + bx^n \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)

[Out] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x} \right)^{\frac{3}{2}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x} \right)^{\frac{3}{2}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)

$$3.379 \quad \int c^2 x^2 \left(\frac{a}{x^2} + bx^n \right)^{3/2} dx$$

Optimal. Leaf size=98

$$-\frac{2a^{3/2}c^2 \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2} + \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)} + \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)

Rubi [A] time = 0.171814, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {12, 2028, 2007, 2029, 206}

$$-\frac{2a^{3/2}c^2 \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2} + \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)} + \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

Antiderivative was successfully verified.

[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2), x]

[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2028

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a*x^j + b*x^n)^p)/(c*p*(n-j)), x] + Dist[a/c^j, Int[(c*x)^(m+j)*(a*x^j + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p+1/2, 0] && NeQ[n, j] && EqQ[Simplify[m+j*p+1], 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int c^2 x^2 \left(\frac{a}{x^2} + b x^n \right)^{3/2} dx &= c^2 \int x^2 \left(\frac{a}{x^2} + b x^n \right)^{3/2} dx \\
 &= \frac{2c^2 x^3 \left(\frac{a}{x^2} + b x^n \right)^{3/2}}{3(2+n)} + (ac^2) \int \sqrt{\frac{a}{x^2} + b x^n} dx \\
 &= \frac{2ac^2 x \sqrt{\frac{a}{x^2} + b x^n}}{2+n} + \frac{2c^2 x^3 \left(\frac{a}{x^2} + b x^n \right)^{3/2}}{3(2+n)} + (a^2 c^2) \int \frac{1}{x^2 \sqrt{\frac{a}{x^2} + b x^n}} dx \\
 &= \frac{2ac^2 x \sqrt{\frac{a}{x^2} + b x^n}}{2+n} + \frac{2c^2 x^3 \left(\frac{a}{x^2} + b x^n \right)^{3/2}}{3(2+n)} - \frac{(2a^2 c^2) \operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x \sqrt{\frac{a}{x^2} + b x^n}} \right)}{2+n} \\
 &= \frac{2ac^2 x \sqrt{\frac{a}{x^2} + b x^n}}{2+n} + \frac{2c^2 x^3 \left(\frac{a}{x^2} + b x^n \right)^{3/2}}{3(2+n)} - \frac{2a^{3/2} c^2 \tanh^{-1} \left(\frac{\sqrt{a}}{x \sqrt{\frac{a}{x^2} + b x^n}} \right)}{2+n}
 \end{aligned}$$

Mathematica [A] time = 0.0678041, size = 94, normalized size = 0.96

$$\frac{2c^2 x \sqrt{\frac{a}{x^2} + b x^n} \left(\sqrt{a + b x^{n+2}} (4a + b x^{n+2}) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b x^{n+2}}}{\sqrt{a}} \right) \right)}{3(n+2) \sqrt{a + b x^{n+2}}}$$

Antiderivative was successfully verified.

[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]

[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(2 + n)])

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int c^2 x^2 \left(\frac{a}{x^2} + b x^n \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c^2*x^2*(1/x^2*a+b*x^n)^(3/2),x)

[Out] int(c^2*x^2*(1/x^2*a+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2 \int \left(b x^n + \frac{a}{x^2} \right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")

[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \left(\int a \sqrt{\frac{a}{x^2} + bx^n} dx + \int bx^2 x^n \sqrt{\frac{a}{x^2} + bx^n} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2), x)

[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x**2 + b*x**n), x))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x^2} \right)^{\frac{3}{2}} c^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2), x, algorithm="giac")

[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)

$$3.380 \quad \int (cx)^{7/2} \left(\frac{a}{x^3} + bx^n \right)^{3/2} dx$$

Optimal. Leaf size=122

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}} + \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3} + \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

[Out] $(2*a*c^2*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^{(9/2)}*(a/x^3 + b*x^n)^{(3/2)})/(3*c*(3 + n)) - (2*a^{(3/2)}*c^4*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(3 + n)*\text{Sqrt}[c*x]$

Rubi [A] time = 0.276027, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}} + \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3} + \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*x)^{(7/2)}*(a/x^3 + b*x^n)^{(3/2)}, x]$

[Out] $(2*a*c^2*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^{(9/2)}*(a/x^3 + b*x^n)^{(3/2)})/(3*c*(3 + n)) - (2*a^{(3/2)}*c^4*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(3 + n)*\text{Sqrt}[c*x]$

Rule 2028

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a*x^j + b*x^n)^p/(c*p*(n-j)), x] + \text{Dist}[a/c^j, \text{Int}[(c*x)^{(m+j)}*(a*x^j + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2031

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j + b$

$x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{IntegerQ}[p + 1/2] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[\text{Simplify}[m + j*p + 1], 0]$

Rule 2029

$\text{Int}[(x_)^{(m_.)}/\text{Sqrt}[(a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)}], x_Symbol] \text{ :> Dist} [-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /; \text{FreeQ}\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2 - 1] \&\& \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int (cx)^{7/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2} dx &= \frac{2(cx)^{9/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}}{3c(3+n)} + (ac^3) \int \sqrt{cx} \sqrt{\frac{a}{x^3} + bx^n} dx \\ &= \frac{2ac^2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{3+n} + \frac{2(cx)^{9/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}}{3c(3+n)} + (a^2c^6) \int \frac{1}{(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx \\ &= \frac{2ac^2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{3+n} + \frac{2(cx)^{9/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}}{3c(3+n)} + \frac{(a^2c^4\sqrt{x}) \int \frac{1}{x^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx}{\sqrt{cx}} \\ &= \frac{2ac^2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{3+n} + \frac{2(cx)^{9/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}}{3c(3+n)} - \frac{(2a^2c^4\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{(3+n)\sqrt{cx}} \\ &= \frac{2ac^2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}{3+n} + \frac{2(cx)^{9/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}}{3c(3+n)} - \frac{2a^{3/2}c^4\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{(3+n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.0839508, size = 100, normalized size = 0.82

$$\frac{2c^2(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n} \left(\sqrt{a + bx^{n+3}} (4a + bx^{n+3}) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}} \right) \right)}{3(n+3)\sqrt{a + bx^{n+3}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]

[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt[a + b*x^(3 + n)])

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}} \left(\frac{a}{x^3} + bx^n \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)

[Out] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x^3} \right)^{\frac{3}{2}} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x^3} \right)^{\frac{3}{2}} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)

$$3.381 \quad \int c^5 x^5 \left(\frac{a}{x^4} + bx^n \right)^{3/2} dx$$

Optimal. Leaf size=100

$$-\frac{2a^{3/2}c^5 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4} + \frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)} + \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(4 + n)

Rubi [A] time = 0.212059, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2028, 2029, 206}

$$-\frac{2a^{3/2}c^5 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4} + \frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)} + \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Antiderivative was successfully verified.

[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2), x]

[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(4 + n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2028

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a*x^j + b*x^n)^p)/(c*p*(n-j)), x] + Dist[a/c^j, Int[(c*x)^(m+j)*(a*x^j + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && IGtQ[p+1/2, 0] && NeQ[n, j] && EqQ[Simplify[m+j*p+1], 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int c^5 x^5 \left(\frac{a}{x^4} + b x^n \right)^{3/2} dx &= c^5 \int x^5 \left(\frac{a}{x^4} + b x^n \right)^{3/2} dx \\
 &= \frac{2c^5 x^6 \left(\frac{a}{x^4} + b x^n \right)^{3/2}}{3(4+n)} + (ac^5) \int x \sqrt{\frac{a}{x^4} + b x^n} dx \\
 &= \frac{2ac^5 x^2 \sqrt{\frac{a}{x^4} + b x^n}}{4+n} + \frac{2c^5 x^6 \left(\frac{a}{x^4} + b x^n \right)^{3/2}}{3(4+n)} + (a^2 c^5) \int \frac{1}{x^3 \sqrt{\frac{a}{x^4} + b x^n}} dx \\
 &= \frac{2ac^5 x^2 \sqrt{\frac{a}{x^4} + b x^n}}{4+n} + \frac{2c^5 x^6 \left(\frac{a}{x^4} + b x^n \right)^{3/2}}{3(4+n)} - \frac{(2a^2 c^5) \text{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^2 \sqrt{\frac{a}{x^4} + b x^n}} \right)}{4+n} \\
 &= \frac{2ac^5 x^2 \sqrt{\frac{a}{x^4} + b x^n}}{4+n} + \frac{2c^5 x^6 \left(\frac{a}{x^4} + b x^n \right)^{3/2}}{3(4+n)} - \frac{2a^{3/2} c^5 \tanh^{-1} \left(\frac{\sqrt{a}}{x^2 \sqrt{\frac{a}{x^4} + b x^n}} \right)}{4+n}
 \end{aligned}$$

Mathematica [A] time = 0.0721212, size = 96, normalized size = 0.96

$$\frac{2c^5 x^2 \sqrt{\frac{a}{x^4} + b x^n} \left(\sqrt{a + b x^{n+4}} (4a + b x^{n+4}) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b x^{n+4}}}{\sqrt{a}} \right) \right)}{3(n+4)\sqrt{a + b x^{n+4}}}$$

Antiderivative was successfully verified.

[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]

[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]])/(3*(4 + n)*Sqrt[a + b*x

$^{(4 + n)]})$

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int c^5 x^5 \left(\frac{a}{x^4} + b x^n \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)`

[Out] `int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^5 \int \left(b x^n + \frac{a}{x^4} \right)^{\frac{3}{2}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")`

[Out] `c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^5 \left(\int ax \sqrt{\frac{a}{x^4} + bx^n} dx + \int bx^5 x^n \sqrt{\frac{a}{x^4} + bx^n} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)

[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/x**4 + b*x**n), x))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x^4} \right)^{\frac{3}{2}} c^5 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)

$$3.382 \quad \int \sqrt{\frac{a+bx}{x^2}} dx$$

Optimal. Leaf size=51

$$2x\sqrt{\frac{a}{x^2} + \frac{b}{x}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + \frac{b}{x}}}\right)$$

[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]

Rubi [A] time = 0.0765409, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1979, 2007, 2013, 620, 206}

$$2x\sqrt{\frac{a}{x^2} + \frac{b}{x}} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + \frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x)/x^2],x]

[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2013

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]

```
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{a+bx}{x^2}} dx &= \int \sqrt{\frac{a}{x^2} + \frac{b}{x}} dx \\
 &= 2\sqrt{\frac{a}{x^2} + \frac{b}{x}}x + a \int \frac{1}{\sqrt{\frac{a}{x^2} + \frac{b}{x}}x^2} dx \\
 &= 2\sqrt{\frac{a}{x^2} + \frac{b}{x}}x - a \operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}} dx, x, \frac{1}{x}\right) \\
 &= 2\sqrt{\frac{a}{x^2} + \frac{b}{x}}x - (2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{\frac{a}{x^2} + \frac{b}{x}}x}\right) \\
 &= 2\sqrt{\frac{a}{x^2} + \frac{b}{x}}x - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2} + \frac{b}{x}}x}\right)
 \end{aligned}$$

Mathematica [A] time = 0.0256985, size = 58, normalized size = 1.14

$$\frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx} - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/x^2], x]
```

[Out] $(2*x*\text{Sqrt}[(a + b*x)/x^2]*(\text{Sqrt}[a + b*x] - \text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*x]/\text{Sqrt}[a]]))/\text{Sqrt}[a + b*x]$

Maple [A] time = 0.007, size = 47, normalized size = 0.9

$$2 \frac{x}{\sqrt{bx+a}} \sqrt{\frac{bx+a}{x^2}} \left(-\sqrt{a} \text{Artanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right) + \sqrt{bx+a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x+a)/x^2)^(1/2),x)`

[Out] $2*((b*x+a)/x^2)^{(1/2)}*x*(-a^{(1/2)}*\text{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})+(b*x+a)^{(1/2)})/(b*x+a)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx+a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt((b*x + a)/x^2), x)`

Fricas [A] time = 0.916763, size = 231, normalized size = 4.53

$$\left[2x\sqrt{\frac{bx+a}{x^2}} + \sqrt{a} \log \left(\frac{bx - 2\sqrt{ax}\sqrt{\frac{bx+a}{x^2}} + 2a}{x} \right), 2x\sqrt{\frac{bx+a}{x^2}} + 2\sqrt{-a} \arctan \left(\frac{\sqrt{-ax}\sqrt{\frac{bx+a}{x^2}}}{a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")`

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((b*x + a)/x^2)/a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a+bx}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

Giac [A] time = 1.12506, size = 88, normalized size = 1.73

$$2 \left(\frac{a \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) + \sqrt{bx+a}}{\sqrt{-a}} \right) \operatorname{sgn}(x) - \frac{2 \left(a \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a} \right) \operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a))*sgn(x) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

$$3.383 \quad \int \sqrt{\frac{a+bx^2}{x^2}} dx$$

Optimal. Leaf size=42

$$x\sqrt{\frac{a}{x^2} + b} - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + b}}\right)$$

[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]

Rubi [A] time = 0.0229586, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1972, 242, 277, 217, 206}

$$x\sqrt{\frac{a}{x^2} + b} - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + b}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x^2)/x^2], x]

[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 242

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBi

nomialQ[a, b, c, n, m, p, x]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{a+bx^2}{x^2}} dx &= \int \sqrt{b + \frac{a}{x^2}} dx \\
 &= -\text{Subst} \left(\int \frac{\sqrt{b+ax^2}}{x^2} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{b + \frac{a}{x^2}} x - a \text{Subst} \left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{b + \frac{a}{x^2}} x - a \text{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{b + \frac{a}{x^2}} x} \right) \\
 &= \sqrt{b + \frac{a}{x^2}} x - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a}}{\sqrt{b + \frac{a}{x^2}} x} \right)
 \end{aligned}$$

Mathematica [A] time = 0.0280174, size = 62, normalized size = 1.48

$$x \sqrt{\frac{a}{x^2} + b} - \frac{\sqrt{ax} \sqrt{\frac{a}{x^2} + b} \tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a}} \right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(a + b*x^2)/x^2], x]

[Out] $\text{Sqrt}[b + a/x^2]*x - (\text{Sqrt}[a]*\text{Sqrt}[b + a/x^2]*x*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/\text{Sqrt}[a + b*x^2]$

Maple [A] time = 0.005, size = 61, normalized size = 1.5

$$x\sqrt{\frac{bx^2 + a}{x^2}} \left(\sqrt{bx^2 + a} - \sqrt{a} \ln \left(2 \frac{\sqrt{a}\sqrt{bx^2 + a} + a}{x} \right) \right) \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((b*x^2+a)/x^2)^{(1/2)}, x)$

[Out] $((b*x^2+a)/x^2)^{(1/2)}*x/(b*x^2+a)^{(1/2)}*((b*x^2+a)^{(1/2)}-a^{(1/2)}*\ln(2*(a^{(1/2)}*(b*x^2+a)^{(1/2)+a}/x))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(((b*x^2+a)/x^2)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 0.85997, size = 259, normalized size = 6.17

$$\left[x\sqrt{\frac{bx^2 + a}{x^2}} + \frac{1}{2}\sqrt{a} \log \left(-\frac{bx^2 - 2\sqrt{ax}\sqrt{\frac{bx^2+a}{x^2}} + 2a}{x^2} \right), x\sqrt{\frac{bx^2 + a}{x^2}} + \sqrt{-a} \arctan \left(\frac{\sqrt{-a}x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2 + a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(((b*x^2+a)/x^2)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out] $[x*\text{sqrt}((b*x^2 + a)/x^2) + 1/2*\text{sqrt}(a)*\log(-(b*x^2 - 2*\text{sqrt}(a)*x*\text{sqrt}((b*x^2 + a)/x^2) + 2*a)/x^2), x*\text{sqrt}((b*x^2 + a)/x^2) + \text{sqrt}(-a)*\text{arctan}(\text{sqrt}(-a)$

```
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a + bx^2}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

Giac [A] time = 1.24815, size = 92, normalized size = 2.19

$$\left(\frac{a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx^2+a} \right) \operatorname{sgn}(x) - \frac{\left(a \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a} \right) \operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a))*sgn(x) - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

$$3.384 \quad \int \sqrt{\frac{a+bx^3}{x^2}} dx$$

Optimal. Leaf size=51

$$\frac{2}{3}x\sqrt{\frac{a}{x^2} + bx} - \frac{2}{3}\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx}}\right)$$

[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x])])/3

Rubi [A] time = 0.0649497, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1979, 2007, 2029, 206}

$$\frac{2}{3}x\sqrt{\frac{a}{x^2} + bx} - \frac{2}{3}\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x^3)/x^2], x]

[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x])])/3

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{a+bx^3}{x^2}} dx &= \int \sqrt{\frac{a}{x^2} + bx} dx \\
 &= \frac{2}{3}x\sqrt{\frac{a}{x^2} + bx} + a \int \frac{1}{x^2\sqrt{\frac{a}{x^2} + bx}} dx \\
 &= \frac{2}{3}x\sqrt{\frac{a}{x^2} + bx} - \frac{1}{3}(2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x\sqrt{\frac{a}{x^2} + bx}}\right) \\
 &= \frac{2}{3}x\sqrt{\frac{a}{x^2} + bx} - \frac{2}{3}\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.0366667, size = 66, normalized size = 1.29

$$\frac{2x\sqrt{\frac{a}{x^2} + bx}\left(\sqrt{a+bx^3} - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2], x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

Maple [A] time = 0.007, size = 55, normalized size = 1.1

$$\frac{2x}{3} \sqrt{\frac{bx^3 + a}{x^2}} \left(-\sqrt{a} \operatorname{Arctanh} \left(\sqrt{bx^3 + a} \frac{1}{\sqrt{a}} \right) + \sqrt{bx^3 + a} \right) \frac{1}{\sqrt{bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)/x^2)^(1/2), x)

[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+ (b*x^3+a)^(1/2))/(b*x^3+a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^3 + a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)/x^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt((b*x^3 + a)/x^2), x)

Fricas [A] time = 0.846674, size = 261, normalized size = 5.12

$$\left[\frac{2}{3} x \sqrt{\frac{bx^3 + a}{x^2}} + \frac{1}{3} \sqrt{a} \log \left(\frac{bx^3 - 2\sqrt{ax} \sqrt{\frac{bx^3 + a}{x^2}} + 2a}{x^3} \right), \frac{2}{3} x \sqrt{\frac{bx^3 + a}{x^2}} + \frac{2}{3} \sqrt{-a} \arctan \left(\frac{\sqrt{-ax} \sqrt{\frac{bx^3 + a}{x^2}}}{a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)/x^2)^(1/2), x, algorithm="fricas")

[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)/x**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.13836, size = 93, normalized size = 1.82

$$\frac{2}{3} \left(\frac{a \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx^3+a} \right) \operatorname{sgn}(x) - \frac{2 \left(a \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a} \right) \operatorname{sgn}(x)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")

[Out] 2/3*(a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^3 + a))*sgn(x)
- 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)

$$3.385 \quad \int \sqrt{\frac{a+bx^n}{x^2}} dx$$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2} + bx^{n-2}}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^{n-2}}}\right)}{n}$$

[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)]/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^(-2 + n)])])/n

Rubi [A] time = 0.0799542, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1979, 2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2} + bx^{n-2}}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^{n-2}}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x^n)/x^2], x]

[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)]/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^(-2 + n)])])/n

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029


```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{a+bx^n}{x^2}} dx &= \int \sqrt{\frac{a}{x^2} + bx^{-2+n}} dx \\
 &= \frac{2x\sqrt{\frac{a}{x^2} + bx^{-2+n}}}{n} + a \int \frac{1}{x^2\sqrt{\frac{a}{x^2} + bx^{-2+n}}} dx \\
 &= \frac{2x\sqrt{\frac{a}{x^2} + bx^{-2+n}}}{n} - \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x\sqrt{\frac{a}{x^2} + bx^{-2+n}}}\right)}{n} \\
 &= \frac{2x\sqrt{\frac{a}{x^2} + bx^{-2+n}}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^{-2+n}}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.0209261, size = 70, normalized size = 1.15

$$\frac{x\sqrt{\frac{a+bx^n}{x^2}} \left(2\sqrt{a+bx^n} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2], x]
```

```
[Out] (x*Sqrt[(a + b*x^n)/x^2]*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

Maple [A] time = 1.267, size = 74, normalized size = 1.2

$$2 \frac{x}{n} \sqrt{\frac{a + be^{n \ln(x)}}{x^2}} - 2 \frac{x\sqrt{a}}{n\sqrt{a + be^{n \ln(x)}}} \operatorname{Arctanh}\left(\frac{\sqrt{a + be^{n \ln(x)}}}{\sqrt{a}}\right) \sqrt{\frac{a + be^{n \ln(x)}}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a+b*x^n)/x^2)^(1/2),x)`

[Out] `2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x)))^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^n + a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt((b*x^n + a)/x^2), x)`

Fricas [A] time = 0.775976, size = 255, normalized size = 4.18

$$\left[\frac{2x\sqrt{\frac{bx^n+a}{x^2}} + \sqrt{a} \log\left(\frac{bx^n - 2\sqrt{ax}\sqrt{\frac{bx^n+a}{x^2}} + 2a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}} + \sqrt{-a} \arctan\left(\frac{\sqrt{-ax}\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")`

[Out] `[(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n + a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt`

```
(-a)*x*sqrt((b*x^n + a)/x^2)/a)/n]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^n + a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

$$3.386 \quad \int \sqrt{\frac{-a+bx}{x^2}} dx$$

Optimal. Leaf size=53

$$2x\sqrt{\frac{b}{x} - \frac{a}{x^2}} + 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x} - \frac{a}{x^2}}}\right)$$

[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x)]

Rubi [A] time = 0.0789457, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1979, 2007, 2013, 620, 203}

$$2x\sqrt{\frac{b}{x} - \frac{a}{x^2}} + 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x} - \frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-a + b*x)/x^2], x]

[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x)]

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2013

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-a+bx}{x^2}} dx &= \int \sqrt{-\frac{a}{x^2} + \frac{b}{x}} dx \\
&= 2\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x - a \int \frac{1}{\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x^2} dx \\
&= 2\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x + a \operatorname{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}} dx, x, \frac{1}{x}\right) \\
&= 2\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x + (2a) \operatorname{Subst}\left(\int \frac{1}{1+ax^2} dx, x, \frac{1}{\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x}\right) \\
&= 2\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x + 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2} + \frac{b}{x}}x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0295023, size = 66, normalized size = 1.25

$$\frac{2x\sqrt{\frac{bx-a}{x^2}}\left(\sqrt{bx-a} - \sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx-a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-a + b*x)/x^2], x]

[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]))/Sqrt[-a + b*x]

Maple [A] time = 0.01, size = 56, normalized size = 1.1

$$-2 \frac{x}{\sqrt{bx-a}} \sqrt{\frac{bx-a}{x^2}} \left(\sqrt{a} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \sqrt{bx-a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x-a)/x^2)^(1/2), x)

[Out] -2*((b*x-a)/x^2)^(1/2)*x*(a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))-(b*x-a)^(1/2))/(b*x-a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx-a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt((b*x - a)/x^2), x)

Fricas [A] time = 0.857931, size = 228, normalized size = 4.3

$$\left[2x\sqrt{\frac{bx-a}{x^2}} + \sqrt{-a} \log\left(\frac{bx - 2\sqrt{-a}x\sqrt{\frac{bx-a}{x^2}} - 2a}{x}\right), 2x\sqrt{\frac{bx-a}{x^2}} - 2\sqrt{a} \arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")

[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)/x^2)/sqrt(a))]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-a + bx}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x**2)**(1/2),x)

[Out] Integral(sqrt((-a + b*x)/x**2), x)

Giac [A] time = 1.28611, size = 82, normalized size = 1.55

$$-2 \left(\sqrt{a} \arctan \left(\frac{\sqrt{bx-a}}{\sqrt{a}} \right) - \sqrt{bx-a} \right) \operatorname{sgn}(x) + 2 \left(\sqrt{a} \arctan \left(\frac{\sqrt{-a}}{\sqrt{a}} \right) - \sqrt{-a} \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")

[Out] -2*(sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x - a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)

$$3.387 \quad \int \sqrt{\frac{-a+bx^2}{x^2}} dx$$

Optimal. Leaf size=43

$$x\sqrt{b-\frac{a}{x^2}} + \sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]

Rubi [A] time = 0.023209, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1972, 242, 277, 217, 203}

$$x\sqrt{b-\frac{a}{x^2}} + \sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-a + b*x^2)/x^2], x]

[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 242

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBi

nomialQ[a, b, c, n, m, p, x]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{-a + bx^2}{x^2}} dx &= \int \sqrt{b - \frac{a}{x^2}} dx \\
 &= -\text{Subst}\left(\int \frac{\sqrt{b - ax^2}}{x^2} dx, x, \frac{1}{x}\right) \\
 &= \sqrt{b - \frac{a}{x^2}}x + a \text{Subst}\left(\int \frac{1}{\sqrt{b - ax^2}} dx, x, \frac{1}{x}\right) \\
 &= \sqrt{b - \frac{a}{x^2}}x + a \text{Subst}\left(\int \frac{1}{1 + ax^2} dx, x, \frac{1}{\sqrt{b - \frac{a}{x^2}}x}\right) \\
 &= \sqrt{b - \frac{a}{x^2}}x + \sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b - \frac{a}{x^2}}x}\right)
 \end{aligned}$$

Mathematica [A] time = 0.0314661, size = 68, normalized size = 1.58

$$x\sqrt{b - \frac{a}{x^2}} - \frac{\sqrt{ax}\sqrt{b - \frac{a}{x^2}} \tan^{-1}\left(\frac{\sqrt{bx^2 - a}}{\sqrt{a}}\right)}{\sqrt{bx^2 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-a + b*x^2)/x^2], x]

[Out] $\text{Sqrt}[b - a/x^2]*x - (\text{Sqrt}[a]*\text{Sqrt}[b - a/x^2]*x*\text{ArcTan}[\text{Sqrt}[-a + b*x^2]/\text{Sqrt}[a]])/\text{Sqrt}[-a + b*x^2]$

Maple [B] time = 0.008, size = 81, normalized size = 1.9

$$x\sqrt{\frac{bx^2 - a}{x^2}} \left(a \ln \left(2 \frac{\sqrt{-a}\sqrt{bx^2 - a} - a}{x} \right) + \sqrt{-a}\sqrt{bx^2 - a} \right) \frac{1}{\sqrt{-a}} \frac{1}{\sqrt{bx^2 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((b*x^2-a)/x^2)^(1/2), x)$

[Out] $((b*x^2-a)/x^2)^(1/2)*x*(a*\ln(2*((-a)^(1/2)*(b*x^2-a)^(1/2)-a)/x)+(-a)^(1/2))*((b*x^2-a)^(1/2))/((-a)^(1/2)/(b*x^2-a)^(1/2))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(((b*x^2-a)/x^2)^(1/2), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 0.75714, size = 259, normalized size = 6.02

$$\left[x\sqrt{\frac{bx^2 - a}{x^2}} + \frac{1}{2}\sqrt{-a}\log\left(-\frac{bx^2 - 2\sqrt{-a}x\sqrt{\frac{bx^2 - a}{x^2}} - 2a}{x^2}\right), x\sqrt{\frac{bx^2 - a}{x^2}} + \sqrt{a}\arctan\left(\frac{\sqrt{ax}\sqrt{\frac{bx^2 - a}{x^2}}}{bx^2 - a}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(((b*x^2-a)/x^2)^(1/2), x, \text{algorithm}="fricas")$

[Out] $[x*\text{sqrt}((b*x^2 - a)/x^2) + 1/2*\text{sqrt}(-a)*\log(-(b*x^2 - 2*\text{sqrt}(-a)*x*\text{sqrt}((b*x^2 - a)/x^2) - 2*a)/x^2), x*\text{sqrt}((b*x^2 - a)/x^2) + \text{sqrt}(a)*\text{arctan}(\text{sqrt}(a)$

```
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-a + bx^2}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

Giac [A] time = 1.22661, size = 86, normalized size = 2.

$$-\left(\sqrt{a} \arctan\left(\frac{\sqrt{bx^2 - a}}{\sqrt{a}}\right) - \sqrt{bx^2 - a}\right) \operatorname{sgn}(x) + \left(\sqrt{a} \arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a)) - sqrt(b*x^2 - a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)
```

$$3.388 \quad \int \sqrt{\frac{-a+bx^3}{x^2}} dx$$

Optimal. Leaf size=53

$$\frac{2}{3}x\sqrt{bx - \frac{a}{x^2}} + \frac{2}{3}\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx - \frac{a}{x^2}}}\right)$$

[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) + b*x]))/3

Rubi [A] time = 0.0679645, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1979, 2007, 2029, 203}

$$\frac{2}{3}x\sqrt{bx - \frac{a}{x^2}} + \frac{2}{3}\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx - \frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-a + b*x^3)/x^2], x]

[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) + b*x]))/3

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-a + bx^3}{x^2}} dx &= \int \sqrt{-\frac{a}{x^2} + bx} dx \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2} + bx} - a \int \frac{1}{x^2\sqrt{-\frac{a}{x^2} + bx}} dx \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2} + bx} + \frac{1}{3}(2a) \operatorname{Subst}\left(\int \frac{1}{1 + ax^2} dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2} + bx}}\right) \\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2} + bx} + \frac{2}{3}\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2} + bx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0403021, size = 73, normalized size = 1.38

$$\frac{2x\sqrt{bx - \frac{a}{x^2}}\left(\sqrt{bx^3 - a} - \sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx^3 - a}}{\sqrt{a}}\right)\right)}{3\sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2], x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

Maple [A] time = 0.019, size = 73, normalized size = 1.4

$$\frac{2x}{3} \sqrt{\frac{bx^3 - a}{x^2}} \left(\sqrt{bx^3 - a} \sqrt{-a} + a \operatorname{Artanh} \left(\sqrt{bx^3 - a} \frac{1}{\sqrt{-a}} \right) \right) \frac{1}{\sqrt{bx^3 - a}} \frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3-a)/x^2)^(1/2),x)

[Out] 2/3*((b*x^3-a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-a)^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^3 - a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((b*x^3 - a)/x^2), x)

Fricas [A] time = 0.821793, size = 258, normalized size = 4.87

$$\left[\frac{2}{3} x \sqrt{\frac{bx^3 - a}{x^2}} + \frac{1}{3} \sqrt{-a} \log \left(\frac{bx^3 - 2 \sqrt{-a} x \sqrt{\frac{bx^3 - a}{x^2}} - 2a}{x^3} \right), \frac{2}{3} x \sqrt{\frac{bx^3 - a}{x^2}} - \frac{2}{3} \sqrt{a} \arctan \left(\frac{x \sqrt{\frac{bx^3 - a}{x^2}}}{\sqrt{a}} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")

[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt((b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3-a)/x**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.30164, size = 88, normalized size = 1.66

$$-\frac{2}{3} \left(\sqrt{a} \arctan \left(\frac{\sqrt{bx^3 - a}}{\sqrt{a}} \right) - \sqrt{bx^3 - a} \right) \operatorname{sgn}(x) + \frac{2}{3} \left(\sqrt{a} \arctan \left(\frac{\sqrt{-a}}{\sqrt{a}} \right) - \sqrt{-a} \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")

[Out] -2/3*(sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a)) - sqrt(b*x^3 - a))*sgn(x) + 2/3*(sqrt(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)

$$3.389 \quad \int \sqrt{\frac{-a+bx^n}{x^2}} dx$$

Optimal. Leaf size=63

$$\frac{2x\sqrt{bx^{n-2} - \frac{a}{x^2}}}{n} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2} - \frac{a}{x^2}}}\right)}{n}$$

[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)]/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) + b*x^(-2 + n)])])/n

Rubi [A] time = 0.0804348, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1979, 2007, 2029, 203}

$$\frac{2x\sqrt{bx^{n-2} - \frac{a}{x^2}}}{n} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2} - \frac{a}{x^2}}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-a + b*x^n)/x^2], x]

[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)]/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) + b*x^(-2 + n)])])/n

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2007

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[j*p + 1], 0]

Rule 2029


```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\frac{-a + bx^n}{x^2}} dx &= \int \sqrt{-\frac{a}{x^2} + bx^{-2+n}} dx \\
 &= \frac{2x\sqrt{-\frac{a}{x^2} + bx^{-2+n}}}{n} - a \int \frac{1}{x^2\sqrt{-\frac{a}{x^2} + bx^{-2+n}}} dx \\
 &= \frac{2x\sqrt{-\frac{a}{x^2} + bx^{-2+n}}}{n} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1+ax^2} dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2} + bx^{-2+n}}}\right)}{n} \\
 &= \frac{2x\sqrt{-\frac{a}{x^2} + bx^{-2+n}}}{n} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2} + bx^{-2+n}}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.0293767, size = 78, normalized size = 1.24

$$\frac{x\sqrt{\frac{bx^n - a}{x^2}} \left(2\sqrt{bx^n - a} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx^n - a}}{\sqrt{a}}\right)\right)}{n\sqrt{bx^n - a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2], x]
```

```
[Out] (x*Sqrt[(-a + b*x^n)/x^2]*(2*Sqrt[-a + b*x^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a +
b*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

Maple [A] time = 0.721, size = 105, normalized size = 1.7

$$-2 \frac{(a - be^{n \ln(x)})x}{n(be^{n \ln(x)} - a)} \sqrt{\frac{be^{n \ln(x)} - a}{x^2}} - 2 \frac{x\sqrt{a}}{n\sqrt{be^{n \ln(x)} - a}} \arctan\left(\frac{\sqrt{be^{n \ln(x)} - a}}{\sqrt{a}}\right) \sqrt{\frac{be^{n \ln(x)} - a}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^n-a)/x^2)^(1/2),x)

[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x
-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x
^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^n - a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((b*x^n - a)/x^2), x)

Fricas [A] time = 0.928918, size = 252, normalized size = 4.

$$\left[\frac{2x\sqrt{\frac{bx^n - a}{x^2}} + \sqrt{-a} \log\left(\frac{bx^n - 2\sqrt{-a}x\sqrt{\frac{bx^n - a}{x^2}} - 2a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n - a}{x^2}} - \sqrt{a} \arctan\left(\frac{x\sqrt{\frac{bx^n - a}{x^2}}}{\sqrt{a}}\right)\right)}{n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")

[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*s

```
qrt((b*x^n - a)/x^2)/sqrt(a))/n]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^n - a}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

$$3.390 \quad \int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}} dx$$

Optimal. Leaf size=62

$$\frac{2x^{-j/2}(cx)^{j/2} \tanh^{-1}\left(\frac{\sqrt{ax}^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{ac}(j-n)}$$

[Out] $(2*(c*x)^{(j/2)}*ArcTanh[(Sqrt[a]*x^{(j/2)})/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(j - n)*x^{(j/2)})$

Rubi [A] time = 0.107995, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2031, 2029, 206}

$$\frac{2x^{-j/2}(cx)^{j/2} \tanh^{-1}\left(\frac{\sqrt{ax}^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{ac}(j-n)}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n], x]

[Out] $(2*(c*x)^{(j/2)}*ArcTanh[(Sqrt[a]*x^{(j/2)})/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(j - n)*x^{(j/2)})$

Rule 2031

```
Int[((c_)*(x_)^(m_))*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}} dx &= \frac{(x^{-j/2}(cx)^{j/2}) \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}} dx}{c} \\ &= \frac{(2x^{-j/2}(cx)^{j/2}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} \\ &= \frac{2x^{-j/2}(cx)^{j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{ac}(j-n)} \end{aligned}$$

Mathematica [A] time = 0.140186, size = 98, normalized size = 1.58

$$\frac{2\sqrt{b}(cx)^{j/2}x^{\frac{n-j}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{ax^{\frac{j-n}{2}}}}{\sqrt{b}}\right)}{\sqrt{ac}(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n], x]
```

```
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int (cx)^{-1+\frac{j}{2}} \frac{1}{\sqrt{ax^j+bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2), x)
```

[Out] `int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)`

[Out] `Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

$$3.391 \quad \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}} dx$$

Optimal. Leaf size=53

$$\frac{2\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 - n)*Sqrt[x])

Rubi [A] time = 0.0987987, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n], x]

[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 - n)*Sqrt[x])

Rule 2031

```
Int[((c_)*(x_)^(m_.))*((a_)*(x_)^(j_.) + (b_)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_)*(x_)^(j_.) + (b_)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx &= \frac{\sqrt{cx} \int \frac{\sqrt{x}}{\sqrt{ax^3 + bx^n}} dx}{\sqrt{x}} \\ &= \frac{(2\sqrt{cx}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^3 + bx^n}}\right)}{(3-n)\sqrt{x}} \\ &= \frac{2\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^{3/2}}}{\sqrt{ax^3 + bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.153705, size = 89, normalized size = 1.68

$$-\frac{2\sqrt{b}\sqrt{cx}x^{\frac{n-1}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{ax^{\frac{3-n}{2}}}}{\sqrt{b}}\right)}{\sqrt{a}(n-3)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n], x]
```

```
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \sqrt{cx} \frac{1}{\sqrt{ax^3 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2), x)
```

[Out] `int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)`

[Out] `Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

$$3.392 \quad \int \frac{1}{\sqrt{ax^2+bx^n}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))

Rubi [A] time = 0.015101, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^2 + b*x^n], x]

[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{ax^2 + bx^n}} dx = \frac{2 \operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2+bx^n}} \right)}{2-n}$$

$$= \frac{2 \tanh^{-1} \left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}} \right)}{\sqrt{a}(2-n)}$$

Mathematica [B] time = 0.0681361, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{bx^{n/2}}\sqrt{\frac{ax^{2-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{ax}^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]

[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^2+b*x^n)^(1/2),x)

[Out] int(1/(a*x^2+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)

[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)

$$3.393 \quad \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx$$

Optimal. Leaf size=51

$$\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*Sqrt[c*x])

Rubi [A] time = 0.0852557, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]

[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*Sqrt[c*x])

Rule 2031

Int[((c_)*(x_)^(m_))*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx &= \frac{\sqrt{x} \int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}} dx}{\sqrt{cx}} \\ &= \frac{(2\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}} \\ &= \frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.119446, size = 87, normalized size = 1.71

$$\frac{2\sqrt{b}x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{ax^{\frac{1}{2}-\frac{n}{2}}}}{\sqrt{b}}\right)}{\sqrt{a}(n-1)\sqrt{cx}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[b]*x^((1+n)/2)*Sqrt[1+(a*x^(1-n))/b]*ArcSinh[(Sqrt[a]*x^(1/2-n/2))/Sqrt[b]])/(Sqrt[a]*(-1+n)*Sqrt[c*x]*Sqrt[a*x+b*x^n])
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}} \frac{1}{\sqrt{ax+bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```


[Out] `int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^n}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)`

[Out] `Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^n} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

$$3.394 \quad \int \frac{1}{cx\sqrt{a+bx^n}} dx$$

Optimal. Leaf size=31

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{acn}}$$

[Out] $(-2*\text{ArcTanh}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*c*n)$

Rubi [A] time = 0.0204733, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 266, 63, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{acn}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(c*x*\text{Sqrt}[a + b*x^n]), x]$

[Out] $(-2*\text{ArcTanh}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*c*n)$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]]$

Rule 266

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 63

$\text{Int}[(a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1) - 1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Den}]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{cx\sqrt{a+bx^n}} dx &= \frac{\int \frac{1}{x\sqrt{a+bx^n}} dx}{c} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^n\right)}{cn} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx^n}\right)}{bcn} \\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{acn}} \end{aligned}$$

Mathematica [A] time = 0.0063857, size = 31, normalized size = 1.

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{acn}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)

Maple [A] time = 0.004, size = 26, normalized size = 0.8

$$-2 \frac{1}{cn\sqrt{a}} \text{Artanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/c/x/(a+b*x^n)^(1/2),x)`

[Out] `-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0.790381, size = 169, normalized size = 5.45

$$\left[\frac{\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a} + 2a}{x^n}\right)}{\sqrt{acn}}, \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] `[log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]`

Sympy [A] time = 1.89404, size = 27, normalized size = 0.87

$$-\frac{2 \operatorname{asinh}\left(\frac{\sqrt{ax}^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{acn}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)

[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*c*n)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + acx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)

$$3.395 \quad \int \frac{1}{(cx)^{3/2} \sqrt{\frac{a}{x} + bx^n}} dx$$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{\sqrt{ac}(n+1)\sqrt{cx}}$$

[Out] $(-2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(\text{Sqrt}[x]*\text{Sqrt}[a/x + b*x^n])]) / (\text{Sqrt}[a]*c*(1 + n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.110832, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{\sqrt{ac}(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((c*x)^{(3/2)}*\text{Sqrt}[a/x + b*x^n]), x]$

[Out] $(-2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(\text{Sqrt}[x]*\text{Sqrt}[a/x + b*x^n])]) / (\text{Sqrt}[a]*c*(1 + n)*\text{Sqrt}[c*x])$

Rule 2031

$\text{Int}[(c_*)(x_)^{(m_*)}*((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:\> \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, j, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[p + 1/2] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[\text{Simplify}[m + j*p + 1], 0]$

Rule 2029

$\text{Int}[(x_)^{(m_*)}/\text{Sqrt}[(a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)}], x_Symbol] :\> \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /;$ $\text{FreeQ}\{a, b, j, n\}, x] \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(cx)^{3/2} \sqrt{\frac{a}{x} + bx^n}} dx &= \frac{\sqrt{x} \int \frac{1}{x^{3/2} \sqrt{\frac{a}{x} + bx^n}} dx}{c\sqrt{cx}} \\ &= -\frac{(2\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{c(1+n)\sqrt{cx}} \\ &= -\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{\sqrt{ac}(1+n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.0480262, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a + bx^{n+1}} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{a}(n+1)(cx)^{3/2} \sqrt{\frac{a}{x} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]

[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{2}} \frac{1}{\sqrt{\frac{a}{x} + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)`

[Out] `int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx)^{\frac{3}{2}} \sqrt{\frac{a}{x} + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)`

[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)

$$3.396 \quad \int \frac{1}{c^2 x^2 \sqrt{\frac{a}{x^2} + b x^n}} dx$$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x \sqrt{\frac{a}{x^2} + b x^n}}\right)}{\sqrt{a} c^2 (n+2)}$$

[Out] $(-2 * \text{ArcTanh}[\text{Sqrt}[a] / (x * \text{Sqrt}[a/x^2 + b * x^n])]) / (\text{Sqrt}[a] * c^2 * (2 + n))$

Rubi [A] time = 0.0737836, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {12, 2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x \sqrt{\frac{a}{x^2} + b x^n}}\right)}{\sqrt{a} c^2 (n+2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(c^2 * x^2 * \text{Sqrt}[a/x^2 + b * x^n]), x]$

[Out] $(-2 * \text{ArcTanh}[\text{Sqrt}[a] / (x * \text{Sqrt}[a/x^2 + b * x^n])]) / (\text{Sqrt}[a] * c^2 * (2 + n))$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2029

$\text{Int}[(x_)^(m_)/\text{Sqrt}[(a_*)(x_)^(j_) + (b_*)(x_)^(n_)], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a * x^2), x], x, x^{(j/2)}/\text{Sqrt}[a * x^j + b * x^n]], x] /; \text{FreeQ}\{a, b, j, n\}, x \ \&\& \ \text{EqQ}[m, j/2 - 1] \ \&\& \ \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{c^2 x^2 \sqrt{\frac{a}{x^2} + b x^n}} dx &= \frac{\int \frac{1}{x^2 \sqrt{\frac{a}{x^2} + b x^n}} dx}{c^2} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{1-a x^2} dx, x, \frac{1}{x \sqrt{\frac{a}{x^2} + b x^n}}\right)}{c^2(2+n)} \\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x \sqrt{\frac{a}{x^2} + b x^n}}\right)}{\sqrt{a} c^2(2+n)} \end{aligned}$$

Mathematica [A] time = 0.0466403, size = 66, normalized size = 1.65

$$-\frac{2\sqrt{a+bx^{n+2}} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]

[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int \frac{1}{c^2 x^2} \frac{1}{\sqrt{\frac{a}{x^2} + b x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/c^2/x^2/(1/x^2*a+b*x^n)^(1/2),x)

[Out] `int(1/c^2/x^2/(1/x^2*a+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{bx^n + \frac{a}{x^2}} x^2} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2 \sqrt{\frac{a}{x^2} + bx^n}} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)`

[Out] `Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^2}c^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)

$$3.397 \quad \int \frac{1}{(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx$$

Optimal. Leaf size=54

$$\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{\sqrt{ac^2(n+3)}\sqrt{cx}}$$

[Out] $(-2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])]) / (\text{Sqrt}[a]*c^2*(3 + n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.135793, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{\sqrt{ac^2(n+3)}\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((c*x)^{(5/2)}*\text{Sqrt}[a/x^3 + b*x^n]), x]$

[Out] $(-2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])]) / (\text{Sqrt}[a]*c^2*(3 + n)*\text{Sqrt}[c*x])$

Rule 2031

$\text{Int}[(c_*)(x_)^{(m_*)}((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $\rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]

Rule 2029

$\text{Int}[(x_)^{(m_*)}/\text{Sqrt}[(a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)}], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /;$ FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx &= \frac{\sqrt{x} \int \frac{1}{x^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx}{c^2 \sqrt{cx}} \\ &= -\frac{(2\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{c^2(3+n)\sqrt{cx}} \\ &= -\frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.0500456, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+3}} \tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{\sqrt{a}(n+3)(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]), x]

[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}} \frac{1}{\sqrt{\frac{a}{x^3} + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)`

[Out] `int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)

$$3.398 \quad \int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}} dx$$

Optimal. Leaf size=107

$$\frac{2x^{-3j/2}(cx)^{3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)} - \frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

[Out] $(-2*(c*x)^{((3*j)/2)})/(a*c*(j-n)*x^j*\text{Sqrt}[a*x^j+b*x^n]) + (2*(c*x)^{((3*j)/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j+b*x^n]])/(a^{(3/2)}*c*(j-n)*x^{(3*j)/2})$

Rubi [A] time = 0.187196, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2031, 2030, 2029, 206}

$$\frac{2x^{-3j/2}(cx)^{3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)} - \frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*x)^{-1+(3*j)/2}/(a*x^j+b*x^n)^{3/2}, x]$

[Out] $(-2*(c*x)^{((3*j)/2)})/(a*c*(j-n)*x^j*\text{Sqrt}[a*x^j+b*x^n]) + (2*(c*x)^{((3*j)/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(j/2)})/\text{Sqrt}[a*x^j+b*x^n]])/(a^{(3/2)}*c*(j-n)*x^{(3*j)/2})$

Rule 2031

$\text{Int}[(c_*)(x_)^{(m_*)}*((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:= \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N eq[n, j] && EqQ[Simplify[m + j*p + 1], 0]

Rule 2030

$\text{Int}[(c_*)(x_)^{(m_*)}*((a_*)(x_)^{(j_*)} + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol]$
 $:= -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[($

$c*x^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x, x] /; \text{FreeQ}[\{a, b, c, j, m, n\}, x]$
 $] \&\& \text{ILtQ}[p + 1/2, 0] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[\text{Simplify}[m + j*p + 1], 0] \&\& (\text{Int}$
 $\text{egerQ}[j] \parallel \text{GtQ}[c, 0])$

Rule 2029

$\text{Int}[(x_)^{(m_)}/\text{Sqrt}[(a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)}], x_Symbol] \text{:> Dist}$
 $[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]],$
 $x] /; \text{FreeQ}[\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2 - 1] \&\& \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2]^{(-1)}, x_Symbol] \text{:> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/$
 $\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{Gt}$
 $\text{Q}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j + bx^n)^{3/2}} dx = \frac{(x^{-3j/2}(cx)^{3j/2}) \int \frac{x^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}} dx}{c}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{(x^{-3j/2}(cx)^{3j/2}) \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}} dx}{ac}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{(2x^{-3j/2}(cx)^{3j/2}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{2x^{-3j/2}(cx)^{3j/2} \tanh^{-1}\left(\frac{\sqrt{ax^{j/2}}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

Mathematica [A] time = 0.17613, size = 117, normalized size = 1.09

$$\frac{2x^{-3j/2}(cx)^{3j/2} \left(\sqrt{ax^{j/2}} - \sqrt{bx^{n/2}} \sqrt{\frac{ax^{j-n}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{ax^{j/2}}}{\sqrt{b}} \right) \right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2), x]

[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n))/b])*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]])/(a^(3/2)*c*(j - n)*x^((3*j)/2)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.37, size = 0, normalized size = 0.

$$\int (cx)^{-1+\frac{3j}{2}} (ax^j + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2), x)

[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)

$$3.399 \quad \int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}} dx$$

Optimal. Leaf size=94

$$\frac{2c^3\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

[Out] $(-2*c^2*(c*x)^{(3/2)})/(a*(3-n)*\text{Sqrt}[a*x^3+b*x^n]) + (2*c^3*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3+b*x^n]])/(a^{(3/2)}*(3-n)*\text{Sqrt}[x])$

Rubi [A] time = 0.159572, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c^3\sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*x)^{(7/2)}/(a*x^3+b*x^n)^{(3/2)},x]$

[Out] $(-2*c^2*(c*x)^{(3/2)})/(a*(3-n)*\text{Sqrt}[a*x^3+b*x^n]) + (2*c^3*\text{Sqrt}[c*x]*\text{ArcTanh}[(\text{Sqrt}[a]*x^{(3/2)})/\text{Sqrt}[a*x^3+b*x^n]])/(a^{(3/2)}*(3-n)*\text{Sqrt}[x])$

Rule 2030

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j+b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j+b*x^n)^{(p+1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c, j, m, n\}, x] \ \&\& \ \text{ILtQ}[p+1/2, 0] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[\text{Simplify}[m+j*p+1], 0] \ \&\& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rule 2031

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $]:> \text{Dist}[(c*\text{IntPart}[m]*(c*x)^{\text{FracPart}[m]})/x^{\text{FracPart}[m]}, \text{Int}[x^m*(a*x^j+b*x^n)^p, x], x] /;$
 $\text{FreeQ}\{a, b, c, j, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[p+1/2] \ \&\& \ N$

eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}} dx &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3 + bx^n}} + \frac{c^3 \int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}} dx}{a} \\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3 + bx^n}} + \frac{(c^3 \sqrt{cx}) \int \frac{\sqrt{x}}{\sqrt{ax^3 + bx^n}} dx}{a\sqrt{x}} \\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3 + bx^n}} + \frac{(2c^3 \sqrt{cx}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x^{3/2}}{\sqrt{ax^3 + bx^n}}\right)}{a(3-n)\sqrt{x}} \\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3 + bx^n}} + \frac{2c^3 \sqrt{cx} \tanh^{-1}\left(\frac{\sqrt{ax^3/2}}{\sqrt{ax^3 + bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.187117, size = 109, normalized size = 1.16

$$\frac{2c^3 \sqrt{cx} \left(\sqrt{ax^3/2} - \sqrt{bx^{n/2}} \sqrt{\frac{ax^3-n}{b}} + 1 \sinh^{-1} \left(\frac{\sqrt{ax^3/2}}{\sqrt{b}} \right) \right)}{a^{3/2}(n-3)\sqrt{x}\sqrt{ax^3 + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x]

[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]])/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr

t[a*x^3 + b*x^n])

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}} (ax^3 + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)

[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(ax^3 + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(ax^3 + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2), x, algorithm="giac")

[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)

$$3.400 \quad \int \frac{c^2 x^2}{(ax^2 + bx^n)^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)} - \frac{2c^2 x}{a(2-n)\sqrt{ax^2+bx^n}}$$

[Out] $(-2*c^2*x)/(a*(2-n)*\text{Sqrt}[a*x^2+b*x^n]) + (2*c^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2+b*x^n]])/(a^{(3/2)}*(2-n))$

Rubi [A] time = 0.0880041, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2030, 2008, 206}

$$\frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)} - \frac{2c^2 x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c^2*x^2)/(a*x^2+b*x^n)^{(3/2)},x]$

[Out] $(-2*c^2*x)/(a*(2-n)*\text{Sqrt}[a*x^2+b*x^n]) + (2*c^2*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[a*x^2+b*x^n]])/(a^{(3/2)}*(2-n))$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2030

$\text{Int}[((c_*)(x_))^{(m_)}*((a_*)(x_)^{(j_)} + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j+b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j+b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, j, m, n\}, x] \ \&\& \ \text{ILtQ}[p+1/2, 0] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{EqQ}[\text{Simplify}[m+j*p+1], 0] \ \&\& \ (\text{IntegerQ}[j] \ || \ \text{GtQ}[c, 0])$

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{c^2 x^2}{(ax^2 + bx^n)^{3/2}} dx &= c^2 \int \frac{x^2}{(ax^2 + bx^n)^{3/2}} dx \\ &= -\frac{2c^2 x}{a(2-n)\sqrt{ax^2 + bx^n}} + \frac{c^2 \int \frac{1}{\sqrt{ax^2 + bx^n}} dx}{a} \\ &= -\frac{2c^2 x}{a(2-n)\sqrt{ax^2 + bx^n}} + \frac{(2c^2) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2 + bx^n}}\right)}{a(2-n)} \\ &= -\frac{2c^2 x}{a(2-n)\sqrt{ax^2 + bx^n}} + \frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{ax^2 + bx^n}}\right)}{a^{3/2}(2-n)} \end{aligned}$$

Mathematica [A] time = 0.138729, size = 91, normalized size = 1.26

$$\frac{2c^2 \left(\sqrt{ax} - \sqrt{bx}^{n/2} \sqrt{\frac{ax^{2-n}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{ax^{1-n/2}}}{\sqrt{b}} \right) \right)}{a^{3/2}(n-2)\sqrt{ax^2 + bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2), x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int c^2 x^2 (ax^2 + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)

[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c^2 \int \frac{x^2}{(ax^2 + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")

[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2 \int \frac{x^2}{ax^2 \sqrt{ax^2 + bx^n} + bx^n \sqrt{ax^2 + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)

[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2 x^2}{(ax^2 + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)

$$3.401 \quad \int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}} dx$$

Optimal. Leaf size=85

$$\frac{2c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}} - \frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

[Out] $(-2*\text{Sqrt}[c*x])/(a*(1-n)*\text{Sqrt}[a*x + b*x^n]) + (2*c*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(a^{(3/2)}*(1-n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.137894, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}} - \frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c*x]/(a*x + b*x^n)^{(3/2)}, x]$

[Out] $(-2*\text{Sqrt}[c*x])/(a*(1-n)*\text{Sqrt}[a*x + b*x^n]) + (2*c*\text{Sqrt}[x]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x])/\text{Sqrt}[a*x + b*x^n]])/(a^{(3/2)}*(1-n)*\text{Sqrt}[c*x])$

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(n-j)*(p+1)), x] + Dist[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), Int[(c*x)^(m-j)*(a*x^j + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && ILtQ[p+1/2, 0] && NeQ[n, j] && EqQ[Simplify[m+j*p+1], 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p+1/2] && NeQ[n, j] && EqQ[Simplify[m+j*p+1], 0]
```

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx}}{(ax + bx^n)^{3/2}} dx &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax + bx^n}} + \frac{c \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} dx}{a} \\ &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax + bx^n}} + \frac{(c\sqrt{x}) \int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}} dx}{a\sqrt{cx}} \\ &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax + bx^n}} + \frac{(2c\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}} \\ &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax + bx^n}} + \frac{2c\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}} \end{aligned}$$

Mathematica [A] time = 0.186556, size = 104, normalized size = 1.22

$$\frac{2\sqrt{cx} \left(\sqrt{a}\sqrt{x} - \sqrt{bx^{n/2}} \sqrt{\frac{ax^{1-n}}{b} + 1} \sinh^{-1} \left(\frac{\sqrt{ax^{2-\frac{n}{2}}}}{\sqrt{b}} \right) \right)}{a^{3/2}(n-1)\sqrt{x}\sqrt{ax + bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2), x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int \sqrt{cx} (ax + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)`

[Out] `int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)

[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)

$$3.402 \quad \int \frac{1}{cx(a+bx^n)^{3/2}} dx$$

Optimal. Leaf size=54

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c*n)

Rubi [A] time = 0.0332688, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 266, 51, 63, 208}

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]

[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c*n)

Rule 12

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :=> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{cx(a+bx^n)^{3/2}} dx &= \frac{\int \frac{1}{x(a+bx^n)^{3/2}} dx}{c} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, x^n\right)}{cn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^n\right)}{acn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx^n}\right)}{abcn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}
\end{aligned}$$

Mathematica [C] time = 0.0140132, size = 40, normalized size = 0.74

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^n}{a} + 1\right)}{acn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^n)/a])/(a*c*n*Sqrt[a + b*x^n])

Maple [A] time = 0.003, size = 42, normalized size = 0.8

$$\frac{1}{cn} \left(-2 \frac{1}{a^{3/2}} \operatorname{Artanh} \left(\frac{\sqrt{a + bx^n}}{\sqrt{a}} \right) + 2 \frac{1}{a\sqrt{a + bx^n}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/c/x/(a+b*x^n)^(3/2),x)

[Out] 1/c/n*(-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))+2/a/(a+b*x^n)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0.846647, size = 335, normalized size = 6.2

$$\left[\frac{\left(\sqrt{abx^n + a^2} \right) \log \left(\frac{bx^n - 2\sqrt{bx^n + a}\sqrt{a+2a}}{x^n} \right) + 2\sqrt{bx^n + aa}}{a^2bcnx^n + a^3cn}, \frac{2 \left(\left(\sqrt{-abx^n} + \sqrt{-aa} \right) \arctan \left(\frac{\sqrt{bx^n + a}\sqrt{-a}}{a} \right) + \sqrt{bx^n + aa} \right)}{a^2bcnx^n + a^3cn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")

[Out]
$$\left[\left(\sqrt{a} b x^n + a^{3/2} \right) \log \left(\left(b x^n - 2 \sqrt{b x^n + a} \sqrt{a} + 2 a \right) / x^n + 2 \sqrt{b x^n + a} a \right) / \left(a^2 b c n x^n + a^3 c n \right), 2 \left(\sqrt{-a} b x^n + \sqrt{-a} a \right) \arctan \left(\sqrt{b x^n + a} \sqrt{-a} / a \right) + \sqrt{b x^n + a} a \right] / \left(a^2 b c n x^n + a^3 c n \right)$$

Sympy [B] time = 3.49617, size = 185, normalized size = 3.43

$$\frac{\frac{2a^3 \sqrt{1 + \frac{bx^n}{a}}}{a^{\frac{9}{2}n + a^{\frac{7}{2}}bnx^n}} + \frac{a^3 \log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}n + a^{\frac{7}{2}}bnx^n}} - \frac{2a^3 \log\left(\sqrt{1 + \frac{bx^n}{a}} + 1\right)}{a^{\frac{9}{2}n + a^{\frac{7}{2}}bnx^n}} + \frac{a^2 bx^n \log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}n + a^{\frac{7}{2}}bnx^n}} - \frac{2a^2 bx^n \log\left(\sqrt{1 + \frac{bx^n}{a}} + 1\right)}{a^{\frac{9}{2}n + a^{\frac{7}{2}}bnx^n}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)

[Out]
$$\left(2 a^{3/2} \sqrt{1 + b x^n / a} / \left(a^{9/2} n + a^{7/2} b n x^n \right) + a^3 \log \left(b x^n / a \right) / \left(a^{9/2} n + a^{7/2} b n x^n \right) - 2 a^{3/2} \log \left(\sqrt{1 + b x^n / a} + 1 \right) / \left(a^{9/2} n + a^{7/2} b n x^n \right) + a^{5/2} b x^n \log \left(b x^n / a \right) / \left(a^{9/2} n + a^{7/2} b n x^n \right) - 2 a^{5/2} b x^n \log \left(\sqrt{1 + b x^n / a} + 1 \right) / \left(a^{9/2} n + a^{7/2} b n x^n \right) \right) / c$$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}} cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)

$$3.403 \quad \int \frac{1}{(cx)^{5/2} \left(\frac{a}{x} + bx^n\right)^{3/2}} dx$$

Optimal. Leaf size=90

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])

Rubi [A] time = 0.193509, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]

[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x]
&& ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
```

$x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& \text{IntegerQ}[p + 1/2] \&\& \text{NeQ}[n, j] \&\& \text{EqQ}[\text{Simplify}[m + j*p + 1], 0]$

Rule 2029

$\text{Int}[(x_)^{(m_.)}/\text{Sqrt}[(a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[-2/(n - j), \text{Subst}[\text{Int}[1/(1 - a*x^2), x], x, x^{(j/2)}/\text{Sqrt}[a*x^j + b*x^n]], x] /; \text{FreeQ}\{a, b, j, n\}, x] \&\& \text{EqQ}[m, j/2 - 1] \&\& \text{NeQ}[n, j]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(cx)^{5/2} \left(\frac{a}{x} + bx^n\right)^{3/2}} dx &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x} + bx^n}} dx}{ac} \\ &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} + \frac{\sqrt{x} \int \frac{1}{x^{3/2}\sqrt{\frac{a}{x} + bx^n}} dx}{ac^2\sqrt{cx}} \\ &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} - \frac{(2\sqrt{x}) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{ac^2(1+n)\sqrt{cx}} \\ &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x} + bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}} \end{aligned}$$

Mathematica [C] time = 0.046005, size = 55, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^{n+1}}{a} + 1\right)}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(1 + n))/a])/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n])

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}} \left(\frac{a}{x} + bx^n \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)

[Out] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)

$$3.404 \quad \int \frac{1}{c^4 x^4 \left(\frac{a}{x^2} + bx^n\right)^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(a^(3/2)*c^4*(2 + n))

Rubi [A] time = 0.155999, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Antiderivative was successfully verified.

[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]

[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(a^(3/2)*c^4*(2 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2030

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int

egerQ[j] || GtQ[c, 0])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{c^4 x^4 \left(\frac{a}{x^2} + bx^n\right)^{3/2}} dx &= \frac{\int \frac{1}{x^4 \left(\frac{a}{x^2} + bx^n\right)^{3/2}} dx}{c^4} \\ &= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2} + bx^n}} + \frac{\int \frac{1}{x^2 \sqrt{\frac{a}{x^2} + bx^n}} dx}{ac^4} \\ &= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2} + bx^n}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{ac^4(2+n)} \\ &= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2} + bx^n}}\right)}{a^{3/2}c^4(2+n)} \end{aligned}$$

Mathematica [C] time = 0.0408202, size = 51, normalized size = 0.71

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^{n+2}}{a} + 1\right)}{ac^4(n+2)x\sqrt{\frac{a}{x^2} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(2 + n))/a])/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n])

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{c^4 x^4} \left(\frac{a}{x^2} + b x^n \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/c^4/x^4/(1/x^2*a+b*x^n)^(3/2),x)

[Out] int(1/c^4/x^4/(1/x^2*a+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\frac{(bx^n + \frac{a}{x^2})^{\frac{3}{2}} x^4}{c^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^2 \sqrt{\frac{a}{x^2} + bx^n} + bx^4 x^n \sqrt{\frac{a}{x^2} + bx^n}} dx$$

$$c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)

[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n)), x)/c**4

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x^2}\right)^{\frac{3}{2}} c^4 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)

$$3.405 \quad \int \frac{1}{(cx)^{11/2} \left(\frac{a}{x^3} + bx^n \right)^{3/2}} dx$$

Optimal. Leaf size=90

$$\frac{2}{ac^4(n+3)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1} \left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} \right)}{a^{3/2} c^5 (n+3) \sqrt{cx}}$$

[Out] $2/(a*c^4*(3+n)*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n]) - (2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(a^{(3/2)}*c^5*(3+n)*\text{Sqrt}[c*x])$

Rubi [A] time = 0.216698, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^4(n+3)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1} \left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} \right)}{a^{3/2} c^5 (n+3) \sqrt{cx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((c*x)^{(11/2)}*(a/x^3 + b*x^n)^{(3/2)}), x]$

[Out] $2/(a*c^4*(3+n)*(c*x)^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n]) - (2*\text{Sqrt}[x]*\text{ArcTanh}[\text{Sqrt}[a]/(x^{(3/2)}*\text{Sqrt}[a/x^3 + b*x^n])])/(a^{(3/2)}*c^5*(3+n)*\text{Sqrt}[c*x])$

Rule 2030

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol]$
 $] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)})/(a*(n-j)*(p+1)), x] + \text{Dist}[(c^j*(m+n*p+n-j+1))/(a*(n-j)*(p+1)), \text{Int}[(c*x)^{(m-j)}*(a*x^j + b*x^n)^{(p+1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c, j, m, n\}, x]$ && $\text{ILtQ}[p + 1/2, 0]$ && $\text{NeQ}[n, j]$ && $\text{EqQ}[\text{Simplify}[m + j*p + 1], 0]$ && $(\text{IntegerQ}[j] \parallel \text{GtQ}[c, 0])$

Rule 2031

```
Int[((c_)*(x_)^(m_))*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(cx)^{11/2} \left(\frac{a}{x^3} + bx^n\right)^{3/2}} dx &= \frac{2}{ac^4(3+n)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} + \frac{\int \frac{1}{(cx)^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx}{ac^3} \\
 &= \frac{2}{ac^4(3+n)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} + \frac{\sqrt{x} \int \frac{1}{x^{5/2} \sqrt{\frac{a}{x^3} + bx^n}} dx}{ac^5 \sqrt{cx}} \\
 &= \frac{2}{ac^4(3+n)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} - \frac{(2\sqrt{x}) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{ac^5(3+n)\sqrt{cx}} \\
 &= \frac{2}{ac^4(3+n)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}} - \frac{2\sqrt{x} \tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}
 \end{aligned}$$

Mathematica [C] time = 0.0519262, size = 55, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^{n+3}}{a} + 1\right)}{ac^4(n+3)(cx)^{3/2} \sqrt{\frac{a}{x^3} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(3 + n))/a])/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{11}{2}} \left(\frac{a}{x^3} + bx^n \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)

[Out] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)

$$3.406 \quad \int \frac{1}{c^7 x^7 \left(\frac{a}{x^4} + bx^n\right)^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4} + bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))

Rubi [A] time = 0.152161, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4} + bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

Antiderivative was successfully verified.

[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]

[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2030

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int

egerQ[j] || GtQ[c, 0])

Rule 2029

Int[(x_)^(m_)/Sqrt[(a_)*(x_)^(j_) + (b_)*(x_)^(n_)], x_Symbol] := Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{c^7 x^7 \left(\frac{a}{x^4} + bx^n\right)^{3/2}} dx &= \frac{\int \frac{1}{x^7 \left(\frac{a}{x^4} + bx^n\right)^{3/2}} dx}{c^7} \\
 &= \frac{2}{ac^7(4+n)x^2 \sqrt{\frac{a}{x^4} + bx^n}} + \frac{\int \frac{1}{x^3 \sqrt{\frac{a}{x^4} + bx^n}} dx}{ac^7} \\
 &= \frac{2}{ac^7(4+n)x^2 \sqrt{\frac{a}{x^4} + bx^n}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{1}{x^2 \sqrt{\frac{a}{x^4} + bx^n}}\right)}{ac^7(4+n)} \\
 &= \frac{2}{ac^7(4+n)x^2 \sqrt{\frac{a}{x^4} + bx^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a}}{x^2 \sqrt{\frac{a}{x^4} + bx^n}}\right)}{a^{3/2}c^7(4+n)}
 \end{aligned}$$

Mathematica [C] time = 0.0420332, size = 51, normalized size = 0.71

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^{n+4}}{a} + 1\right)}{ac^7(n+4)x^2 \sqrt{\frac{a}{x^4} + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(4 + n))/a])/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n])

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int \frac{1}{c^7 x^7} \left(\frac{a}{x^4} + b x^n \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)

[Out] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")

[Out] Timed out

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^3 \sqrt{\frac{a}{x^4} + bx^n} + bx^7 x^n \sqrt{\frac{a}{x^4} + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)

[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n)), x)/c**7

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^7 x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)

$$3.407 \quad \int \frac{1}{\sqrt{\frac{a+bx^3}{x}}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])

Rubi [A] time = 0.0144815, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^3)/x], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{LtQ}[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a+bx^3}{x}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x} + bx^2}} dx \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x} + bx^2}} \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x} + bx^2}} \right)}{3\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0238737, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^3} \tanh^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a+bx^3}} \right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a + b*x^3)/x], x]

[Out] (2*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt[x]*Sqrt[(a + b*x^3)/x])

Maple [C] time = 0.014, size = 477, normalized size = 14.9

$$-4 \frac{(bx^3 + a)(-1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})^2}{b^2 \sqrt{(bx^3 + a)x} (i\sqrt{3} - 3)} \sqrt{\frac{(i\sqrt{3} - 3)xb}{(-1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a} + 2bx + \sqrt[3]{-b^2a}}{(1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a} - 2bx}{(-1 + i\sqrt{3})(-bx + \sqrt[3]{-b^2a})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^3+a)/x)^(1/2), x)


```
[Out] -4*(b*x^3+a)*(-1+I*3^(1/2))*(-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3))^(1/2)*(-b*x+(-b^2*a)^(1/3))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2)/b^2*(EllipticF((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-EllipticPi((-I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)))/(-b*x+(-b^2*a)^(1/3))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2)))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))/((b*x^3+a)/x)^(1/2)/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^3+a}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

Fricas [A] time = 1.1653, size = 236, normalized size = 7.38

$$\left[\frac{\log\left(-8b^2x^6 - 8abx^3 - a^2 - 4(2bx^5 + ax^2)\sqrt{b}\sqrt{\frac{bx^3+a}{x}}\right)}{6\sqrt{b}}, -\frac{\sqrt{-b} \arctan\left(\frac{2\sqrt{-b}x^2\sqrt{\frac{bx^3+a}{x}}}{2bx^3+a}\right)}{3b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
```

$/x)/(2*b*x^3 + a)/b]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x**3+a)/x)**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.408 \quad \int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx$$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])

Rubi [A] time = 0.0159634, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1979, 2008, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^4)/x^2], x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_)*(x_)^2 + (b_)*(x_)^(n_)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x^2} + bx^2}} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x^2} + bx^2}} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^2} + bx^2}} \right)}{2\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0207978, size = 59, normalized size = 1.84

$$\frac{\sqrt{a+bx^4} \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}} \right)}{2\sqrt{bx} \sqrt{\frac{a+bx^4}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a + b*x^4)/x^2], x]

[Out] (Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[(a + b*x^4)/x^2])

Maple [A] time = 0.051, size = 49, normalized size = 1.5

$$\frac{1}{2x} \sqrt{bx^4 + a} \ln \left(x^2 \sqrt{b} + \sqrt{bx^4 + a} \right) \frac{1}{\sqrt{\frac{bx^4 + a}{x^2}}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^4+a)/x^2)^(1/2), x)

[Out] $\frac{1}{2} \left(\frac{(bx^4+a)/x^2}{b} \right)^{1/2} / x \cdot (bx^4+a)^{1/2} \cdot \ln(x^2 \cdot b^{1/2} + (bx^4+a)^{1/2}) / b^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{x^5}{(bx^4+a)^{\frac{3}{2}}} dx + \frac{x^2}{2\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")`

[Out] `b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)`

Fricas [A] time = 0.835654, size = 196, normalized size = 6.12

$$\left[\frac{\log\left(-2bx^4 - 2\sqrt{b}x^3\sqrt{\frac{bx^4+a}{x^2}} - a\right)}{4\sqrt{b}}, -\frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.409 \quad \int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])

Rubi [A] time = 0.0158393, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^5)/x^3], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x^3} + bx^2}} dx \\ &= \frac{2}{5} \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x^3} + bx^2}} \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^3} + bx^2}} \right)}{5\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0236792, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^5} \tanh^{-1} \left(\frac{\sqrt{bx^{5/2}}}{\sqrt{a+bx^5}} \right)}{5\sqrt{bx^{3/2}} \sqrt{\frac{a+bx^5}{x^3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a + b*x^5)/x^3], x]

[Out] (2*Sqrt[a + b*x^5]*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b]*x^(3/2)*Sqrt[(a + b*x^5)/x^3])

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^5+a)/x^3)^(1/2), x)

[Out] int(1/((b*x^5+a)/x^3)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)

Fricas [A] time = 2.59082, size = 244, normalized size = 7.62

$$\left[\frac{\log\left(-8b^2x^{10} - 8abx^5 - a^2 - 4(2bx^9 + ax^4)\sqrt{b}\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\sqrt{b}}, -\frac{\sqrt{-b} \arctan\left(\frac{2\sqrt{-b}x^4\sqrt{\frac{bx^5+a}{x^3}}}{2bx^5+a}\right)}{5b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")

[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt((b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5 + a)/x^3)/(2*b*x^5 + a))/b]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.410 \quad \int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{bn}}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)])/(Sqrt[b]*n)

Rubi [A] time = 0.0239837, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{bn}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)])/(Sqrt[b]*n)

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} dx &= \int \frac{1}{\sqrt{bx^2+ax^{2-n}}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n} \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{bn}} \end{aligned}$$

Mathematica [B] time = 0.0387811, size = 76, normalized size = 2.05

$$\frac{2\sqrt{ax}^{1-\frac{n}{2}} \sqrt{\frac{bx^n}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{bx^{n/2}}}{\sqrt{a}}\right)}{\sqrt{bn}\sqrt{x^{2-n}(a+bx^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)], x]

[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(2-n)*(a+b*x^n))^(1/2), x)

[Out] int(1/(x^(2-n)*(a+b*x^n))^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^n + a)x^{-n+2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)`

Fricas [A] time = 0.911825, size = 216, normalized size = 5.84

$$\left[\frac{\log\left(\frac{2bx^n + ax + 2\sqrt{bx^n}\sqrt{\frac{bx^2x^n + ax^2}{x^n}}}{x}\right)}{\sqrt{bn}}, \frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{bx^2x^n + ax^2}{x^n}}}{bx}\right)}{bn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")`

[Out] `[log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sqrt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/(b*n)]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^n + a)x^{-n+2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

$$3.411 \quad \int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx$$

Optimal. Leaf size=33

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])

Rubi [A] time = 0.0140644, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a - b*x^3)/x], x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x} - bx^2}} dx \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x} - bx^2}} \right) \\ &= \frac{2 \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x} - bx^2}} \right)}{3\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0259932, size = 66, normalized size = 2.

$$\frac{2\sqrt{a-bx^3} \tan^{-1} \left(\frac{\sqrt{bx^{3/2}}}{\sqrt{a-bx^3}} \right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a - b*x^3)/x], x]

[Out] (2*Sqrt[a - b*x^3]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a - b*x^3]])/(3*Sqrt[b]*Sqrt[x]*Sqrt[(a - b*x^3)/x])

Maple [C] time = 0.566, size = 471, normalized size = 14.3

$$4 \frac{(bx^3 - a)(1 + i\sqrt{3})(-bx + \sqrt[3]{b^2a})^2}{b^2 \sqrt{-(bx^3 - a)x} (i\sqrt{3} + 3)} \sqrt{\frac{(i\sqrt{3} + 3)xb}{(1 + i\sqrt{3})(-bx + \sqrt[3]{b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{b^2a} - 2bx - \sqrt[3]{b^2a}}{(-1 + i\sqrt{3})(-bx + \sqrt[3]{b^2a})}} \sqrt{\frac{i\sqrt{3}\sqrt[3]{b^2a} + 2bx + \sqrt[3]{b^2a}}{(1 + i\sqrt{3})(-bx + \sqrt[3]{b^2a})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-b*x^3+a)/x)^(1/2), x)


```
[Out] 4*(b*x^3-a)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2)))/(-b*x+(b^2*a)^(1/3))^(1/2)*(-b*x+(b^2*a)^(1/3))^(1/2)*((I*3^(1/2)*(b^2*a)^(1/3)-2*b*x-(b^2*a)^(1/3))/(-1+I*3^(1/2)))/(-b*x+(b^2*a)^(1/3))^(1/2)*((I*3^(1/2)*(b^2*a)^(1/3)+2*b*x+(b^2*a)^(1/3))/(1+I*3^(1/2)))/(-b*x+(b^2*a)^(1/3))^(1/2)/b^2*(EllipticF((-I*3^(1/2)+3)*x*b/(1+I*3^(1/2)))/(-b*x+(b^2*a)^(1/3))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2)))/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2))-EllipticPi((-I*3^(1/2)+3)*x*b/(1+I*3^(1/2)))/(-b*x+(b^2*a)^(1/3))^(1/2),(1+I*3^(1/2))/(I*3^(1/2)+3),((I*3^(1/2)-3)*(1+I*3^(1/2)))/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2)))/(-b*x^3-a)/x)^(1/2)/(-b*x^3-a)*x)^(1/2)/(I*3^(1/2)+3)/(-1/b^2*x*(-b*x+(b^2*a)^(1/3))*(I*3^(1/2)*(b^2*a)^(1/3)-2*b*x-(b^2*a)^(1/3))*(I*3^(1/2)*(b^2*a)^(1/3)+2*b*x+(b^2*a)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

Fricas [A] time = 1.54122, size = 240, normalized size = 7.27

$$\left[\frac{\sqrt{-b} \log\left(-8b^2x^6 + 8abx^3 - a^2 + 4(2bx^5 - ax^2)\sqrt{-b}\sqrt{-\frac{bx^3-a}{x}}\right)}{6b}, \frac{\arctan\left(\frac{2\sqrt{b}x^2\sqrt{-\frac{bx^3-a}{x}}}{2bx^3-a}\right)}{3\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
```

$$\frac{1}{(2bx^3 - a)\sqrt{b}}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.412 \quad \int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} dx$$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])

Rubi [A] time = 0.0149095, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1979, 2008, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a - b*x^4)/x^2], x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x^2} - bx^2}} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x^2} - bx^2}} \right) \\ &= \frac{\tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^2} - bx^2}} \right)}{2\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0219543, size = 62, normalized size = 1.88

$$\frac{\sqrt{a-bx^4} \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a-bx^4}} \right)}{2\sqrt{bx} \sqrt{\frac{a-bx^4}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a - b*x^4)/x^2], x]

[Out] (Sqrt[a - b*x^4]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]*x*Sqrt[(a - b*x^4)/x^2])

Maple [B] time = 0.013, size = 53, normalized size = 1.6

$$\frac{1}{2x} \sqrt{-bx^4 + a} \arctan \left(x^2 \sqrt{b} \frac{1}{\sqrt{-bx^4 + a}} \right) \frac{1}{\sqrt{-\frac{bx^4 - a}{x^2}}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-b*x^4+a)/x^2)^(1/2), x)

[Out] $1/2/(-b*x^4-a)/x^2)^{(1/2)}/x*(-b*x^4+a)^{(1/2)}/b^{(1/2)}*\arctan(x^2*b^{(1/2)}/(-b*x^4+a)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{x^5}{(bx^4 - a)\sqrt{-bx^4 + a}} dx + \frac{x^2}{2\sqrt{-bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")`

[Out] `b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 + a)`

Fricas [A] time = 1.02574, size = 198, normalized size = 6.

$$\left[\frac{\sqrt{-b} \log\left(2bx^4 - 2\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}} - a\right)}{4b}, \frac{\arctan\left(\frac{\sqrt{b}x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")`

[Out] `[-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b, -1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.413 \quad \int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx$$

Optimal. Leaf size=33

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^3} - bx^2}} \right)}{5\sqrt{b}}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])

Rubi [A] time = 0.0161781, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^3} - bx^2}} \right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a - b*x^5)/x^3], x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] := Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x^3} - bx^2}} dx \\ &= \frac{2}{5} \text{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x^3} - bx^2}} \right) \\ &= \frac{2 \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{\frac{a}{x^3} - bx^2}} \right)}{5\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.0248042, size = 66, normalized size = 2.

$$\frac{2\sqrt{a-bx^5} \tan^{-1} \left(\frac{\sqrt{bx^{5/2}}}{\sqrt{a-bx^5}} \right)}{5\sqrt{bx^{3/2}} \sqrt{\frac{a-bx^5}{x^3}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a - b*x^5)/x^3], x]

[Out] (2*Sqrt[a - b*x^5]*ArcTan[(Sqrt[b]*x^(5/2))/Sqrt[a - b*x^5]])/(5*Sqrt[b]*x^(3/2)*Sqrt[(a - b*x^5)/x^3])

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{-bx^5+a}{x^3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-b*x^5+a)/x^3)^(1/2), x)

[Out] int(1/((-b*x^5+a)/x^3)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)

Fricas [A] time = 3.25213, size = 248, normalized size = 7.52

$$\left[\frac{\sqrt{-b} \log\left(-8b^2x^{10} + 8abx^5 - a^2 + 4(2bx^9 - ax^4)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10b}, \frac{\arctan\left(\frac{2\sqrt{bx^4}\sqrt{-\frac{bx^5-a}{x^3}}}{2bx^5-a}\right)}{5\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")

[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqrt(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a)/x^3)/(2*b*x^5 - a))/sqrt(b)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.414 \quad \int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}} dx$$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{bn}}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)

Rubi [A] time = 0.0226047, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{bn}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)], x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}} dx &= \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{bn}} \end{aligned}$$

Mathematica [B] time = 0.0428816, size = 78, normalized size = 2.05

$$\frac{2\sqrt{ax}^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\sin^{-1}\left(\frac{\sqrt{bx^{n/2}}}{\sqrt{a}}\right)}{\sqrt{bn}\sqrt{x^{2-n}(a-bx^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)], x]

[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(2-n)*(a-b*x^n))^(1/2), x)

[Out] int(1/(x^(2-n)*(a-b*x^n))^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(bx^n - a)x^{-n+2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)`

Fricas [A] time = 1.00405, size = 219, normalized size = 5.76

$$\left[\frac{\sqrt{-b} \log\left(\frac{2bx^n - ax - 2\sqrt{-b}x^n \sqrt{\frac{-bx^2x^n - ax^2}{x^n}}}{x}\right)}{bn}, \frac{2 \arctan\left(\frac{\sqrt{\frac{-bx^2x^n - ax^2}{x^n}}}{\sqrt{bx}}\right)}{\sqrt{bn}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")`

[Out] `[-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqrt(b)*n)]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(bx^n - a)x^{-n+2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

$$3.415 \quad \int \frac{1}{\sqrt{x^n(a+bx^{2-n})}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0233351, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^n (a + bx^{2-n})}} dx &= \int \frac{1}{\sqrt{bx^2 + ax^n}} dx \\ &= \frac{2 \operatorname{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+ax^n}} \right)}{2-n} \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{bx^2+ax^n}} \right)}{\sqrt{b}(2-n)} \end{aligned}$$

Mathematica [B] time = 0.0754614, size = 78, normalized size = 2.11

$$\frac{2\sqrt{ax^{n/2}}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^n (a + bx^{2-n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)

[Out] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^{-n+2} + a)x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^{-n+2} + a)x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

$$3.416 \quad \int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0193817, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}} dx &= \int \frac{1}{\sqrt{bx^2+ax^n}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n} \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)} \end{aligned}$$

Mathematica [B] time = 0.021973, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{ax^{n/2}}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))], x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])

Maple [F] time = 2.16, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(b+a*x^(-2+n)))^(1/2), x)

[Out] int(1/(x^2*(b+a*x^(-2+n)))^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-2} + b)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)

Fricas [A] time = 1.02494, size = 252, normalized size = 6.81

$$\left[\frac{\sqrt{b} \log\left(\frac{ax^{n-2} + 2bx - 2\sqrt{ax^2x^{n-2} + bx^2}\sqrt{b}}{xx^{n-2}}\right)}{bn - 2b}, \frac{2\sqrt{-b} \arctan\left(\frac{\sqrt{ax^2x^{n-2} + bx^2}\sqrt{-b}}{bx}\right)}{bn - 2b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")

[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-2} + b)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)

$$3.417 \quad \int \frac{1}{\sqrt{x(bx+ax^{-1+n})}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0183542, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1979, 2008, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x(bx + ax^{-1+n})}} dx &= \int \frac{1}{\sqrt{bx^2 + ax^n}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n} \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)} \end{aligned}$$

Mathematica [B] time = 0.0220208, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{ax^{n/2}}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(bx + ax^{-1+n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)

[Out] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} + bx)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} + bx)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

$$3.418 \quad \int \frac{1}{\sqrt{x^n(a-bx^{2-n})}} dx$$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0209641, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}} dx &= \int \frac{1}{\sqrt{-bx^2 + ax^n}} dx \\
&= \frac{2 \operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{-bx^2+ax^n}} \right)}{2-n} \\
&= \frac{2 \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{-bx^2+ax^n}} \right)}{\sqrt{b}(2-n)}
\end{aligned}$$

Mathematica [B] time = 0.0795574, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{ax^{n/2}}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)

[Out] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(bx^{-n+2} - a)x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(bx^{-n+2} - a)x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

$$3.419 \quad \int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}} dx$$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0198648, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}} dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)} \end{aligned}$$

Mathematica [B] time = 0.0224835, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))], x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])

Maple [F] time = 0.813, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2), x)

[Out] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-2} - b)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)

Fricas [A] time = 1.01108, size = 251, normalized size = 6.61

$$\left[\frac{\sqrt{-b} \log\left(\frac{ax^{n-2} - 2bx - 2\sqrt{ax^2x^{n-2} - bx^2}\sqrt{-b}}{x^{n-2}}\right)}{bn - 2b}, \frac{2\sqrt{b} \arctan\left(\frac{\sqrt{ax^2x^{n-2} - bx^2}}{\sqrt{bx}}\right)}{bn - 2b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")

[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqrt(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) - b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-2} - b)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)

$$3.420 \quad \int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}} dx$$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rubi [A] time = 0.0188031, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1979, 2008, 203}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2008

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x(-bx + ax^{-1+n})}} dx &= \int \frac{1}{\sqrt{-bx^2 + ax^n}} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)} \end{aligned}$$

Mathematica [B] time = 0.0233586, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{bx}^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))], x]

[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(-bx + ax^{-1+n})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2), x)

[Out] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} - bx)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} - bx)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

$$3.421 \quad \int (cx)^m (ax^j + bx^n)^{3/2} dx$$

Optimal. Leaf size=107

$$\frac{2bx^{n+1}(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{m+\frac{3n}{2}+1}{j-n}; \frac{m+\frac{3n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

[Out] (2*b*x^(1+n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1+m+(3*n)/2)/(j-n), 1+(1+m+(3*n)/2)/(j-n), -((a*x^(j-n))/b)])/((2+2*m+3*n)*Sqrt[1+(a*x^(j-n))/b])

Rubi [A] time = 0.102115, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 365, 364}

$$\frac{2bx^{n+1}(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{m+\frac{3n}{2}+1}{j-n}; \frac{m+\frac{3n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]

[Out] (2*b*x^(1+n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1+m+(3*n)/2)/(j-n), 1+(1+m+(3*n)/2)/(j-n), -((a*x^(j-n))/b)])/((2+2*m+3*n)*Sqrt[1+(a*x^(j-n))/b])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p]]/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n-j))^FracPart[p]], Int[x^(m+j*p)*(a + b*x^(n-j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n-j]

Rule 365

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (cx)^m (ax^j + bx^n)^{3/2} dx &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{3n}{2}} (b + ax^{j-n})^{3/2} dx}{\sqrt{b + ax^{j-n}}} \\ &= \frac{\left(bx^{-m-\frac{n}{2}}(cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{3n}{2}} \left(1 + \frac{ax^{j-n}}{b}\right)^{3/2} dx}{\sqrt{1 + \frac{ax^{j-n}}{b}}} \\ &= \frac{2bx^{1+n}(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1+m+\frac{3n}{2}}{j-n}; 1 + \frac{1+m+\frac{3n}{2}}{j-n}; -\frac{ax^{j-n}}{b}\right)}{(2 + 2m + 3n)\sqrt{1 + \frac{ax^{j-n}}{b}}} \end{aligned}$$

Mathematica [B] time = 0.355433, size = 218, normalized size = 2.04

$$\frac{2(cx)^m \left(3a^2(j-n)^2 x^{2j+1} \sqrt{\frac{ax^{j-n}}{b}} + 1 {}_2F_1\left(\frac{1}{2}, \frac{4j+2m-n+2}{2j-2n}; \frac{6j+2m-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) + x^{-m}(4j+2m-n+2)(ax^j + bx^n)(a(-j+2m-n+2))\right)}{(2m+3n+2)(4j+2m-n+2)(2j+2m+n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2), x]
```

```
[Out] (2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^(
1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*x^
(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*m -
n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((
```


$(2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*\text{Sqrt}[a*x^j + b*x^n]$
 $)$

Maple [F] time = 0.692, size = 0, normalized size = 0.

$$\int (cx)^m (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)`

[Out] `int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)

3.422 $\int (cx)^m \sqrt{ax^j + bx^n} dx$

Optimal. Leaf size=100

$$\frac{2x(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{m+\frac{n}{2}+1}{j-n}; \frac{2m+n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

[Out] $(2*x*(c*x)^m*\text{Sqrt}[a*x^j + b*x^n]*\text{Hypergeometric2F1}[-1/2, (1 + m + n/2)/(j - n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*\text{Sqrt}[1 + (a*x^(j - n))/b])$

Rubi [A] time = 0.094578, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{m+\frac{n}{2}+1}{j-n}; \frac{2m+n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c*x)^m*\text{Sqrt}[a*x^j + b*x^n], x]$

[Out] $(2*x*(c*x)^m*\text{Sqrt}[a*x^j + b*x^n]*\text{Hypergeometric2F1}[-1/2, (1 + m + n/2)/(j - n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*\text{Sqrt}[1 + (a*x^(j - n))/b])$

Rule 2032

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*)(x_*)^{(j_*)} + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(c^{\text{IntPart}[m]}*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]}]/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n-j)})^{\text{FracPart}[p]}), \text{Int}[x^{(m+j*p)}*(a + b*x^{(n-j)})^p, x], x] /; \text{FreeQ}\{a, b, c, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 365

$\text{Int}[(c_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*\text{IntPart}[p]*(a + b*x^n)^{\text{FracPart}[p]}]/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(c*x)^m$

$m*(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0]$
 $\&\& \text{!(ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

$\text{Int}[\{(c_)*(x_)\}^{(m_)}*\{(a_)+(b_)*(x_)\}^{(n_)}\}^{(p_)}, x_Symbol] \text{:> Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/ (c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rubi steps

$$\begin{aligned} \int (cx)^m \sqrt{ax^j + bx^n} dx &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{n}{2}} \sqrt{b + ax^{j-n}} dx}{\sqrt{b + ax^{j-n}}} \\ &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{n}{2}} \sqrt{1 + \frac{ax^{j-n}}{b}} dx}{\sqrt{1 + \frac{ax^{j-n}}{b}}} \\ &= \frac{2x(cx)^m \sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1+m+\frac{n}{2}}{j-n}; 1 + \frac{2+2m+n}{2j-2n}; -\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1 + \frac{ax^{j-n}}{b}}} \end{aligned}$$

Mathematica [A] time = 0.191334, size = 156, normalized size = 1.56

$$\frac{2x(cx)^m \left((2j+2m-n+2)(ax^j + bx^n) - a(j-n)x^j \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{1}{2}, \frac{2j+2m-n+2}{2j-2n}; \frac{4j+2m-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) \right)}{(2m+n+2)(2j+2m-n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]

[Out] $(2*x*(c*x)^m*((2+2*j+2*m-n)*(a*x^j+b*x^n)-a*(j-n)*x^j*\text{Sqrt}[1+(a*x^{(j-n)})/b]*\text{Hypergeometric2F1}[1/2,(2+2*j+2*m-n)/(2*j-2*n),(2+4*j+2*m-3*n)/(2*j-2*n),-((a*x^{(j-n)})/b)]))/((2+2*j+2*m-n)*(2+2*m+n)*\text{Sqrt}[a*x^j+b*x^n])$

Maple [F] time = 0.512, size = 0, normalized size = 0.

$$\int (cx)^m \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)

[Out] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (cx)^m \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

$$3.423 \quad \int \frac{(cx)^m}{\sqrt{ax^j+bx^n}} dx$$

Optimal. Leaf size=102

$$\frac{2x(cx)^m \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m-\frac{n}{2}+1}{j-n}; \frac{m-\frac{n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sqrt[a*x^j + b*x^n])

Rubi [A] time = 0.097152, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m-\frac{n}{2}+1}{j-n}; \frac{m-\frac{n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m/Sqrt[a*x^j + b*x^n], x]

[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sqrt[a*x^j + b*x^n])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^

$m*(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0]$
 $\&\& \text{!(ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

$\text{Int}[\{(c_)*(x_)\}^{(m_)}*\{(a_)+(b_)*(x_)\}^{(n_)}\}^{(p_)}, x_Symbol] \text{:> Simp}[(a^$
 $p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a$
 $)]/(c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILt}$
 $Q[p, 0] \parallel \text{GtQ}[a, 0])$

Rubi steps

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{b+ax^{j-n}}\right) \int \frac{x^{m-\frac{n}{2}}}{\sqrt{b+ax^{j-n}}} dx}{\sqrt{ax^j + bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{1+\frac{ax^{j-n}}{b}}\right) \int \frac{x^{m-\frac{n}{2}}}{\sqrt{1+\frac{ax^{j-n}}{b}}} dx}{\sqrt{ax^j + bx^n}}$$

$$= \frac{2x(cx)^m \sqrt{1+\frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{1}{2}, \frac{1+m-\frac{n}{2}}{j-n}; 1+\frac{1+m-\frac{n}{2}}{j-n}; -\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j + bx^n}}$$

Mathematica [A] time = 0.0811581, size = 106, normalized size = 1.04

$$\frac{2x(cx)^m \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{2m-n+2}{2j-2n}; \frac{2m-n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]

[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n)/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.37, size = 0, normalized size = 0.

$$\int (cx)^m \frac{1}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)

[Out] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)

[Out] Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)

$$3.424 \quad \int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{2x^{1-n}(cx)^m \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{3}{2}, \frac{m-\frac{3n}{2}+1}{j-n}; \frac{m-\frac{3n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j + bx^n}}$$

[Out] (2*x^(1-n)*(c*x)^m*Sqrt[1+(a*x^(j-n))/b]*Hypergeometric2F1[3/2, (1+m-(3*n)/2)/(j-n), 1+(1+m-(3*n)/2)/(j-n), -(a*x^(j-n))/b])/(b*(2+2*m-3*n)*Sqrt[a*x^j+b*x^n])

Rubi [A] time = 0.107765, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 365, 364}

$$\frac{2x^{1-n}(cx)^m \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{3}{2}, \frac{m-\frac{3n}{2}+1}{j-n}; \frac{m-\frac{3n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m/(a*x^j + b*x^n)^(3/2), x]

[Out] (2*x^(1-n)*(c*x)^m*Sqrt[1+(a*x^(j-n))/b]*Hypergeometric2F1[3/2, (1+m-(3*n)/2)/(j-n), 1+(1+m-(3*n)/2)/(j-n), -(a*x^(j-n))/b])/(b*(2+2*m-3*n)*Sqrt[a*x^j+b*x^n])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n-j))^FracPart[p]), Int[x^(m+j*p)*(a + b*x^(n-j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n-j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^

$m*(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0]$
 $\&\& \text{!(ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

$\text{Int}[\{(c_)*(x_)\}^{(m_)}*\{(a_)+(b_)*(x_)\}^{(n_)\}^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{m+1}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/ (c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}} dx &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{b+ax^{j-n}}\right) \int \frac{x^{m-\frac{3n}{2}}}{(b+ax^{j-n})^{3/2}} dx}{\sqrt{ax^j + bx^n}} \\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{1+\frac{ax^{j-n}}{b}}\right) \int \frac{x^{m-\frac{3n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}} dx}{b\sqrt{ax^j + bx^n}} \\ &= \frac{2x^{1-n}(cx)^m \sqrt{1+\frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{3}{2}, \frac{1+m-\frac{3n}{2}}{j-n}; 1+\frac{1+m-\frac{3n}{2}}{j-n}; -\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j + bx^n}} \end{aligned}$$

Mathematica [A] time = 0.128102, size = 116, normalized size = 1.05

$$\frac{2x^{1-j}(cx)^m \left(\sqrt{\frac{ax^{j-n}}{b}} + 1 {}_2F_1\left(\frac{1}{2}, \frac{-2j+2m-n+2}{2j-2n}; \frac{2m-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) - 1 \right)}{a(j-n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2), x]

[Out] (2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b])*Hypergeometric2F1[1/2, (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int (cx)^m (ax^j + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)

[Out] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)

[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)

$$3.425 \quad \int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}} dx$$

Optimal. Leaf size=111

$$\frac{2x^{1-2n}(cx)^m \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{5}{2}, \frac{m-\frac{5n}{2}+1}{j-n}; \frac{m-\frac{5n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j + bx^n}}$$

[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 + m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -(a*x^(j - n))/b]) / (b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])

Rubi [A] time = 0.106485, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 365, 364}

$$\frac{2x^{1-2n}(cx)^m \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{5}{2}, \frac{m-\frac{5n}{2}+1}{j-n}; \frac{m-\frac{5n}{2}+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m/(a*x^j + b*x^n)^(5/2), x]

[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 + m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -(a*x^(j - n))/b]) / (b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^

$m*(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0]$
 $\&\& \text{!(ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

$\text{Int}[\{(c_)*(x_)\}^{(m_)}*\{(a_)+(b_)*(x_)\}^{(n_)}\}^{(p_)}, x_Symbol] \text{:> Simp}[(a^p*(c*x)^{m+1}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/(c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rubi steps

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}} dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{b + ax^{j-n}}\right) \int \frac{x^{m-\frac{5n}{2}}}{(b+ax^{j-n})^{5/2}} dx}{\sqrt{ax^j + bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m \sqrt{1 + \frac{ax^{j-n}}{b}}\right) \int \frac{x^{m-\frac{5n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}} dx}{b^2 \sqrt{ax^j + bx^n}}$$

$$= \frac{2x^{1-2n}(cx)^m \sqrt{1 + \frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{5}{2}, \frac{1+m-\frac{5n}{2}}{j-n}; 1 + \frac{1+m-\frac{5n}{2}}{j-n}; -\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j + bx^n}}$$

Mathematica [A] time = 0.376667, size = 166, normalized size = 1.5

$$\frac{2x^{1-2j}(cx)^m \left(-2j-2m+3n-2\right) \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{1}{2}, \frac{-4j+2m-n+2}{2j-2n}; \frac{-2j+2m-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) - \frac{a(j-n)x^j}{ax^j+bx^n} + 2j-2m+3n-2}{3a^2(j-n)^2 \sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2), x]

[Out] $(2*x^{(1-2*j)}*(c*x)^m*(-2+2*j-2*m+3*n-(a*(j-n)*x^j)/(a*x^j+b*x^n)) - (-2+2*j-2*m+3*n)*\text{Sqrt}[1+(a*x^{(j-n)})/b]*\text{Hypergeometric2F1}[1/2, (2-4*j+2*m-n)/(2*j-2*n), (2-2*j+2*m-3*n)/(2*j-2*n), -((a*x^{(j-n)})/b)])/(3*a^2*(j-n)^2*\text{Sqrt}[a*x^j+b*x^n])$

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int (cx)^m (ax^j + bx^n)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)

[Out] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.426 $\int (ax^j + bx^n)^{3/2} dx$

Optimal. Leaf size=97

$$\frac{2bx^{n+1}\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{3n}{2}+1; \frac{2j+n+2}{2(j-n)}; -\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n))/2]/(j - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)]/((2 + 3*n)*Sqrt[1 + (a*x^(j - n))/b])

Rubi [A] time = 0.05521, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2011, 365, 364}

$$\frac{2bx^{n+1}\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{3n}{2}+1; \frac{2j+n+2}{2(j-n)}; -\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^j + b*x^n)^(3/2), x]

[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n))/2]/(j - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)]/((2 + 3*n)*Sqrt[1 + (a*x^(j - n))/b])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^

$m*(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0]$
 $\&\& \text{!(ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

$\text{Int}[\{(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/(c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rubi steps

$$\begin{aligned} \int (ax^j + bx^n)^{3/2} dx &= \frac{(x^{-n/2}\sqrt{ax^j + bx^n}) \int x^{3n/2} (b + ax^{j-n})^{3/2} dx}{\sqrt{b + ax^{j-n}}} \\ &= \frac{(bx^{-n/2}\sqrt{ax^j + bx^n}) \int x^{3n/2} \left(1 + \frac{ax^{j-n}}{b}\right)^{3/2} dx}{\sqrt{1 + \frac{ax^{j-n}}{b}}} \\ &= \frac{2bx^{1+n}\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1+\frac{3n}{2}}{j-n}, \frac{2+2j+n}{2(j-n)}, -\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1 + \frac{ax^{j-n}}{b}}} \end{aligned}$$

Mathematica [A] time = 0.194312, size = 177, normalized size = 1.82

$$\frac{2x \left(3a^2(j-n)^2 x^{2j} \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{1}{2}, \frac{4j-n+2}{2j-2n}, \frac{6j-3n+2}{2j-2n}, -\frac{ax^{j-n}}{b}\right) + (4j-n+2)(ax^j + bx^n)(a(-j+4n+2)x^j + b(2j+n+2)x^n) \right)}{(3n+2)(4j-n+2)(2j+n+2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^j + b*x^n)^(3/2), x]

[Out] $(2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)*x^n) + 3*a^2*(j - n)^2*x^{(2*j)}*\text{Sqrt}[1 + (a*x^{(j - n)})/b]*\text{Hypergeometric2F1}[1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^{(j - n)})/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*\text{Sqrt}[a*x^j + b*x^n])$

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^j+b*x^n)^(3/2),x)

[Out] int((a*x^j+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x^j + b*x^n)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

3.427 $\int \sqrt{ax^j + bx^n} dx$

Optimal. Leaf size=87

$$\frac{2x\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)]/((2 + n)*Sqrt[1 + (a*x^(j - n))/b])

Rubi [A] time = 0.0525132, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^j + b*x^n], x]

[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)]/((2 + n)*Sqrt[1 + (a*x^(j - n))/b])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]

&& !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{ax^j + bx^n} dx &= \frac{\left(x^{-n/2}\sqrt{ax^j + bx^n}\right) \int x^{n/2}\sqrt{b + ax^{j-n}} dx}{\sqrt{b + ax^{j-n}}} \\ &= \frac{\left(x^{-n/2}\sqrt{ax^j + bx^n}\right) \int x^{n/2}\sqrt{1 + \frac{ax^{j-n}}{b}} dx}{\sqrt{1 + \frac{ax^{j-n}}{b}}} \\ &= \frac{2x\sqrt{ax^j + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{2+n}{2(j-n)}; 1 + \frac{2+n}{2j-2n}; -\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1 + \frac{ax^{j-n}}{b}}} \end{aligned}$$

Mathematica [A] time = 0.142623, size = 134, normalized size = 1.54

$$\frac{2x \left(a(j-n)x^j \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{1}{2}, \frac{2j-n+2}{2j-2n}; \frac{4j-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) - (2j-n+2)(ax^j + bx^n) \right)}{(n+2)(-2j+n-2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^j + b*x^n], x]

[Out] (2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.387, size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^j+b*x^n)^(1/2),x)

[Out] int((a*x^j+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^j + b*x^n), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**j + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

$$3.428 \quad \int \frac{1}{\sqrt{ax^j+bx^n}} dx$$

Optimal. Leaf size=93

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2(j-n)}; \frac{1-\frac{n}{2}}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)), 1 + (1 - n/2)/(j - n), -(a*x^(j - n))/b])/((2 - n)*Sqrt[a*x^j + b*x^n])

Rubi [A] time = 0.0511551, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2(j-n)}; \frac{1-\frac{n}{2}}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^j + b*x^n], x]

[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)), 1 + (1 - n/2)/(j - n), -(a*x^(j - n))/b])/((2 - n)*Sqrt[a*x^j + b*x^n])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^(m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax^j + bx^n}} dx &= \frac{\left(x^{n/2} \sqrt{b + ax^{j-n}}\right) \int \frac{x^{-n/2}}{\sqrt{b + ax^{j-n}}} dx}{\sqrt{ax^j + bx^n}} \\ &= \frac{\left(x^{n/2} \sqrt{1 + \frac{ax^{j-n}}{b}}\right) \int \frac{x^{-n/2}}{\sqrt{1 + \frac{ax^{j-n}}{b}}} dx}{\sqrt{ax^j + bx^n}} \\ &= \frac{2x \sqrt{1 + \frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2(j-n)}; 1 + \frac{1-n}{j-n}; -\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j + bx^n}} \end{aligned}$$

Mathematica [A] time = 0.0510112, size = 88, normalized size = 0.95

$$\frac{2x \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2(n-j)}; \frac{n-2}{2(n-j)} + 1; -\frac{ax^{j-n}}{b}\right)}{(n-2)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^j + b*x^n], x]

[Out] (-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n)), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*x^j+b*x^n)^(1/2),x)`

[Out] `int(1/(a*x^j+b*x^n)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(a*x^j + b*x^n), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x**j+b*x**n)**(1/2),x)`

[Out] Integral(1/sqrt(a*x**j + b*x**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(a*x^j + b*x^n), x)

$$3.429 \quad \int \frac{1}{(ax^j + bx^n)^{3/2}} dx$$

Optimal. Leaf size=101

$$\frac{2x^{1-n} \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{3}{2}, \frac{1-3n}{j-n}; \frac{1-3n}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j + bx^n}}$$

[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n))/2]/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)]/(b*(2 - 3*n)*Sqrt[a*x^j + b*x^n])

Rubi [A] time = 0.0568401, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2011, 365, 364}

$$\frac{2x^{1-n} \sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{3}{2}, \frac{1-3n}{j-n}; \frac{1-3n}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^j + b*x^n)^(-3/2), x]

[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n))/2]/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)]/(b*(2 - 3*n)*Sqrt[a*x^j + b*x^n])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]

&& !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^j + bx^n)^{3/2}} dx &= \frac{\left(x^{n/2} \sqrt{b + ax^{j-n}}\right) \int \frac{x^{-3n/2}}{(b+ax^{j-n})^{3/2}} dx}{\sqrt{ax^j + bx^n}} \\ &= \frac{\left(x^{n/2} \sqrt{1 + \frac{ax^{j-n}}{b}}\right) \int \frac{x^{-3n/2}}{\left(1 + \frac{ax^{j-n}}{b}\right)^{3/2}} dx}{b\sqrt{ax^j + bx^n}} \\ &= \frac{2x^{1-n} \sqrt{1 + \frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{3}{2}, \frac{1-3n}{j-n}; 1 + \frac{1-3n}{j-n}; -\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j + bx^n}} \end{aligned}$$

Mathematica [A] time = 0.0827265, size = 104, normalized size = 1.03

$$\frac{2x^{1-j} \left(\sqrt{\frac{ax^{j-n}}{b} + 1} {}_2F_1\left(\frac{1}{2}, -\frac{2j+n-2}{2(j-n)}; \frac{2-3n}{2j-2n}; -\frac{ax^{j-n}}{b}\right) - 1 \right)}{a(j-n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^j + b*x^n)^(-3/2), x]

[Out] (2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b])*Hypergeometric2F1[1/2, -(-2 + 2*j + n)/(2*(j - n)), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)])/(a*(j - n)*Sqrt[a*x^j + b*x^n])

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^j+b*x^n)^(3/2),x)

[Out] int(1/(a*x^j+b*x^n)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x^j + b*x^n)^(-3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**j+b*x**n)**(3/2),x)

[Out] Integral((a*x**j + b*x**n)**(-3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")

[Out] integrate((a*x^j + b*x^n)^(-3/2), x)

$$3.430 \quad \int \frac{1}{(ax^j + bx^n)^{5/2}} dx$$

Optimal. Leaf size=101

$$\frac{2x^{1-2n} \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{5}{2}, \frac{1-5n}{j-n}; \frac{1-5n}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j + bx^n}}$$

[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*Sqrt[a*x^j + b*x^n])

Rubi [A] time = 0.0565193, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2011, 365, 364}

$$\frac{2x^{1-2n} \sqrt{\frac{ax^{j-n}}{b}} + {}_2F_1\left(\frac{5}{2}, \frac{1-5n}{j-n}; \frac{1-5n}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j + bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^j + b*x^n)^(-5/2), x]

[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*Sqrt[a*x^j + b*x^n])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]

&& !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^j + bx^n)^{5/2}} dx &= \frac{\left(x^{n/2} \sqrt{b + ax^{j-n}}\right) \int \frac{x^{-5n/2}}{(b+ax^{j-n})^{5/2}} dx}{\sqrt{ax^j + bx^n}} \\ &= \frac{\left(x^{n/2} \sqrt{1 + \frac{ax^{j-n}}{b}}\right) \int \frac{x^{-5n/2}}{\left(1 + \frac{ax^{j-n}}{b}\right)^{5/2}} dx}{b^2 \sqrt{ax^j + bx^n}} \\ &= \frac{2x^{1-2n} \sqrt{1 + \frac{ax^{j-n}}{b}} {}_2F_1\left(\frac{5}{2}, \frac{1-5n}{j-n}; 1 + \frac{1-5n}{j-n}; -\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j + bx^n}} \end{aligned}$$

Mathematica [A] time = 0.243232, size = 185, normalized size = 1.83

$$\frac{2x^{1-2j} \left((8j^2 + 2j(7n-6) + 3n^2 - 8n + 4) \sqrt{\frac{ax^{j-n}}{b} + 1} (ax^j + bx^n) {}_2F_1\left(\frac{1}{2}, -\frac{4j+n-2}{2(j-n)}; \frac{-2j-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right) - (4j+n-2)(a(j+4n) + b) \sqrt{ax^j + bx^n} \right)}{3a^2(-4j-n+2)(j-n)^2 (ax^j + bx^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^j + b*x^n)^(-5/2), x]

[Out] (2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n)*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n))/b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -(-2 + 4*j + n)/(2*(j - n)), (2 - 2*j - 3*n)/(2*j - 2*n), -(a*x^(j - n))/b])))/(3*a^2*(2 - 4*j - n)*(j - n)^2*(a*x^j + b*x^n)^(3/2))

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^j+b*x^n)^(5/2),x)

[Out] int(1/(a*x^j+b*x^n)^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate((a*x^j + b*x^n)^(-5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

$$3.431 \quad \int \sqrt{\frac{1+x}{x^5}} dx$$

Optimal. Leaf size=18

$$-\frac{2}{3} \left(\frac{1}{x^4} + \frac{1}{x^5} \right)^{3/2} x^6$$

[Out] $(-2*(x^{(-5)} + x^{(-4)})^{(3/2)}*x^6)/3$

Rubi [A] time = 0.006895, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1979, 2000}

$$-\frac{2}{3} \left(\frac{1}{x^4} + \frac{1}{x^5} \right)^{3/2} x^6$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x)/x^5], x]

[Out] $(-2*(x^{(-5)} + x^{(-4)})^{(3/2)}*x^6)/3$

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1+x}{x^5}} dx &= \int \sqrt{\frac{1}{x^5} + \frac{1}{x^4}} dx \\ &= -\frac{2}{3} \left(\frac{1}{x^5} + \frac{1}{x^4} \right)^{3/2} x^6 \end{aligned}$$

Mathematica [A] time = 0.0052456, size = 19, normalized size = 1.06

$$-\frac{2}{3}x(x+1)\sqrt{\frac{x+1}{x^5}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x)/x^5], x]

[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3

Maple [A] time = 0.024, size = 16, normalized size = 0.9

$$-\frac{2x(1+x)}{3}\sqrt{\frac{1+x}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)/x^5)^(1/2), x)

[Out] -2/3*x*(1+x)*((1+x)/x^5)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{x^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x^5)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt((x + 1)/x^5), x)

Fricas [A] time = 0.92097, size = 46, normalized size = 2.56

$$-\frac{2}{3}(x^2+x)\sqrt{\frac{x+1}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")`

[Out] `-2/3*(x^2 + x)*sqrt((x + 1)/x^5)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{x^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x)/x**5)**(1/2),x)`

[Out] `Integral(sqrt((x + 1)/x**5), x)`

Giac [B] time = 1.2883, size = 68, normalized size = 3.78

$$\frac{2 \left(3 \left(x - \sqrt{x^2 + x} \right)^2 \operatorname{sgn}(x) + 3 \left(x - \sqrt{x^2 + x} \right) \operatorname{sgn}(x) + \operatorname{sgn}(x) \right)}{3 \left(x - \sqrt{x^2 + x} \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")`

[Out] `2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x))/ (x - sqrt(x^2 + x))^3`

3.432

$$\int \sqrt{x + x^{5/2}} dx$$

Optimal. Leaf size=20

$$\frac{4(x^{5/2} + x)^{3/2}}{9x^{3/2}}$$

[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))

Rubi [A] time = 0.003314, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2000}

$$\frac{4(x^{5/2} + x)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + x^(5/2)],x]

[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int \sqrt{x + x^{5/2}} dx = \frac{4(x + x^{5/2})^{3/2}}{9x^{3/2}}$$

Mathematica [A] time = 0.0099119, size = 20, normalized size = 1.

$$\frac{4(x^{5/2} + x)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + x^(5/2)],x]

[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$\frac{4}{9} \sqrt{x + x^{\frac{5}{2}}} \left(1 + x^{\frac{3}{2}}\right) \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+x^(5/2))^(1/2),x)

[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(1+x^(3/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{5}{2}} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^(5/2) + x), x)

Fricas [A] time = 1.1921, size = 55, normalized size = 2.75

$$\frac{4 \sqrt{x^{\frac{5}{2}} + x} (x^2 + \sqrt{x})}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")

[Out] $4/9*\sqrt{x^{5/2} + x}*(x^2 + \sqrt{x})/x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{5}{2}} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+x**(5/2))**(1/2),x)`

[Out] `Integral(sqrt(x**(5/2) + x), x)`

Giac [A] time = 1.27574, size = 15, normalized size = 0.75

$$\frac{4}{9} \left(x^{\frac{3}{2}} + 1 \right)^{\frac{3}{2}} - \frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+x^(5/2))^(1/2),x, algorithm="giac")`

[Out] $4/9*(x^{3/2} + 1)^{3/2} - 4/9$

$$3.433 \quad \int \frac{1}{\sqrt{x+x^{3/2}}} dx$$

Optimal. Leaf size=8

$$2 \tan^{-1}(\sqrt{x})$$

[Out] 2*ArcTan[Sqrt[x]]

Rubi [A] time = 0.003541, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 63, 203}

$$2 \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] + x^(3/2))^(-1), x]

[Out] 2*ArcTan[Sqrt[x]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} + x^{3/2}} dx &= \int \frac{1}{\sqrt{x}(1+x)} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{x} \right) \\ &= 2 \tan^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.0018928, size = 8, normalized size = 1.

$$2 \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] + x^(3/2))^(-1), x]

[Out] 2*ArcTan[Sqrt[x]]

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)+x^(1/2)), x)

[Out] 2*arctan(x^(1/2))

Maxima [A] time = 1.71752, size = 8, normalized size = 1.

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(3/2)+x^(1/2)), x, algorithm="maxima")

[Out] 2*arctan(sqrt(x))

Fricas [A] time = 0.958751, size = 26, normalized size = 3.25

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")

[Out] 2*arctan(sqrt(x))

Sympy [A] time = 0.231157, size = 7, normalized size = 0.88

$$2 \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**(3/2)+x**(1/2)),x)

[Out] 2*atan(sqrt(x))

Giac [A] time = 1.32652, size = 8, normalized size = 1.

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")

[Out] 2*arctan(sqrt(x))

3.434 $\int x\sqrt{x^2(a+bx^3)} dx$

Optimal. Leaf size=25

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

[Out] $(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)$

Rubi [A] time = 0.010443, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[x^2*(a + b*x^3)],x]`

[Out] $(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)$

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{x^2(a+bx^3)} dx = \frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.0119565, size = 25, normalized size = 1.

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]

[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)

Maple [A] time = 0.005, size = 29, normalized size = 1.2

$$\frac{2bx^3 + 2a}{9bx} \sqrt{x^2(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^2*(b*x^3+a))^(1/2),x)

[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x

Maxima [A] time = 1.24923, size = 19, normalized size = 0.76

$$\frac{2(bx^3 + a)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")

[Out] 2/9*(b*x^3 + a)^(3/2)/b

Fricas [A] time = 0.908743, size = 58, normalized size = 2.32

$$\frac{2\sqrt{bx^5 + ax^2}(bx^3 + a)}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")

[Out] $2/9*\text{sqrt}(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2*(b*x**3+a))**(1/2),x)`

[Out] Timed out

Giac [A] time = 1.45822, size = 36, normalized size = 1.44

$$\frac{2(bx^3 + a)^{\frac{3}{2}}\text{sgn}(x)}{9b} - \frac{2a^{\frac{3}{2}}\text{sgn}(x)}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")`

[Out] $2/9*(b*x^3 + a)^{(3/2)}*\text{sgn}(x)/b - 2/9*a^{(3/2)}*\text{sgn}(x)/b$

$$3.435 \quad \int x\sqrt{ax^2 + bx^5} dx$$

Optimal. Leaf size=25

$$\frac{2(ax^2 + bx^5)^{3/2}}{9bx^3}$$

[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)

Rubi [A] time = 0.0105747, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$\frac{2(ax^2 + bx^5)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a*x^2 + b*x^5],x]

[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{ax^2 + bx^5} dx = \frac{2(ax^2 + bx^5)^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.0023885, size = 25, normalized size = 1.

$$\frac{2(x^2(a + bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]

[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)

Maple [A] time = 0.002, size = 29, normalized size = 1.2

$$\frac{2bx^3 + 2a}{9bx} \sqrt{bx^5 + ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^5+a*x^2)^(1/2),x)

[Out] 2/9*(b*x^3+a)*(b*x^5+a*x^2)^(1/2)/b/x

Maxima [A] time = 1.13338, size = 19, normalized size = 0.76

$$\frac{2(bx^3 + a)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")

[Out] 2/9*(b*x^3 + a)^(3/2)/b

Fricas [A] time = 0.937805, size = 58, normalized size = 2.32

$$\frac{2\sqrt{bx^5 + ax^2}(bx^3 + a)}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $2/9*\sqrt{b*x^5 + a*x^2}*(b*x^3 + a)/(b*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2(a+bx^3)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**5+a*x**2)**(1/2),x)`

[Out] `Integral(x*sqrt(x**2*(a + b*x**3)), x)`

Giac [A] time = 1.17149, size = 36, normalized size = 1.44

$$\frac{2(bx^3 + a)^{\frac{3}{2}}\operatorname{sgn}(x)}{9b} - \frac{2a^{\frac{3}{2}}\operatorname{sgn}(x)}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

[Out] $2/9*(b*x^3 + a)^{(3/2)}*\operatorname{sgn}(x)/b - 2/9*a^{(3/2)}*\operatorname{sgn}(x)/b$

3.436 $\int \sqrt{x^4 (a + bx^3)} dx$

Optimal. Leaf size=25

$$\frac{2(ax^4 + bx^7)^{3/2}}{9bx^6}$$

[Out] $(2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)$

Rubi [A] time = 0.0078477, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1979, 2000}

$$\frac{2(ax^4 + bx^7)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x^4*(a + b*x^3)],x]`

[Out] $(2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)$

Rule 1979

`Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]`

Rule 2000

`Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]`

Rubi steps

$$\begin{aligned} \int \sqrt{x^4 (a + bx^3)} dx &= \int \sqrt{ax^4 + bx^7} dx \\ &= \frac{2(ax^4 + bx^7)^{3/2}}{9bx^6} \end{aligned}$$

Mathematica [A] time = 0.010894, size = 25, normalized size = 1.

$$\frac{2(x^4(a + bx^3))^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^4*(a + b*x^3)], x]

[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)

Maple [A] time = 0.004, size = 29, normalized size = 1.2

$$\frac{2bx^3 + 2a}{9bx^2} \sqrt{x^4(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(b*x^3+a))^(1/2), x)

[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2

Maxima [A] time = 1.1652, size = 19, normalized size = 0.76

$$\frac{2(bx^3 + a)^{3/2}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4*(b*x^3+a))^(1/2), x, algorithm="maxima")

[Out] 2/9*(b*x^3 + a)^(3/2)/b

Fricas [A] time = 0.918432, size = 61, normalized size = 2.44

$$\frac{2\sqrt{bx^7 + ax^4}(bx^3 + a)}{9bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")

[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4*(b*x**3+a))**(1/2),x)

[Out] Timed out

Giac [A] time = 1.24237, size = 19, normalized size = 0.76

$$\frac{2(bx^3 + a)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")

[Out] 2/9*(b*x^3 + a)^(3/2)/b

$$3.437 \quad \int \frac{1}{\sqrt[3]{a}\sqrt[3]{x+bx^{2/3}}} dx$$

Optimal. Leaf size=988

result too large to display

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-(b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3))*(-(
b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3))*(-(b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^(
2)*(-(b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
- 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/
3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)], -7 + 4*Sqrt[3]]/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)]/(1 -
Sqrt[3] - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3))*(-(
b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3))*(-(b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3)]/(1 - Sqrt[3] - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^(
2)*(-(b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
- 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/
3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)], -7 + 4*Sqrt[3]]/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3)]/(1 -
Sqrt[3] - 2^(2/3))*(-(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Rubi [A] time = 2.24545, antiderivative size = 988, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.526, Rules used = {2011, 341, 50, 61, 622, 619, 235, 304, 219, 1879}

$$\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b(\sqrt[3]{xa+bx^{2/3}})}{a^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}}{-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}}\right)}{28\sqrt[3]{2}b^3\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b(a+b\sqrt[3]{x})\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}(a+2b\sqrt[3]{x})\sqrt[3]{\sqrt[3]{xa+bx^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^(1/3) + b*x^(2/3))^(1/3), x]

[Out]
$$\frac{(-45a^2(a + 2bx^{1/3}) * (-((b(a x^{1/3} + b x^{2/3}))/a^2))^{1/3}) / (14 * 2^{1/3} * b^3 * (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3})) * (a x^{1/3} + b x^{2/3})^{1/3} - (45a * (a + b x^{1/3}) * x^{1/3}) / (28 * b^2 * (a x^{1/3} + b x^{2/3})^{1/3}) + (9 * (a + b x^{1/3}) * x^{2/3}) / (7 * b * (a x^{1/3} + b x^{2/3})^{1/3}) - (45 * 3^{1/4} * \sqrt{2 + \sqrt{3}}] * a^4 * (1 - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) * \sqrt{(1 + 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3} + 2 * 2^{1/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{2/3})} / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}))^2 * (-((b(a x^{1/3} + b x^{2/3}))/a^2))^{1/3} * \text{EllipticE}[\text{ArcSin}[(1 + \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3})], -7 + 4 * \sqrt{3}]] / (28 * 2^{1/3} * b^3 * \sqrt{-(1 - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}))^2} * (a + 2 * b * x^{1/3}) * (a x^{1/3} + b x^{2/3})^{1/3} + (15 * 3^{3/4} * a^4 * (1 - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) * \sqrt{(1 + 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3} + 2 * 2^{1/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{2/3})} / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}))^2 * (-((b(a x^{1/3} + b x^{2/3}))/a^2))^{1/3} * \text{EllipticF}[\text{ArcSin}[(1 + \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3})], -7 + 4 * \sqrt{3}]] / (7 * 2^{5/6} * b^3 * \sqrt{-(1 - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}) / (1 - \sqrt{3} - 2^{2/3} * (-((b(a + b x^{1/3}) * x^{1/3})/a^2))^{1/3}))^2} * (a + 2 * b * x^{1/3}) * (a x^{1/3} + b x^{2/3})^{1/3})$$

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x

$(j*p)*(a + b*x^{(n - j)})^p, x], x] /;$ FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 341

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 61

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Dist[(a + b*x)^m*(c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]

Rule 622

Int[((b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 235

Int[((a_) + (b_)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} dx &= \frac{\left(\sqrt[3]{a + b\sqrt[3]{x}\sqrt[9]{x}}\right) \int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}\sqrt[9]{x}}} dx}{\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= \frac{\left(3\sqrt[3]{a + b\sqrt[3]{x}\sqrt[9]{x}}\right) \text{Subst}\left(\int \frac{x^{5/3}}{\sqrt[3]{a+bx}} dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} - \frac{\left(15a\sqrt[3]{a + b\sqrt[3]{x}\sqrt[9]{x}}\right) \text{Subst}\left(\int \frac{x^{2/3}}{\sqrt[3]{a+bx}} dx, x, \sqrt[3]{x}\right)}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{a + b\sqrt[3]{x}\sqrt[9]{x}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{a+bx}} dx, x, \sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{(15a^2) \text{Subst}\left(\int \frac{1}{\sqrt[3]{ax+bx^2}} dx, x, \sqrt[3]{x}\right)}{14b^2} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{bx}{a} - \frac{b^2x^2}{a^2}}} dx, x, \sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} - \frac{\left(15a^4\sqrt[3]{-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{a^2x^2}{b^2}}} dx, x, -\frac{b(a+bx^{2/3})}{a}\right)}{14\sqrt[3]{2}b^4\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} - \frac{\left(45a^4\sqrt[3]{-\frac{(a+2b\sqrt[3]{x})^2}{a^2}}\sqrt[3]{-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}}\right) \text{Subst}\left(\int \frac{x}{\sqrt{-1+x^2}} dx, x, \sqrt[3]{x}\right)}{28\sqrt[3]{2}b^3(a + 2b\sqrt[3]{x})\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{\left(45a^4\sqrt[3]{-\frac{(a+2b\sqrt[3]{x})^2}{a^2}}\sqrt[3]{-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}}\right) \text{Subst}\left(\int \frac{1+\sqrt{3-x}}{\sqrt{-1+x^2}} dx, x, \sqrt[3]{x}\right)}{28\sqrt[3]{2}b^3(a + 2b\sqrt[3]{x})\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} \\
&= -\frac{45a^2(a + 2b\sqrt[3]{x})\sqrt[3]{-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}}}{14\sqrt[3]{2}b^3\left(1 - \sqrt{3} - \sqrt[3]{1 - \frac{(a+2b\sqrt[3]{x})^2}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} - \frac{45a(a + b\sqrt[3]{x})\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}} + \frac{9(a + b\sqrt[3]{x})x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x} + bx^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.0185253, size = 61, normalized size = 0.06

$$\frac{9x\sqrt[3]{\frac{b\sqrt[3]{x}}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{11}{3}; -\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\sqrt[3]{x}(a + b\sqrt[3]{x})}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(1/3), x]

[Out] (9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -(b*x^(1/3)/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x} + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3), x)

[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3), x, algorithm="maxima")

[Out] integrate((b*x^(2/3) + a*x^(1/3))^(1/3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x} + bx^{\frac{2}{3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)

[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")

[Out] integrate((b*x^(2/3) + a*x^(1/3))^(1/3), x)

$$3.438 \quad \int \frac{1}{(a\sqrt[3]{x}+bx^{2/3})^{2/3}} dx$$

Optimal. Leaf size=487

$$6\sqrt[3]{23}^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}\right)^{2/3}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}\right)}{\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1\right)^2}}(a+2b\sqrt[3]{x})(a\sqrt[3]{x}+bx^{2/3})^{2/3}}}\right)}{5b^3\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1\right)^2}}(a+2b\sqrt[3]{x})(a\sqrt[3]{x}+bx^{2/3})^{2/3}}}$$

[Out] $(-18*a*(a + b*x^{(1/3)})*x^{(1/3)})/(5*b^2*(a*x^{(1/3)} + b*x^{(2/3)})^{(2/3)}) + (9*(a + b*x^{(1/3)})*x^{(2/3)})/(5*b*(a*x^{(1/3)} + b*x^{(2/3)})^{(2/3)}) + (6*2^{(1/3)}*3^{(3/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^4*(1 - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)}))*\text{Sqrt}[(1 + 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)} + 2*2^{(1/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(2/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)})^2]*(-((b*(a*x^{(1/3)} + b*x^{(2/3)}))/a^2))^{(2/3)}*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(5*b^3*\text{Sqrt}[-((1 - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*(-((b*(a + b*x^{(1/3)})*x^{(1/3)})/a^2))^{(1/3)})^2)]*(a + 2*b*x^{(1/3)})*(a*x^{(1/3)} + b*x^{(2/3)})^{(2/3)})$

Rubi [A] time = 0.973875, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2011, 341, 50, 61, 622, 619, 236, 219}

$$6\sqrt[3]{23}^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}\right)^{2/3}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}}{-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1}\right)}{5b^3\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}(a+b\sqrt[3]{x})}{a^2}}-\sqrt{3}+1\right)^2}}(a+2b\sqrt[3]{x})(a\sqrt[3]{x}+bx^{2/3})^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^(1/3) + b*x^(2/3))^(2/3), x]

[Out]
$$\begin{aligned} & (-18*a*(a + b*x^{1/3})*x^{1/3})/(5*b^2*(a*x^{1/3} + b*x^{2/3})^{2/3}) + (9*(a + b*x^{1/3})*x^{2/3})/(5*b*(a*x^{1/3} + b*x^{2/3})^{2/3}) + (6*2^{1/3}*3^{3/4}*\sqrt{2 - \sqrt{3}}*a^4*(1 - 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})*\sqrt{(1 + 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3} + 2*2^{1/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{2/3})/(1 - \sqrt{3} - 2^{2/3}) * (-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})^2 * (-((b*(a*x^{1/3} + b*x^{2/3}))/a^2))^{2/3} * \text{EllipticF}[\text{ArcSin}[(1 + \sqrt{3} - 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})/(1 - \sqrt{3} - 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})], -7 + 4*\sqrt{3}]/(5*b^3*\sqrt{-((1 - 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})/(1 - \sqrt{3} - 2^{2/3})*(-((b*(a + b*x^{1/3})*x^{1/3})/a^2))^{1/3})^2}*(a + 2*b*x^{1/3})*(a*x^{1/3} + b*x^{2/3})^{2/3}) \end{aligned}$$

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 341

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 61

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[(a + b*x)^m*(c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,

0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]

Rule 622

Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[-((c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 236

Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]

Rule 219

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a\sqrt[3]{x} + bx^{2/3})^{2/3}} dx &= \frac{\left((a + b\sqrt[3]{x})^{2/3} x^{2/9}\right) \int \frac{1}{(a+b\sqrt[3]{x})^{2/3} x^{2/9}} dx}{(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= \frac{\left(3(a + b\sqrt[3]{x})^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{x^{4/3}}{(a+bx)^{2/3}} dx, x, \sqrt[3]{x}\right)}{(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} - \frac{\left(12a(a + b\sqrt[3]{x})^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{\sqrt[3]{x}}{(a+bx)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{\left(6a^2(a + b\sqrt[3]{x})^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{1}{x^{2/3}(a+bx)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{(6a^2) \text{Subst}\left(\int \frac{1}{(ax+bx^2)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b^2} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{\left(6a^2\left(-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(-\frac{bx}{a} - \frac{b^2x^2}{a^2}\right)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} - \frac{\left(6\sqrt[3]{2}a^4\left(-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{a^2x^2}{b^2}\right)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b^4(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} - \frac{\left(9\sqrt[3]{2}a^4\sqrt{-\frac{(a+2b\sqrt[3]{x})^2}{a^2}}\left(-\frac{b(a\sqrt[3]{x}+bx^{2/3})}{a^2}\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{a^2x^2}{b^2}\right)^{2/3}} dx, x, \sqrt[3]{x}\right)}{5b^3(a + 2b\sqrt[3]{x})(a\sqrt[3]{x} + bx^{2/3})^{2/3}} \\
&= -\frac{18a(a + b\sqrt[3]{x}) \sqrt[3]{x}}{5b^2(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{9(a + b\sqrt[3]{x}) x^{2/3}}{5b(a\sqrt[3]{x} + bx^{2/3})^{2/3}} + \frac{6\sqrt[3]{2}3^{3/4}\sqrt{2 - \sqrt{3}}a^4\left(1 - \sqrt[3]{1 - \frac{(a+2b\sqrt[3]{x})^2}{a^2}}\right)}{5b^3(a + 2b\sqrt[3]{x})(a\sqrt[3]{x} + bx^{2/3})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0159076, size = 61, normalized size = 0.13

$$\frac{9x \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^{2/3} {}_2F_1 \left(\frac{2}{3}, \frac{7}{3}; \frac{10}{3}; -\frac{b\sqrt[3]{x}}{a} \right)}{7 \left(\sqrt[3]{x} (a + b\sqrt[3]{x}) \right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(2/3),x]

[Out] (9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -(b*x^(1/3)/a)])/ (7*((a + b*x^(1/3))*x^(1/3))^(2/3))

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \left(a\sqrt[3]{x} + bx^{\frac{2}{3}} \right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)

[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}} \right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")

[Out] integrate((b*x^(2/3) + a*x^(1/3))^(2/3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sqrt[3]{x} + bx^{\frac{2}{3}}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)

[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")

[Out] integrate((b*x^(2/3) + a*x^(1/3))^(2/3), x)

3.439 $\int x^m (ax^j + bx^n)^p dx$

Optimal. Leaf size=89

$$\frac{x^{m+1} (a + bx^{n-j}) (ax^j + bx^n)^p {}_2F_1\left(1, p + \frac{m+jp+1}{n-j} + 1; \frac{m+jp+1}{n-j} + 1; -\frac{bx^{n-j}}{a}\right)}{a(jp + m + 1)}$$

[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*(a + b*x^(-j + n))*Hypergeometric2F1[1, 1 + p + (1 + m + j*p)/(-j + n), 1 + (1 + m + j*p)/(-j + n), -((b*x^(-j + n))/a)])/(a*(1 + m + j*p))

Rubi [A] time = 0.0683331, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2032, 365, 364}

$$\frac{x^{m+1} \left(\frac{ax^{j-n}}{b} + 1\right)^{-p} (ax^j + bx^n)^p {}_2F_1\left(-p, \frac{m+np+1}{j-n}; \frac{m+np+1}{j-n} + 1; -\frac{ax^{j-n}}{b}\right)}{m + np + 1}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a*x^j + b*x^n)^p,x]

[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j - n))/b)^p)

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
```

&& !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int x^m (ax^j + bx^n)^p dx &= \left(x^{-np} (b + ax^{j-n})^{-p} (ax^j + bx^n)^p \right) \int x^{m+np} (b + ax^{j-n})^p dx \\ &= \left(x^{-np} \left(1 + \frac{ax^{j-n}}{b} \right)^{-p} (ax^j + bx^n)^p \right) \int x^{m+np} \left(1 + \frac{ax^{j-n}}{b} \right)^p dx \\ &= \frac{x^{1+m} \left(1 + \frac{ax^{j-n}}{b} \right)^{-p} (ax^j + bx^n)^p {}_2F_1 \left(-p, \frac{1+m+np}{j-n}; 1 + \frac{1+m+np}{j-n}; -\frac{ax^{j-n}}{b} \right)}{1 + m + np} \end{aligned}$$

Mathematica [A] time = 0.103634, size = 92, normalized size = 1.03

$$\frac{x^{m+1} \left(\frac{ax^{j-n}}{b} + 1 \right)^{-p} (ax^j + bx^n)^p {}_2F_1 \left(-p, \frac{m+np+1}{j-n}; \frac{m+np+1}{j-n} + 1; -\frac{ax^{j-n}}{b} \right)}{m + np + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a*x^j + b*x^n)^p,x]

[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1 + (1 + m + n*p)/(j - n), -(a*x^(j - n))/b])/((1 + m + n*p)*(1 + (a*x^(j - n))/b)^p)

Maple [F] time = 0.582, size = 0, normalized size = 0.

$$\int x^m (ax^j + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(a*x^j+b*x^n)^p,x)`

[Out] `int(x^m*(a*x^j+b*x^n)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^p x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")`

[Out] `integrate((a*x^j + b*x^n)^p*x^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ax^j + bx^n\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")`

[Out] `integral((a*x^j + b*x^n)^p*x^m, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m (ax^j + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(a*x**j+b*x**n)**p,x)`

[Out] `Integral(x**m*(a*x**j + b*x**n)**p, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^j + bx^n)^p x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

$$3.440 \quad \int x^{-1-pq} (bx^n + ax^q)^p dx$$

Optimal. Leaf size=69

$$-\frac{x^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p {}_2F_1\left(1, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a(p+1)(n-q)}$$

[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))

Rubi [A] time = 0.0683911, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2032, 266, 65}

$$-\frac{x^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p {}_2F_1\left(1, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a(p+1)(n-q)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]

[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-pq} (bx^n + ax^q)^p dx &= (x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p) \int \frac{(a + bx^{n-q})^p}{x} dx \\ &= \frac{(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p) \operatorname{Subst}\left(\int \frac{(a+bx)^p}{x} dx, x, x^{n-q}\right)}{n-q} \\ &= -\frac{x^{-pq} (a + bx^{n-q}) (bx^n + ax^q)^p {}_2F_1\left(1, 1 + p; 2 + p; 1 + \frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)} \end{aligned}$$

Mathematica [A] time = 0.116537, size = 73, normalized size = 1.06

$$\frac{x^{-pq} (ax^q + bx^n)^p \left(\frac{ax^{q-n}}{b} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1 - p; -\frac{ax^{q-n}}{b}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]

[Out] ((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/(p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)

Maple [F] time = 0.632, size = 0, normalized size = 0.

$$\int x^{-pq-1} (bx^n + ax^q)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)

[Out] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-pq-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")

[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^n + ax^q)^p x^{-pq-1}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")

[Out] integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-pq-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

3.441 $\int x^{-1-np} (bx^n + ax^q)^p dx$

Optimal. Leaf size=66

$$\frac{x^{-np} (a + bx^{n-q}) (ax^q + bx^n)^p {}_2F_1\left(1, 1; 1 - p; -\frac{bx^{n-q}}{a}\right)}{ap(n - q)}$$

[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1, 1 - p, -((b*x^(n - q))/a)])/(a*p*(n - q)*x^(n*p)))

Rubi [A] time = 0.0675878, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2032, 365, 364}

$$\frac{x^{-np} \left(\frac{bx^{n-q}}{a} + 1\right)^{-p} (ax^q + bx^n)^p {}_2F_1\left(-p, -p; 1 - p; -\frac{bx^{n-q}}{a}\right)}{p(n - q)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]

[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]], Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-np} (bx^n + ax^q)^p dx &= \left(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p \right) \int x^{-1-np+pq} (a + bx^{n-q})^p dx \\ &= \left(x^{-pq} \left(1 + \frac{bx^{n-q}}{a} \right)^{-p} (bx^n + ax^q)^p \right) \int x^{-1-np+pq} \left(1 + \frac{bx^{n-q}}{a} \right)^p dx \\ &= -\frac{x^{-np} \left(1 + \frac{bx^{n-q}}{a} \right)^{-p} (bx^n + ax^q)^p {}_2F_1\left(-p, -p; 1-p; -\frac{bx^{n-q}}{a}\right)}{p(n-q)} \end{aligned}$$

Mathematica [A] time = 0.0955013, size = 74, normalized size = 1.12

$$-\frac{x^{-np} \left(\frac{bx^{n-q}}{a} + 1 \right)^{-p} (ax^q + bx^n)^p {}_2F_1\left(-p, -p; 1-p; -\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F] time = 0.511, size = 0, normalized size = 0.

$$\int x^{-np-1} (bx^n + ax^q)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-np-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")

[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^n + ax^q)^p x^{-np-1}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")

[Out] integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-np-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

$$3.442 \quad \int x^{-1-n-(-1+p)q} (bx^n + ax^q)^p dx$$

Optimal. Leaf size=69

$$\frac{bx^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p {}_2F_1\left(2, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a^2(p+1)(n-q)}$$

[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))

Rubi [A] time = 0.077541, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2032, 266, 65}

$$\frac{bx^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p {}_2F_1\left(2, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a^2(p+1)(n-q)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]

[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-n-(-1+p)q} (bx^n + ax^q)^p dx &= (x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p) \int x^{-1-n-(-1+p)q+pq} (a + bx^{n-q})^p dx \\ &= \frac{(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p) \text{Subst}\left(\int \frac{(a+bx)^p}{x^2} dx, x, x^{n-q}\right)}{n - q} \\ &= \frac{bx^{-pq} (a + bx^{n-q}) (bx^n + ax^q)^p {}_2F_1\left(2, 1 + p; 2 + p; 1 + \frac{bx^{n-q}}{a}\right)}{a^2(1 + p)(n - q)} \end{aligned}$$

Mathematica [A] time = 0.159438, size = 82, normalized size = 1.19

$$\frac{x^{-n-pq+q} (ax^q + bx^n)^p \left(\frac{ax^{q-n}}{b} + 1\right)^{-p} {}_2F_1\left(1 - p, -p; 2 - p; -\frac{ax^{q-n}}{b}\right)}{(p - 1)(n - q)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]

[Out] (x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x^(-n + q))/b])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int x^{-1-n-(p-1)q} (bx^n + ax^q)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)

[Out] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-(p-1)q-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")

[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + ax^q)^p x^{-(p-1)q-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")

[Out] integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-(p-1)q-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

3.443 $\int x^{-1-n(-1+p)-q} (bx^n + ax^q)^p dx$

Optimal. Leaf size=84

$$\frac{x^{n(-p)+n-q} \left(\frac{bx^{n-q}}{a} + 1\right)^{-p} (ax^q + bx^n)^p {}_2F_1\left(1-p, -p; 2-p; -\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

[Out] $(x^{(n - n*p - q)}*(b*x^n + a*x^q))^p \text{Hypergeometric2F1}[1 - p, -p, 2 - p, -((b*x^{(n - q)})/a)]/(((1 - p)*(n - q)*(1 + (b*x^{(n - q)})/a))^p)$

Rubi [A] time = 0.081219, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2032, 365, 364}

$$\frac{x^{n(-p)+n-q} \left(\frac{bx^{n-q}}{a} + 1\right)^{-p} (ax^q + bx^n)^p {}_2F_1\left(1-p, -p; 2-p; -\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(-1 - n*(-1 + p) - q)}*(b*x^n + a*x^q)^p, x]$

[Out] $(x^{(n - n*p - q)}*(b*x^n + a*x^q))^p \text{Hypergeometric2F1}[1 - p, -p, 2 - p, -((b*x^{(n - q)})/a)]/(((1 - p)*(n - q)*(1 + (b*x^{(n - q)})/a))^p)$

Rule 2032

$\text{Int}[\left((c_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(j_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}\right), x_Symbol] \rightarrow \text{Dist}[\left(c \text{IntPart}[m]*(c*x)^{\text{FracPart}[m]}*(a*x^j + b*x^n)^{\text{FracPart}[p]}\right)/(x^{(\text{FracPart}[m] + j*\text{FracPart}[p])}*(a + b*x^{(n - j)})^{\text{FracPart}[p]})], \text{Int}[x^{(m + j*p)}*(a + b*x^{(n - j)})^p, x], x] /; \text{FreeQ}[\{a, b, c, j, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{NeQ}[n, j] \&\& \text{PosQ}[n - j]$

Rule 365

$\text{Int}[\left((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}\right), x_Symbol] \rightarrow \text{Dist}[\left(a \text{IntPart}[p]*(a + b*x^n)^{\text{FracPart}[p]}\right)/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[\left(c*x^{m*(1 + (b*x^n)/a)}\right)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& !\text{IGtQ}[p, 0] \&\& !(\text{ILtQ}[p, 0] \parallel \text{GtQ}[a, 0])$

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-n(-1+p)-q} (bx^n + ax^q)^p dx &= (x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p) \int x^{-1-n(-1+p)-q+pq} (a + bx^{n-q})^p dx \\ &= \left(x^{-pq} \left(1 + \frac{bx^{n-q}}{a} \right)^{-p} (bx^n + ax^q)^p \right) \int x^{-1-n(-1+p)-q+pq} \left(1 + \frac{bx^{n-q}}{a} \right)^p dx \\ &= \frac{x^{n-np-q} \left(1 + \frac{bx^{n-q}}{a} \right)^{-p} (bx^n + ax^q)^p {}_2F_1 \left(1 - p, -p; 2 - p; -\frac{bx^{n-q}}{a} \right)}{(1-p)(n-q)} \end{aligned}$$

Mathematica [A] time = 0.131903, size = 83, normalized size = 0.99

$$\frac{x^{n(-p)+n-q} \left(\frac{bx^{n-q}}{a} + 1 \right)^{-p} (ax^q + bx^n)^p {}_2F_1 \left(1 - p, -p; 2 - p; -\frac{bx^{n-q}}{a} \right)}{(p-1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F] time = 0.528, size = 0, normalized size = 0.

$$\int x^{-1-n(p-1)-q} (bx^n + ax^q)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-n(p-1)-q-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")

[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + ax^q)^p x^{-np+n-q-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")

[Out] integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-n(p-1)-q-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

$$3.444 \quad \int (ax^m + bx^{1+m+mp})^p dx$$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+1})^{p+1}}{b(p+1)(mp+1)}$$

[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))

Rubi [A] time = 0.0154702, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2000}

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+1})^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m + b*x^(1 + m + m*p))^p, x]

[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p+1)/(b*(n-j)*(p+1)*x^(n-1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int (ax^m + bx^{1+m+mp})^p dx = \frac{x^{-m(1+p)} (ax^m + bx^{1+m+mp})^{1+p}}{b(1+p)(1+mp)}$$

Mathematica [A] time = 0.0350637, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)} (x^m (a + bx^{mp+1}))^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p, x]

[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))

Maple [F] time = 0.547, size = 0, normalized size = 0.

$$\int (ax^m + bx^{mp+m+1})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m+b*x^(m*p+m+1))^p,x)

[Out] int((a*x^m+b*x^(m*p+m+1))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^{mp+m+1} + ax^m)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")

[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)

Fricas [A] time = 0.754974, size = 147, normalized size = 3.34

$$\frac{(bxx^{mp+m+1} + axx^m)(bx^{mp+m+1} + ax^m)^p}{(bmp^2 + (bm + b)p + b)x^{mp+m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")

[Out] $(b*x*x^{(m*p + m + 1)} + a*x*x^m)*(b*x^{(m*p + m + 1)} + a*x^m)^p / ((b*m*p^2 + (b*m + b)*p + b)*x^{(m*p + m + 1)})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x**m+b*x**(m*p+m+1))**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^{mp+m+1} + ax^m)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")`

[Out] `integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)`

$$3.445 \quad \int \left(x^m (a + bx^{1+mp}) \right)^p dx$$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+1})^{p+1}}{b(p+1)(mp+1)}$$

[Out] $(a*x^m + b*x^{(1+m+mp)})^{(1+p)}/(b*(1+p)*(1+mp)*x^{(m*(1+p))})$

Rubi [A] time = 0.0150384, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1979, 2000}

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+1})^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^m(a + b*x^{(1+m*mp)}))^p, x]$

[Out] $(a*x^m + b*x^{(1+m+mp)})^{(1+p)}/(b*(1+p)*(1+mp)*x^{(m*(1+p))})$

Rule 1979

$\text{Int}[(u_)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandToSum}[u, x]^{(p)}, x] /;$ FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

$\text{Int}[(a_)*(x_)^{(j_)} + (b_)*(x_)^{(n_)}]^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a*x^j + b*x^n)^{(p+1)}/(b*(n-j)*(p+1)*x^{(n-1)}), x] /;$ FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned} \int \left(x^m (a + bx^{1+mp}) \right)^p dx &= \int (ax^m + bx^{1+m+mp})^p dx \\ &= \frac{x^{-m(1+p)} (ax^m + bx^{1+m+mp})^{1+p}}{b(1+p)(1+mp)} \end{aligned}$$

Mathematica [A] time = 0.0025564, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m (a + bx^{mp+1}) \right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p, x]

[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int (x^m (a + bx^{mp+1}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*(a+b*x^(m*p+1)))^p, x)

[Out] int((x^m*(a+b*x^(m*p+1)))^p, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^{mp+1} + a)x^m)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^m*(a+b*x^(m*p+1)))^p, x, algorithm="maxima")

[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)

Fricas [A] time = 0.758676, size = 131, normalized size = 2.98

$$\frac{(bx^{mp+1} + ax)(bx^{mp+1}x^m + ax^m)^p}{(bmp^2 + (bm + b)p + b)x^{mp+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^{mp+1} + a)x^m)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

$$3.446 \quad \int x^n \left(x^m \left(a + bx^{1+n+mp} \right) \right)^p dx$$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1} \right)^{p+1}}{b(p+1)(mp+n+1)}$$

[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Rubi [A] time = 0.0753067, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1980, 2014}

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1} \right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]

[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Rule 1980

Int[(u_)^(p_)*((c_)*(x_))^(m_), x_Symbol] := Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2014

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\int x^n (x^m (a + bx^{1+n+mp}))^p dx = \int x^n (ax^m + bx^{1+m+n+mp})^p dx$$

$$= \frac{x^{-m(1+p)} (ax^m + bx^{1+m+n+mp})^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A] time = 0.041979, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} (x^m (a + bx^{mp+n+1}))^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]

[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Maple [F] time = 0.542, size = 0, normalized size = 0.

$$\int x^n (x^m (a + bx^{mp+n+1}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)

[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^{mp+n+1} + a)x^m)^p x^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")

[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)

Fricas [A] time = 0.767607, size = 174, normalized size = 3.78

$$\frac{(bx^{mp+n+1}x^n + ax^n)(bx^{mp+n+1}x^m + ax^m)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")

[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^{mp+n+1} + a)x^m)^p x^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")

[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)

$$3.447 \quad \int x^n (ax^m + bx^{1+m+n+mp})^p dx$$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+n+1})^{p+1}}{b(p+1)(mp+n+1)}$$

[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Rubi [A] time = 0.0453976, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2014}

$$\frac{x^{-m(p+1)} (ax^m + bx^{mp+m+n+1})^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]

[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^n (ax^m + bx^{1+m+n+mp})^p dx = \frac{x^{-m(1+p)} (ax^m + bx^{1+m+n+mp})^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A] time = 0.0044973, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m (a + bx^{mp+n+1}) \right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]

[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))

Maple [F] time = 0.542, size = 0, normalized size = 0.

$$\int x^n (ax^m + bx^{mp+m+n+1})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)

[Out] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^{mp+m+n+1} + ax^m)^p x^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")

[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)

Fricas [A] time = 0.996988, size = 190, normalized size = 4.13

$$\frac{(bxx^{mp+m+n+1}x^n + axx^m x^n)(bx^{mp+m+n+1} + ax^m)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+m+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^n*(a*x^m+b*x^(m*p+m+n+1)))^p,x, algorithm="fricas")`

[Out] $(b*x*x^{(m*p + m + n + 1)}*x^n + a*x*x^m*x^n)*(b*x^{(m*p + m + n + 1)} + a*x^m)^p / ((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^{(m*p + m + n + 1)})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**n*(a*x**m+b*x**(m*p+m+n+1)))**p,x`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^{mp+m+n+1} + ax^m)^p x^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^n*(a*x^m+b*x^(m*p+m+n+1)))^p,x, algorithm="giac")`

[Out] `integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)`

$$3.448 \quad \int \sqrt{x^{2(-1+n)} (a + bx^n)} dx$$

Optimal. Leaf size=44

$$\frac{2x^{3(1-n)} (ax^{-2(1-n)} + bx^{3n-2})^{3/2}}{3bn}$$

[Out] $(2*x^{3*(1-n)}*(a/x^{2*(1-n)} + b*x^{(-2+3*n)})^{(3/2)})/(3*b*n)$

Rubi [A] time = 0.0181042, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{2x^{3(1-n)} (ax^{-2(1-n)} + bx^{3n-2})^{3/2}}{3bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^(2*(-1+n))*(a + b*x^n)],x]

[Out] $(2*x^{3*(1-n)}*(a/x^{2*(1-n)} + b*x^{(-2+3*n)})^{(3/2)})/(3*b*n)$

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p+1)/(b*(n-j)*(p+1)*x^(n-1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{x^{2(-1+n)} (a + bx^n)} dx &= \int \sqrt{ax^{2(-1+n)} + bx^{2(-1+n)+n}} dx \\ &= \frac{2x^{3(1-n)} (ax^{-2(1-n)} + bx^{-2+3n})^{3/2}}{3bn} \end{aligned}$$

Mathematica [A] time = 0.0306752, size = 36, normalized size = 0.82

$$\frac{2x^{3-3n} (x^{2n-2} (a + bx^n))^{3/2}}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)], x]

[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)

Maple [A] time = 0.028, size = 40, normalized size = 0.9

$$\frac{(2a + 2bx^n)x}{3bx^n n} \sqrt{\frac{(x^n)^2 (a + bx^n)}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(-2+2*n)*(a+b*x^n))^(1/2), x)

[Out] 2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n

Maxima [A] time = 1.17945, size = 23, normalized size = 0.52

$$\frac{2(bx^n + a)^{3/2}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2), x, algorithm="maxima")

[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)

Fricas [A] time = 0.775232, size = 88, normalized size = 2.

$$\frac{2(bxx^n + ax)\sqrt{\frac{bx^{3n} + ax^{2n}}{x^2}}}{3bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")

[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(bx^n + a)x^{2n-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)

$$3.449 \quad \int \sqrt[3]{x^{3(-1+n)} (a + bx^n)} dx$$

Optimal. Leaf size=44

$$\frac{3x^{4(1-n)} (ax^{-3(1-n)} + bx^{4n-3})^{4/3}}{4bn}$$

[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)

Rubi [A] time = 0.0172021, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{3x^{4(1-n)} (ax^{-3(1-n)} + bx^{4n-3})^{4/3}}{4bn}$$

Antiderivative was successfully verified.

[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3), x]

[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{x^{3(-1+n)} (a + bx^n)} dx &= \int \sqrt[3]{ax^{3(-1+n)} + bx^{3(-1+n)+n}} dx \\ &= \frac{3x^{4(1-n)} (ax^{-3(1-n)} + bx^{-3+4n})^{4/3}}{4bn} \end{aligned}$$

Mathematica [A] time = 0.0298387, size = 36, normalized size = 0.82

$$\frac{3x^{4-4n} (x^{3n-3} (a + bx^n))^{4/3}}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3), x]

[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)

Maple [A] time = 0.019, size = 40, normalized size = 0.9

$$\frac{3x(a + bx^n)^3 \sqrt[3]{\frac{(x^n)^3 (a + bx^n)}{x^3}}}{4bx^n n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(-3+3*n))*(a+b*x^n))^(1/3), x)

[Out] 3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n

Maxima [A] time = 1.19037, size = 23, normalized size = 0.52

$$\frac{3(bx^n + a)^{4/3}}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-3+3*n))*(a+b*x^n))^(1/3), x, algorithm="maxima")

[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)

Fricas [A] time = 0.678657, size = 90, normalized size = 2.05

$$\frac{3(bxx^n + ax) \left(\frac{bx^{4n} + ax^{3n}}{x^3} \right)^{1/3}}{4bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")`

[Out] $3/4*(b*x*x^n + a*x)*((b*x^{4*n} + a*x^{3*n})/x^3)^{1/3}/(b*n*x^n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^n + a)x^{3n-3})^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")`

[Out] `integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)`

$$3.450 \quad \int \sqrt[4]{x^{4(-1+n)} (a + bx^n)} dx$$

Optimal. Leaf size=44

$$\frac{4x^{5(1-n)} (ax^{-4(1-n)} + bx^{5n-4})^{5/4}}{5bn}$$

[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)

Rubi [A] time = 0.0183645, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{4x^{5(1-n)} (ax^{-4(1-n)} + bx^{5n-4})^{5/4}}{5bn}$$

Antiderivative was successfully verified.

[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4), x]

[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\begin{aligned} \int \sqrt[4]{x^{4(-1+n)} (a + bx^n)} dx &= \int \sqrt[4]{ax^{4(-1+n)} + bx^{4(-1+n)+n}} dx \\ &= \frac{4x^{5(1-n)} (ax^{-4(1-n)} + bx^{-4+5n})^{5/4}}{5bn} \end{aligned}$$

Mathematica [A] time = 0.030217, size = 36, normalized size = 0.82

$$\frac{4x^{5-5n} (x^{4n-4} (a + bx^n))^{5/4}}{5bn}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4), x]

[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)

Maple [A] time = 0.018, size = 40, normalized size = 0.9

$$\frac{4x(a + bx^n)}{5bx^n} \sqrt[4]{\frac{(x^n)^4(a + bx^n)}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(-4+4*n)*(a+b*x^n))^(1/4), x)

[Out] 4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n

Maxima [A] time = 1.06904, size = 23, normalized size = 0.52

$$\frac{4(bx^n + a)^{5/4}}{5bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4), x, algorithm="maxima")

[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)

Fricas [A] time = 0.93438, size = 90, normalized size = 2.05

$$\frac{4(bxx^n + ax) \left(\frac{bx^{5n} + ax^{4n}}{x^4} \right)^{1/4}}{5bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(-4+4*n))*(a+b*x^n))^(1/4),x, algorithm="fricas")`

[Out] $4/5*(b*x*x^n + a*x)*((b*x^{5*n} + a*x^{4*n})/x^4)^{1/4}/(b*n*x^n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**(-4+4*n))*(a+b*x**n))**(1/4),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^n + a)x^{4n-4})^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(-4+4*n))*(a+b*x^n))^(1/4),x, algorithm="giac")`

[Out] `integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)`

$$3.451 \quad \int \left(x^{(-1+n)p} (a + bx^n) \right)^{\frac{1}{p}} dx$$

Optimal. Leaf size=57

$$\frac{px^{(1-n)(p+1)} \left(ax^{-(1-n)p} + bx^{n-(1-n)p} \right)^{\frac{1}{p}+1}}{bn(p+1)}$$

[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1)))/(b*n*(1 + p))

Rubi [A] time = 0.0277425, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{px^{(1-n)(p+1)} \left(ax^{-(1-n)p} + bx^{n-(1-n)p} \right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1), x]

[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1)))/(b*n*(1 + p))

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p+1)/(b*(n-j)*(p+1)*x^(n-1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int (x^{(-1+n)p} (a + bx^n))^{\frac{1}{p}} dx = \int (ax^{(-1+n)p} + bx^{n+(-1+n)p})^{\frac{1}{p}} dx$$

$$= \frac{px^{(1-n)(1+p)} (ax^{-(1-n)p} + bx^{n-(1-n)p})^{1+\frac{1}{p}}}{bn(1+p)}$$

Mathematica [A] time = 0.021195, size = 47, normalized size = 0.82

$$\frac{x^{1-n} (a + bx^n) (x^{(n-1)p} (a + bx^n))^{\frac{1}{p}}}{bn \left(\frac{1}{p} + 1\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1), x]

[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1)))

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \sqrt[p]{x^{(-1+n)p} (a + bx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^((-1+n)*p)*(a+b*x^n))^(1/p), x)

[Out] int((x^((-1+n)*p)*(a+b*x^n))^(1/p), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^n + a)x^{(n-1)p})^{\left(\frac{1}{p}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")

[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)

Fricas [A] time = 0.918842, size = 101, normalized size = 1.77

$$\frac{(bpx^n + apx) \left((bx^n + a)x^{(n-1)p} \right)^{\left(\frac{1}{p} \right)}}{(bnp + bn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")

[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((bx^n + a)x^{(n-1)p} \right)^{\left(\frac{1}{p} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")

[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)

$$3.452 \quad \int \left(x^{\frac{-1+n}{p}} (a + bx^n) \right)^p dx$$

Optimal. Leaf size=61

$$\frac{x^{\frac{(1-n)(p+1)}{p}} \left(ax^{\frac{-1-n}{p}} + bx^{n-\frac{1-n}{p}} \right)^{p+1}}{bn(p+1)}$$

[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(b*n*(1 + p))

Rubi [A] time = 0.0248845, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{x^{\frac{(1-n)(p+1)}{p}} \left(ax^{\frac{-1-n}{p}} + bx^{n-\frac{1-n}{p}} \right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]

[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(b*n*(1 + p))

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int \left(x^{\frac{-1+n}{p}} (a + bx^n) \right)^p dx = \int \left(bx^{n+\frac{-1+n}{p}} + ax^{\frac{-1+n}{p}} \right)^p dx$$

$$= \frac{x^{\frac{(1-n)(1+p)}{p}} \left(bx^{n-\frac{1-n}{p}} + ax^{\frac{-1-n}{p}} \right)^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.0250275, size = 45, normalized size = 0.74

$$\frac{x^{1-n} (a + bx^n) \left(x^{\frac{n-1}{p}} (a + bx^n) \right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]

[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int \left(x^{\frac{-1+n}{p}} (a + bx^n) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^((-1+n)/p)*(a+b*x^n))^p,x)

[Out] int((x^((-1+n)/p)*(a+b*x^n))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((bx^n + a)x^{\frac{n-1}{p}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")

[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)

Fricas [A] time = 0.907711, size = 107, normalized size = 1.75

$$\frac{(bx^n + ax)\left(bx^n x^{\frac{n-1}{p}} + ax^{\frac{n-1}{p}}\right)^p}{(bnp + bn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")

[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^n)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((bx^n + a)x^{\frac{n-1}{p}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")

[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)

$$3.453 \quad \int x^{-1+n-p(1+q)} (ax^n + bx^p)^q dx$$

Optimal. Leaf size=39

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))

Rubi [A] time = 0.0498496, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {2014}

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q, x]

[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :-> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q dx = \frac{x^{-p(1+q)} (ax^n + bx^p)^{1+q}}{a(n-p)(1+q)}$$

Mathematica [A] time = 0.0240465, size = 40, normalized size = 1.03

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(p-n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^{(-1 + n - p*(1 + q))}*(a*xⁿ + b*x^p)^q, x]

[Out] -((a*xⁿ + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^{(p*(1 + q))}))

Maple [F] time = 0.53, size = 0, normalized size = 0.

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^{(-1+n-p*(1+q))}*(a*xⁿ+b*x^p)^q, x)

[Out] int(x^{(-1+n-p*(1+q))}*(a*xⁿ+b*x^p)^q, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n-p*(1+q))}*(a*xⁿ+b*x^p)^q, x, algorithm="maxima")

[Out] integrate((a*xⁿ + b*x^p)^q*x^{(-p*(q + 1) + n - 1)}, x)

Fricas [A] time = 0.87875, size = 155, normalized size = 3.97

$$\frac{(axx^{-pq+n-p-1}x^n + bxx^{-pq+n-p-1}x^p)(ax^n + bx^p)^q}{(an - ap + (an - ap)q)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n-p*(1+q))}*(a*xⁿ+b*x^p)^q, x, algorithm="fricas")

[Out] $(a*x*x^{(-p*q + n - p - 1)} + b*x*x^{(-p*q + n - p - 1)*x^p})*(a*x^n + b*x^p)^q / ((a*n - a*p + (a*n - a*p)*q)*x^n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")`

[Out] `integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)`

$$3.454 \quad \int x^{-1-nq-p(1+q)} (x^n (a + bx^p))^q dx$$

Optimal. Leaf size=40

$$-\frac{x^{(q+1)(-n+p)} (ax^n + bx^{n+p})^{q+1}}{ap(q+1)}$$

[Out] $-\left((a*x^n + b*x^{(n+p)})^{(1+q)} / (a*p*(1+q)*x^{((n+p)*(1+q))})\right)$

Rubi [A] time = 0.0738037, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1980, 2014}

$$-\frac{x^{(q+1)(-n+p)} (ax^n + bx^{n+p})^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(-1 - n*q - p*(1 + q))}*(x^n*(a + b*x^p))^{q,x}]$

[Out] $-\left((a*x^n + b*x^{(n+p)})^{(1+q)} / (a*p*(1+q)*x^{((n+p)*(1+q))})\right)$

Rule 1980

$\text{Int}[(u_)^{(p_.)}*((c_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[(c*x)^m*\text{ExpandToSum}[u, x]^{p}, x] /;$ FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2014

$\text{Int}[((c_.)*(x_))^{(m_.)}*((a_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(c^{(j-1)}*(c*x)^{(m-j+1)}*(a*x^j + b*x^n)^{(p+1)}) / (a*(n-j)*(p+1)), x] /;$ FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\int x^{-1-nq-p(1+q)} (x^n (a + bx^p))^q dx = \int x^{-1-nq-p(1+q)} (ax^n + bx^{n+p})^q dx$$

$$= -\frac{x^{-(n+p)(1+q)} (ax^n + bx^{n+p})^{1+q}}{ap(1+q)}$$

Mathematica [A] time = 0.0397407, size = 38, normalized size = 0.95

$$-\frac{x^{(q+1)(-n+p)} (x^n (a + bx^p))^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]

[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))

Maple [F] time = 0.47, size = 0, normalized size = 0.

$$\int x^{-1-nq-p(1+q)} (x^n (a + bx^p))^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x)

[Out] int(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^p + a)x^n)^q x^{-p(q+1)-nq-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")

[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)

Fricas [A] time = 0.858222, size = 135, normalized size = 3.38

$$\frac{\left(bxx^{-(n+p)q-p-1}x^p + axx^{-(n+p)q-p-1}\right)(bx^nx^p + ax^n)^q}{apq + ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")

[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p + a*x^n)^q/(a*p*q + a*p)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int ((bx^p + a)x^n)^q x^{-p(q+1)-nq-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")

[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+'') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

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93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```



```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```